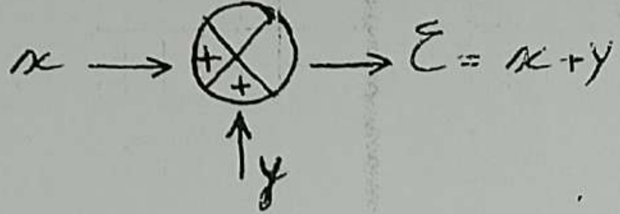
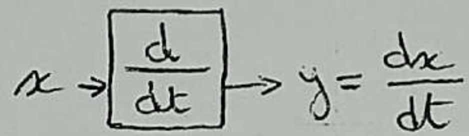
Sommateur :

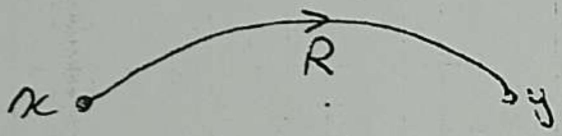



Dérivation :

$$x' = \frac{dx}{dt} = \dot{x}$$




Le graphe de fluence :




Exemple :

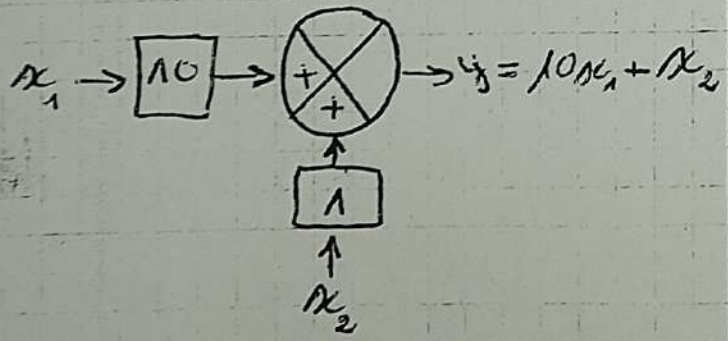
Représenter le schéma fonctionnelle des relations suivantes

1) $y = 10x_1 + x_2$ 2) $y = 5x_1 - 2x_2$

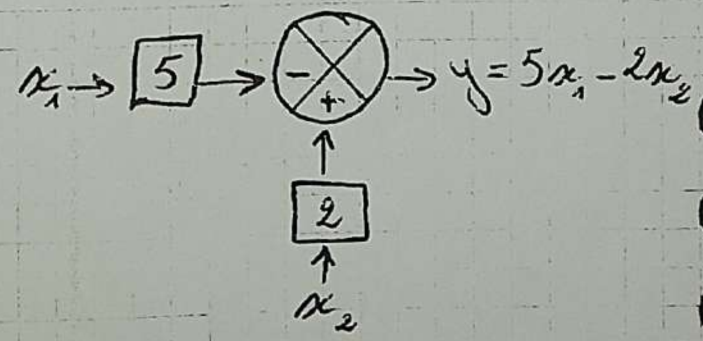
3) $y = 2x'_1 + x'_2$

- Solution :

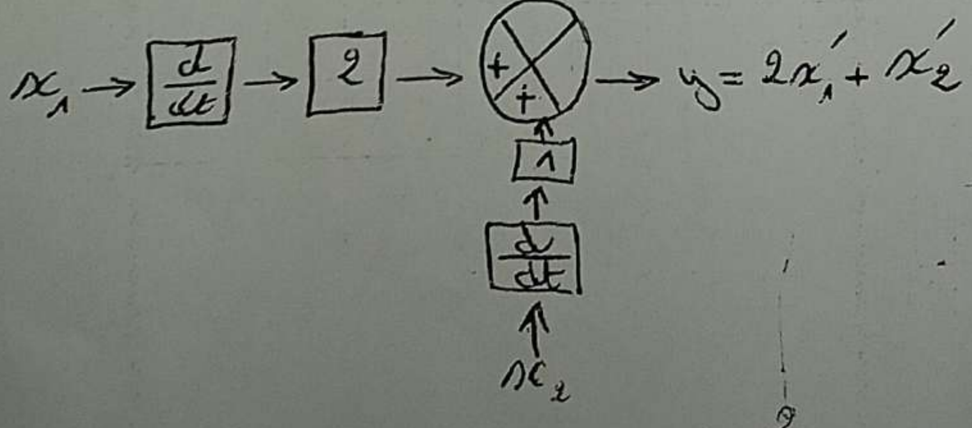
1) $y = 10x_1 + x_2$




2) $y = 5x_1 - 2x_2$




3) $y = 2x'_1 + x'_2$



... représenter le schéma fonctionnel des relations suivantes :-

① $y = 3x_1 - 2x_2 + x_3 - \frac{1}{2}x_4$ — ③ $y = \sqrt{2}\int x_1\,dt - \frac{dx_2}{dt}$

