In this chapter, the bus admittance matrix of the node-voltage equation is formulated, and a *MATLAB* function named **ybus** is developed for the systematic formation of the bus admittance matrix. Next, two commonly used iterative techniques, namely Gauss-Seidel and Newton-Raphson methods for the solution of nonlinear algebraic equations, are discussed. These techniques are employed in the solution of power flow problems. Three programs **lfgauss**, **lfnewton**, and **decouple** are developed for the solution of power flow problems by Gauss-Seidel, Newton-Raphson, and the fast decoupled power flow, respectively.

6.2 BUS ADMITTANCE MATRIX

In order to obtain the node-voltage equations, consider the simple power system shown in Figure 6.1 where impedances are expressed in per unit on a common MVA base and for simplicity resistances are neglected. Since the nodal solution is based upon Kirchhoff's current law, impedances are converted to admittance, i.e.,

$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$

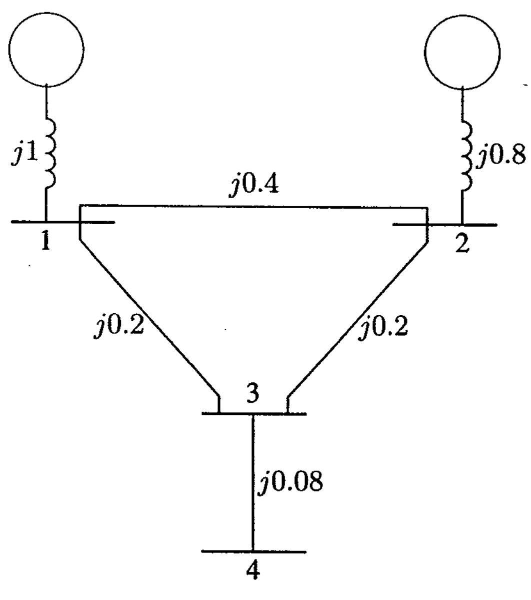

FIGURE 6.1
The impedance diagram of a simple system.

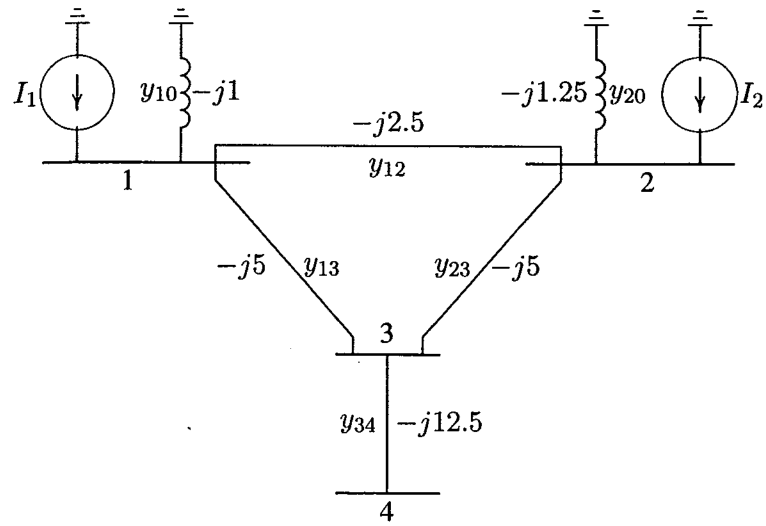

FIGURE 6.2
The admittance diagram for system of Figure 6.1.

The circuit has been redrawn in Figure 6.2 in terms of admittances and transformation to current sources. Node 0 (which is normally ground) is taken as reference. Applying KCL to the independent nodes 1 through 4 results in

$$\begin{aligned}
I_1 &= y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3) \\
I_2 &= y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3) \\
0 &= y_{23}(V_3 - V_2) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4) \\
0 &= y_{34}(V_4 - V_3)
\end{aligned}$$

Rearranging these equations yields

$$\begin{aligned}
I_1 &= (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3 \\
I_2 &= -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3 \\
0 &= -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4 \\
0 &= -y_{34}V_3 + y_{34}V_4
\end{aligned}$$

We introduce the following admittances

$$\begin{aligned}
Y_{11} &= y_{10} + y_{12} + y_{13} \\
Y_{22} &= y_{20} + y_{12} + y_{23}
\end{aligned}$$

$$Y_{33} = y_{13} + y_{23} + y_{34}$$
$$Y_{44} = y_{34}$$
$$Y_{12} = Y_{21} = -y_{12}$$
$$Y_{13} = Y_{31} = -y_{13}$$
$$Y_{23} = Y_{32} = -y_{23}$$
$$Y_{34} = Y_{43} = -y_{34}$$

The node equation reduces to

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$
$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$
$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$

In the above network, since there is no connection between bus 1 and 4, $Y_{14} = Y_{41} = 0$; similarly $Y_{24} = Y_{42} = 0$.

Extending the above relation to an n bus system, the node-voltage equation in matrix form is

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \tag{6.1}$$

or

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\,\mathbf{V}_{bus} \tag{6.2}$$

where $\mathbf{I}_{bus}$ is the vector of the injected bus currents (i.e., external current sources). The current is positive when flowing towards the bus, and it is negative if flowing away from the bus. $\mathbf{V}_{bus}$ is the vector of bus voltages measured from the reference node (i.e., node voltages). $\mathbf{Y}_{bus}$ is known as the *bus admittance matrix.* The diagonal element of each node is the sum of admittances connected to it. It is known as the *self-admittance* or *driving point admittance*, i.e.,

$$Y_{ii} = \sum_{j=0}^{n} y_{ij} \qquad j \neq i \tag{6.3}$$

The off-diagonal element is equal to the negative of the admittance between the nodes. It is known as the *mutual admittance* or *transfer admittance*, i.e.,

$$Y_{ij} = Y_{ji} = -y_{ij} \tag{6.4}$$