

السؤال الأول (12): لتكن $D = \{(x, y) \in \mathbb{R}^2 /, x^2 + y^2 - 9 \leq 0, (x-1)^2 + y^2 \geq 1\}$

احسب (x_G, y_G) مركز D حيث $x_G = \frac{\iint_D x dx dy}{\iint_D dx dy}$ و $y_G = \frac{\iint_D y dx dy}{\iint_D dx dy}$

1..... الاجابة: 1- من $(x-1)^2 + y^2 \geq 0 \Rightarrow r^2 \geq 2r \cos \theta \Rightarrow r \geq 2 \cos \theta$

1.5..... $0 \leq \theta \leq 2\pi$ و $2 \cos \theta \leq r \leq 3$ و منه $x^2 + y^2 - 9 \leq 0 \Rightarrow r^2 \leq 9 \Rightarrow r \leq 3$

1..... وبالتالي $D' = \{(r, \theta) \in \mathbb{R} / 2 \cos \theta \leq r \leq 3, 0 \leq \theta \leq 2\pi\}$

$$1.5... \iint_D dx dy = \iint_{D'} r dr d\theta = \int_0^{2\pi} \left(\int_{2 \cos \theta}^3 r dr \right) d\theta = \frac{1}{2} \int_0^{2\pi} (9 - 4 \cos^2 \theta) d\theta = 7\pi \quad -2$$

$$1.5... \iint_D x dx dy = \iint_{D'} r^2 \cos \theta dr d\theta = \int_0^{2\pi} \cos \theta d\theta \left(\int_{2 \cos \theta}^3 r^2 dr \right)$$

$$1.5... = \int_0^{2\pi} 9 \cos \theta d\theta - \frac{8}{3} \int_0^{2\pi} \cos^4 \theta d\theta = -\frac{2}{3} \int_0^{2\pi} (1 + \cos 2\theta)^2 d\theta = -2\pi$$

$$1.5... \iint_D y dx dy = \iint_{D'} r^2 \sin \theta dr d\theta = \int_0^{2\pi} \sin \theta d\theta \left(\int_{2 \cos \theta}^3 r^2 dr \right)$$

$$1.5... \int_0^{2\pi} \sin \theta \left(9 - \frac{8}{3} \cos^3 \theta \right) d\theta = \left[9 \cos \theta + \frac{2}{3} \cos^4 \theta \right]_0^{2\pi} = 0$$

1..... ومنه $(x_G, y_G) = (-2/7, 0)$

السؤال الثاني (08): لتكن $V = \{(x, y, z) \in \mathbb{R}^3 /, x^2 + z \leq 1, y^2 + z \leq 1\}$

ثم احسب التكامل الثلاثي التالي: $\iiint_V z dx dy dz$

2..... الاجابة: $V = \{(x, y, z) \in \mathbb{R}^3 /, 0 \leq x \leq \sqrt{1-z}, 0 \leq y \leq \sqrt{1-z}, 0 \leq z \leq 1\}$

$$2... \iiint_V z dx dy dz = \int_0^1 \left(\int_0^{\sqrt{1-z}} \left(\int_0^{\sqrt{1-z}} z dx \right) dy \right) dz$$

$$4... = \int_0^1 \left(\int_0^{\sqrt{1-z}} z \sqrt{1-z} dy \right) dz = \int_0^1 z(1-z) dz = 1/6$$