② $y = \frac{3}{2}\frac{dx_1}{dt} - \frac{dx_2}{dt} + x_3$ — ④ $y = \frac{1}{2}\frac{d^2x_1}{dt^2} + 2\frac{dx_2}{dt} - \int x_3$

⑤ $y = \frac{1}{2}\int(x_1+x_2)\,dt - 2\frac{d}{dt}(x_3 - x_4)$ — ⑥ $y = \frac{1}{2}\int\left(x_1 - \frac{1}{2}x_2 + 3x_3\right)dt + \frac{d}{dt}(x_4 - x_5)$

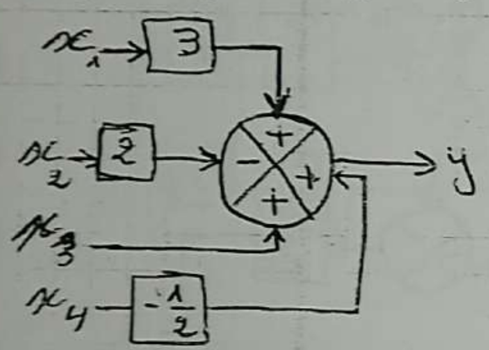
⑦ $y = 5\frac{d^3x_1}{dt^3} - 2\frac{d^2x_2}{dt^2} + \frac{dx_3}{dt}$

⑧ $y = \frac{d}{dt}\left(\frac{2}{3}x_1 - x_2\right) - x_3$ — ⑨ $y = \int x_1\,dt + \int \frac{1}{3}x_2\,dt + \frac{dx_3}{dt}$

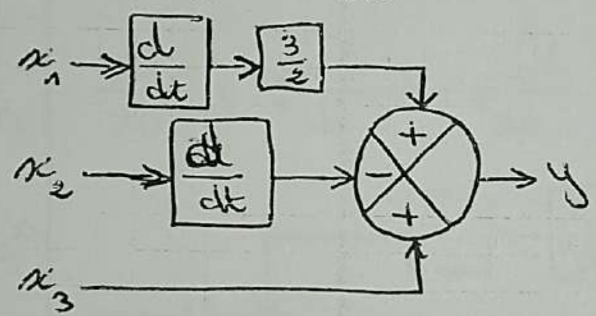
⑩ $y = \frac{1}{\sqrt{2}}x_1' - \frac{3}{2}x_2'' + 4x_3'''$

* La solution :-

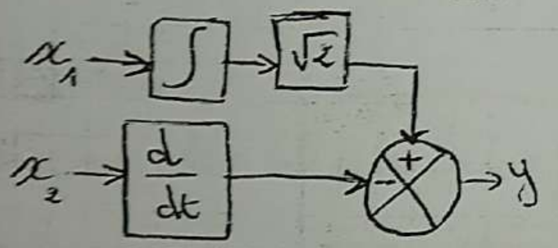
① $y = 3x_1 - 2x_2 + x_3 - \frac{1}{2}x_4$



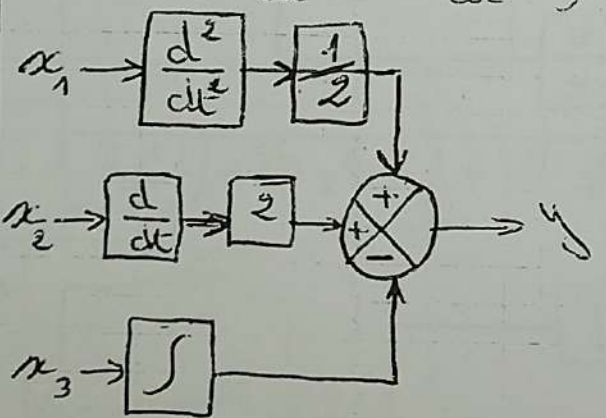

② $y = \frac{3}{2}\frac{dx_1}{dt} - \frac{dx_2}{dt} + x_3$




③ $y = \sqrt{2}\int x_1\,dt - \frac{dx_2}{dt}$

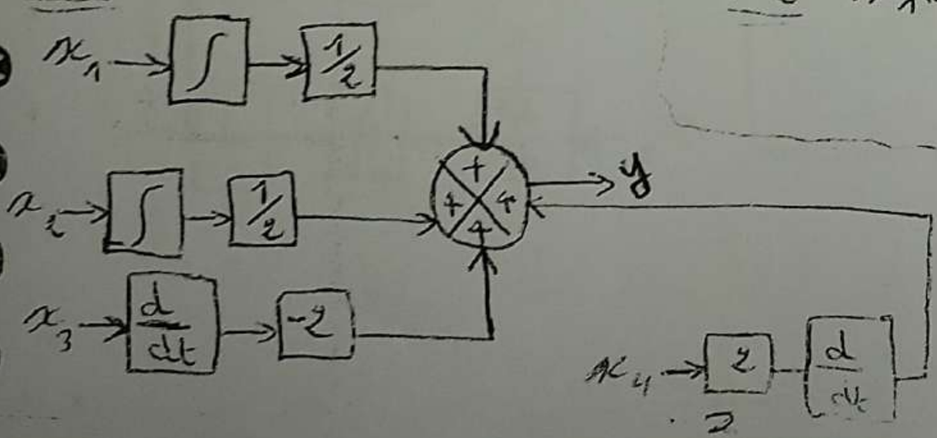



④ $y = \frac{1}{2}\frac{d^2x_1}{dt^2} + 2\frac{dx_2}{dt} - \int x_3\,dt$

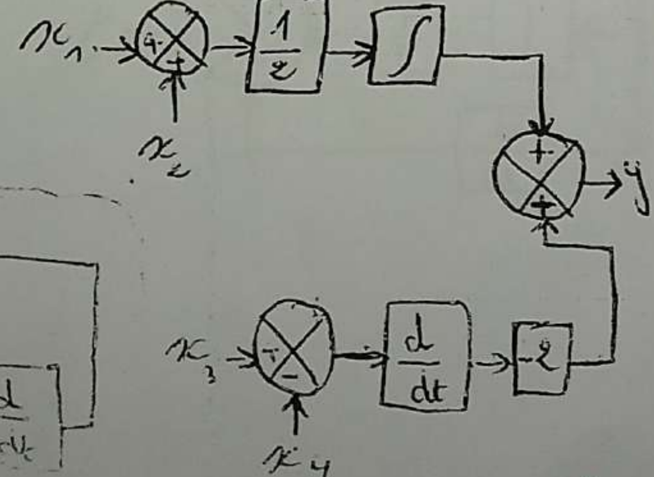



⑤ $y = \frac{1}{2}\int(x_1+x_2)\,dt - 2\frac{d}{dt}(x_3 - x_4)$

$M_1$

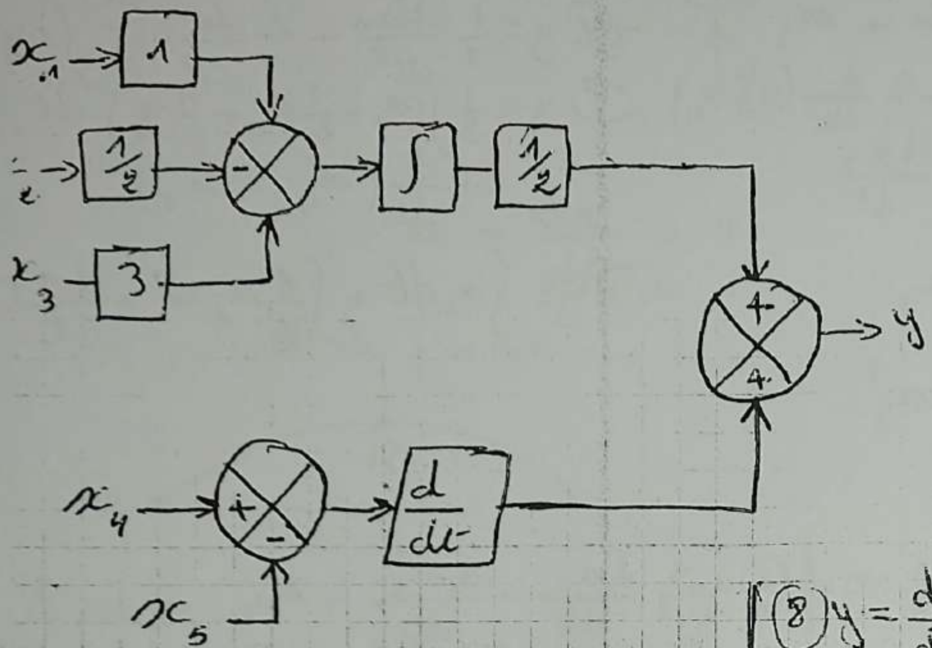



$M_2$

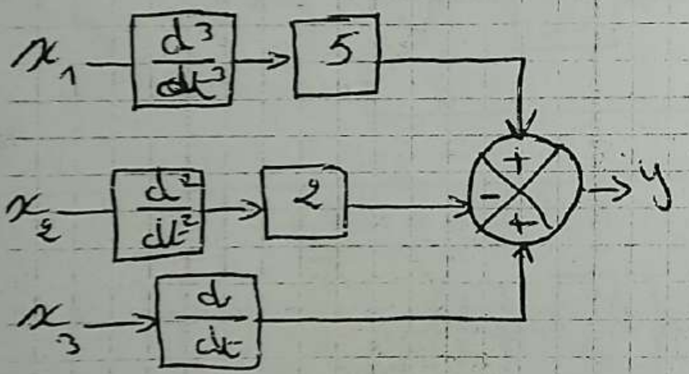



2

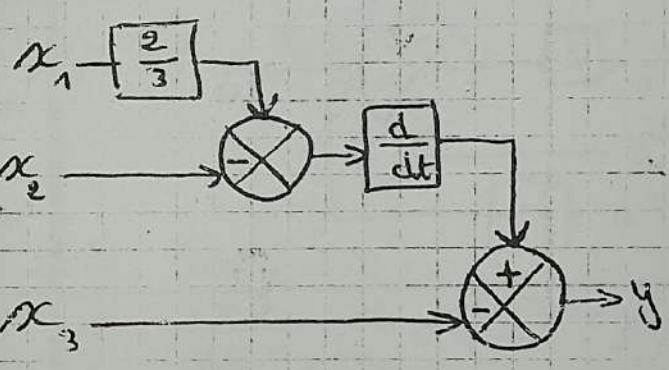
$y = \frac{1}{2} \int (x_1 - \frac{1}{2} x_2 + 3x_3)\, dt + \frac{d}{dt}(x_4 - x_5)$



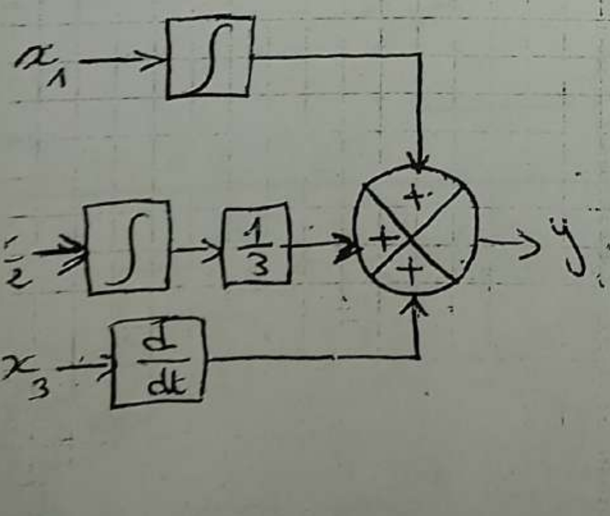
7) $y = 5 \frac{d^3 x_1}{dt^3} - 2 \frac{d^2 x_2}{dt^2} + \frac{dx_3}{dt}$



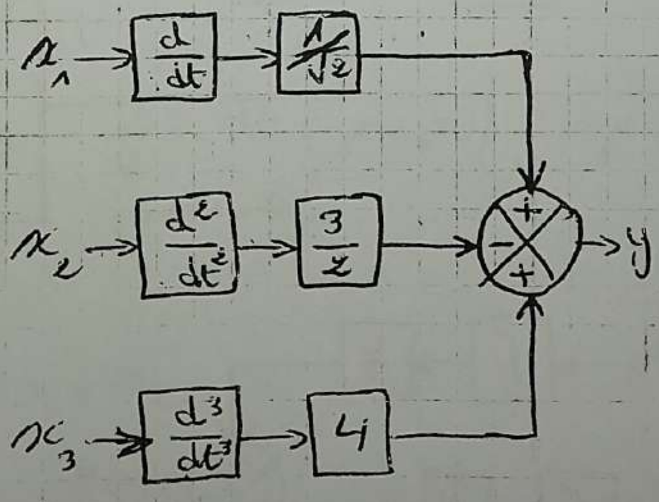
8) $y = \frac{d}{dt}\left(\frac{2}{3} x_1 - x_2\right) - x_3$



9) $y = \int x_1\, dt + \int \frac{1}{3} x_2\, dt + \frac{dx_3}{dt}$



10) $y =$



Ex 02 : $u(t)=t$ , $u(t)=t^2$ , $u(t)=\frac{A}{2}t+Bt^2$ .

Ex 01 : $f(t)=t^n \Rightarrow F(P)=\mathcal{L}[f(t)]=\int_0^{+\infty} f(t)e^{-Pt}\,dt$

$\Rightarrow F(P)=\int_0^{\infty} \underbrace{t^n}_{u}\underbrace{e^{-Pt}dt}_{dv}$ Donc $\begin{cases} U=t^n \\ du=nt^{n-1}\,dt \end{cases}$ et $\begin{cases} dv=e^{-Pt}\,dt \\ v=-\frac{1}{P}e^{-Pt} \end{cases}$

$\int_0^{+\infty} u\,dv=[u.v]_0^{+\infty}-\int_0^{+\infty} du.v$ où $[U.v]_0^{+\infty}\to 0$ (Toujours)

$F(P)=\left[t^n\left(-\frac{1}{P}\right)e^{-Pt}\right]_0^{\infty}+\int_0^{\infty}\frac{n}{P}t^{n-1}e^{-Pt}\,dt=\frac{n}{P}\int_0^{\infty}t^{n-1}e^{-Pt}\,dt$

Donc $F(P)=\mathcal{L}(t^n)=\frac{n!}{P^{n+1}}$ $\quad F(P)=\mathcal{L}(t^n e^{-at})=\frac{n!}{(P+a)^{n+1}}$

Ex 2 : $f(t)=t \Rightarrow F(P)=\mathcal{L}[f(t)]=\int_0^{\infty} f(t)e^{-Pt}\,dt = \int_0^{\infty} t e^{-Pt}\,dt$

où $\begin{cases} u=t \\ du=dt \end{cases}$ et $\begin{cases} dV=e^{-Pt}\,dt \\ V=-\frac{1}{P}e^{-Pt} \end{cases}$ Donc

$\int_0^{\infty} U\,dV=[UV]_0^{\infty}-\int_0^{\infty} du.V \Rightarrow \int_0^{\infty} t e^{-Pt}\,dt=\left[t\left(-\frac{1}{P}\right)e^{-Pt}\right]_0^{\infty}$

$-\int_0^{\infty} 1\left(-\frac{1}{P}\right)e^{-Pt}\,dt=\frac{1}{P}\int_0^{\infty} e^{-Pt}\,dt=\frac{1}{P}\left[-\frac{1}{P}\left(e^{-Pt}\right)\right]_0^{\infty}=-\frac{1}{P}\left(\frac{1}{P}\right)(-1)$

$\Rightarrow F(P)=\mathcal{L}[f(t)]=\mathcal{L}[t]=\frac{1}{P^2}$

Ex 3 : $f(t)=t^2$

$F(P)=\mathcal{L}[f(t)]=\int_0^{\infty} t^2 e^{-Pt}\,dt \Rightarrow \begin{cases} u=t^2 \\ du=2t\,dt \end{cases}$ et $\begin{cases} dV=e^{-Pt}\,dt \\ V=-\frac{1}{P}e^{-Pt} \end{cases}$

$\Rightarrow \int_0^{\infty} U.dV=\int_0^{\infty} t^2 e^{-Pt}\,dt=\left[t^2\left(-\frac{1}{P}\right)e^{-Pt}\right]_0^{\infty}-\int_0^{\infty} 2t\left(-\frac{1}{P}\right)e^{-Pt}\,dt$

$=\frac{2}{P}\left[te^{-Pt}\right]_0^{\infty}=\frac{2}{P}\times\left(\frac{1}{P^2}\right)=\frac{2}{P^3} \Rightarrow F(P)=\mathcal{L}(t^2)=\frac{2}{P^3}$

Donc $\mathcal{L}[x(t)] = X(P) = P \cdot Y(P) - 1 - \ldots + 3Y(P) - 5 + \ldots$

$$\Rightarrow X(P) = Y(P)\left[P^2 + 3P + 1\right] - P - 5$$

Exc 3

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} + 6y(t) = \frac{d^2x(t)}{dt^2} - x(t)$$

où $\frac{d^3y(t)}{dt^3} = P^3 Y(P) - P^2 y(0) - Py'(0) - y''(0)$ et les conditions initiales $\begin{cases} y(0)=0, \ y'(0)=0, \ y''(0)=1 \end{cases}$

La solution :

$$\frac{d^3y(t)}{dt^3} = P^3Y(P) - P^2y(0) - Py'(0) - y''(0) = P^3Y(P) - P.0 - P.0 - 1 = P^3Y(P) - 1$$

$$\frac{d^2y(t)}{dt^2} = P^2Y(P) - Py(0) - y'(0) = P^2Y(P) - P.0 - 0 = P^2Y(P)$$

$$\frac{dy(t)}{dt^2} = PY(P) - y(0) = PY(P) - 0 = PY(P) \ / \ y(t) = Y(P)$$

$$\frac{d^2x(t)}{dt^2} = P^2X(P) - P.x(0) - x'(0) = P^2X(P) \ ; \ x(t) = X(P)$$ Donc

$$P^3Y(P) - 1 + 3P^2Y(P) - PY(P) + 6Y(P) = P^2X(P) - X(P)$$

$$\Leftrightarrow Y(P)\left[P^3 + 3P^2 + (-P+6)\right] - 1 = X(P)\left[P^2 - 1\right]$$

Exc4 : Determiner de T.P pour :

$$2\frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} + x(t) = \frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t)$$

La solution

$$\frac{d^2x(t)}{dt^2} = P^2X(P) - Px(0) - x'(0) = P^2X(P) \ / \ x(t) = X(P)$$

$$\frac{dx(t)}{dt} = PX(P) - x(0) = PX(P)$$

$$\frac{d^2y(t)}{dt^2} = P^2Y(P) - Py(0) - y'(0) = P^2Y(P) \ / \ y(t) = Y(P)$$

$$\frac{dy(t)}{dt} = PY(P) - y(0) = PY(P)$$

Donc : $2P^2X(P) + PX(P) + X(P) = P^2Y(P) + 2PY(P) + 3Y(P)$

$$\Leftrightarrow X(P) = \left[2P^2 + P + 1\right] = Y(P)\left[P^2 + 2P + 3\right]$$

Exo 5 :

Déterminer la T.P de l'équation diff :

$\ddot{y}(t) + 4\dot{y}(t) + 20y(t) = 4$ où $\begin{cases} y(0) = -2 \\ \dot{y}(0) = 0 \end{cases}$

Solution

$\mathcal{L}[\ddot{y}(t)] = P^2Y(P) - Py(0) - y'(0) = P^2Y(P) - P(-2) - 0 = P^2Y(P) + 2P$

$\mathcal{L}[\dot{y}(t)] = PY(P) - y(0) - y'(0) = PY(P) - (-2) - 0 = PY(P) + 2$

$\mathcal{L}[y(t)] = Y(P)$ et $4 \Rightarrow \mathcal{L}[4] = 4/P$

Donc $P^2Y(P) + 2P + 4(PY(P) + 2) + 20(Y(P)) = \frac{4}{P}$

$\Rightarrow P^2Y(P) + 2P + 4PY(P) + 8 + 20Y(P) = \frac{4}{P}$

$\Rightarrow Y(P)[P^2 + 4P + 20] + 2P + 8 = \frac{4}{P}$

$\Rightarrow Y(P)[P^2 + 4P + 20] = \frac{4}{P} - 2P - 8$

$\Leftrightarrow Y(P)[P^2 + 4P + 20] = \frac{4 - 2P^2 - 8P}{P} \Leftrightarrow Y(P) = \frac{-2P^2 - 8P + 4}{P[P^2 + 4P + 20]}$

Exo 6 :

$\frac{d^3y(t)}{dt^3} + 5\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} = 0$ où $\begin{cases} y(0) = 3 \\ \dot{y}(0) = -2 \\ \ddot{y}(0) = 7 \end{cases}$

Solution

$\mathcal{L}\left[\frac{d^3y(t)}{dt^3}\right] = P^3Y(P) - P^2y(0) - P\dot{y}(0) - \ddot{y}(0) = P^3Y(P) - 3P^2 + 2P - 7$

$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = P^2Y(P) - Py(0) - \dot{y}(0) = P^2Y(P) - 3P + 2$

$\mathcal{L}\left[\frac{dy(t)}{dt}\right] = PY(P) - y(0) = PY(P) - 3$ Donc

$P^3Y(P) - 3P^2 + 2P - 7 + 5P^2Y(P) - 15P + 10 + 6PY(P) - 18 = 0$

$\Rightarrow Y(P)[P^3 + 5P^2 + 6P] - 3P^2 - 13P - 15 = 0$

$\Rightarrow Y(P) = \frac{3P^2 + 13P + 5}{P^3 + 5P^2 + 6}$

Ex 07 :

$$\frac{d^2y(t)}{dt^2} + y(t) = \frac{3}{2} \sin(2t) \quad \text{où} \quad \begin{cases} y(0) = 1 \\ \dot{y}(0) = 2 \end{cases}$$

Solution

$$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = P^2Y(P) - Py(0) - \dot{y}(0) = P^2Y(P) - P - 2$$

$$\mathcal{L}[y(t)] = Y(P) \quad \text{et} \quad \mathcal{L}[\sin(2t)] = \frac{(2)}{P^2+(2)^2} = \frac{2}{P^2+4}$$

$$P^2Y(P) - P - 2 + Y(P) = \frac{3}{2}\left[\frac{2}{P^2+4}\right]$$

$$\Leftrightarrow Y(P)\left[P^2+1\right] - P - 2 = \frac{3}{P^2+4} \Leftrightarrow Y(P)\left[P^2+1\right] = \frac{3}{P^2+4} + P + 2$$

$$\Leftrightarrow Y(P)\left[P^2+1\right] = \frac{3 + P(P^2+4) + 2(P^2+4)}{(P^2+4)}$$

$$\Leftrightarrow \boxed{Y(P) = \frac{P^3+2P^2+4P+11}{(P^2+1)(P^2+4)}}$$

Ex 08 :

même Exercice 07 mais les conditions initiales sont $\begin{cases} y(0) = -3 \\ \dot{y}(0) = -1 \end{cases}$

$$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = P^2Y(P) - Py(0) - \dot{y}(0) = P^2Y(P) + 3P + 1$$

$$\mathcal{L}[y(t)] = Y(P), \quad \mathcal{L}\left[\frac{3}{2}\sin(2t)\right] = \frac{3}{P^2+4} \quad \text{Donc}$$

$$P^2Y(P) + 3P + 1 + Y(P) = \frac{3}{P^2+4} \Leftrightarrow Y(P)\left[P^2+1\right] = -3P - 1 + \frac{3}{P^2+4}$$

$$\text{où} \; -3P - 1 + \frac{3}{P^2+4} = \frac{-3P^3 - 12P - P^2 - 4 + 3}{P^2+4} = \frac{-3P^3 - 12P - P^2 - 1}{P^2+4}$$

$$\Rightarrow Y(P)(P^2+1) = \frac{-3P^3 - P^2 - 12P - 1}{P^2+4} \Leftrightarrow -3P^3 - P^2 - 12P - 1 = Y(P)(P^2+1)(P^2+4)$$

$$\Leftrightarrow \boxed{Y(P) = \frac{-3P^3 - P^2 - 12P - 1}{(P^2+1)(P^2+4)}}$$