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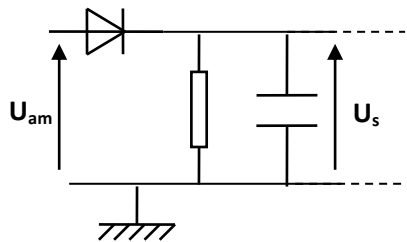
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Communications Analogiques

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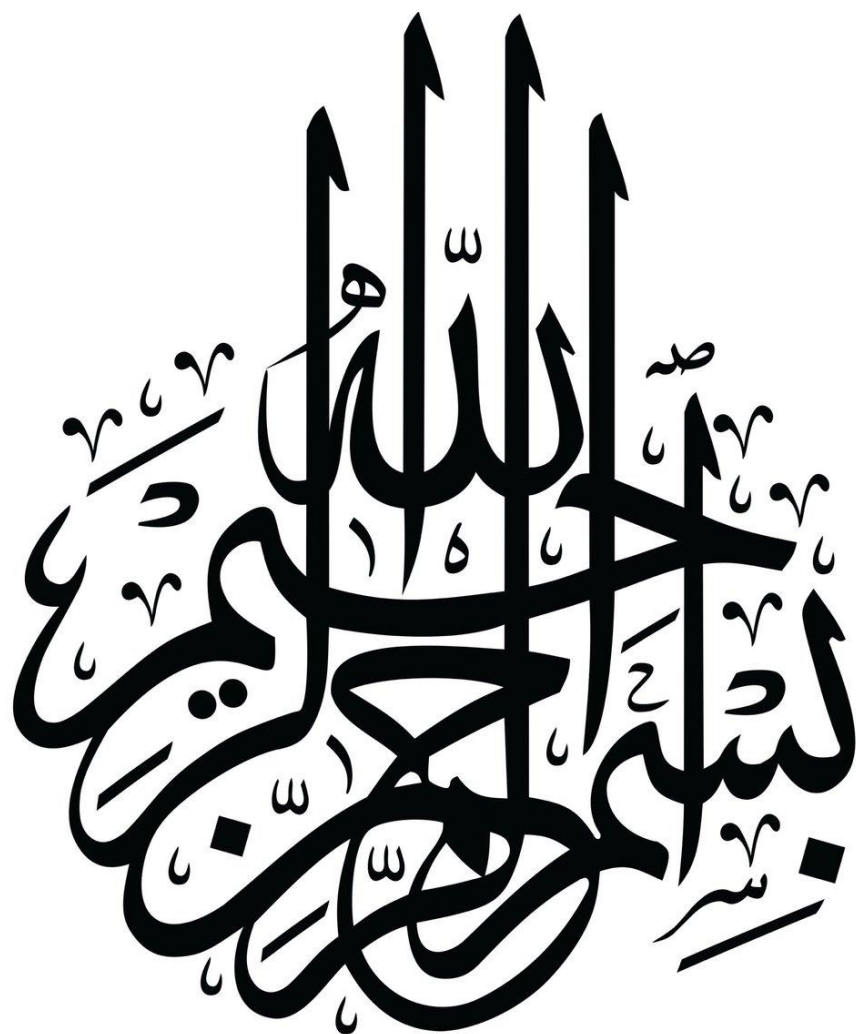


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حكمتي لطلبتي الأعزاء-

«لو تقتل منافسك الكفاء, فقد حطمت أحد محركات عزيمتك»

-

-My wisdom to my dear students-

*“If you kill your competent competitor,
you broke one of your will engines”*

CH. I

Radio Frequency Basics

I

CHAPTER

Radio Frequency Basics

1. Analog transmission chain

1.1 Information

Information is the heart of the message to be transmitted using: telephony, radar, internet...etc. We are surrounded by many technological tools that transmit messages carrying information. As an example, machines associated with computer tools are subject to a flow of data from sensors, where communication performance and security must be ensured. Information are thus everywhere in our daily lives. Beyond the information processing that is made to message by both the human and the machine, it is interesting to transmit the information in all fidelity to the recipient. In other words, it is imperative that the message received is the exact replica of the message sent!

1.2 Transmission Chain

The information transmission chain with its simplest functional structure, consists of:

1. Transmitter; 2. Transmission channel; 3. Receiver



1.2.1 Transducer at the transmitter

The transducer at the transmitter allows converting the original (Physical) signal such as (voice, image, etc.) into an electrical signal which is useful for the transmitter.

Transducer	Original Signal
Microphone	Human voice
Keyboard	Pressed touch

1.2.2 Transmitter

The transmitter has the function of adapting the signal from the transducer to transmit it over the transmission channel. It can simultaneously carry out several functions:

- Modulating;
- Amplifying

1.2.3 Transmission channel

The transmission channel is the medium that allows conducting and receiving the information transmitted by the transmitter.

Many medium types are used for the channel:

- Medium with physical guide (cables, fibers, ...); ;
- Medium without physical guide (radio waves, light waves).

These different mediums are chosen taking into account:

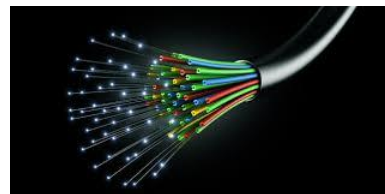
- The data rate of information to be transmitted;
- The signal characteristics (bandwidth, coding ...);
- The signal range which is the distance between the transmitter and the receiver;
- The possibility of implementation.

Regarding the quality of the mediums, we can distinguish:

- Twisted pair electrical cables that are the least reliable, followed by coaxial cables;
- Optical fiber currently offers the best compromise between reliability and performance.



Twisted pair cable



Cable of Optical fiber

1.2.4 The Receiver

The role of the receiver is to receive the transmitted signal as well as to make it compatible with the transducer (example: loudspeaker) used for reception. The actions performed by the receiver are then as follows:

- Filtering the received signal (eliminate the unnecessary part of the received signal so as to keep only the information);
- Demodulating;
- Amplify the signal to make it usable by the output transducer.

1.2.5 Transducer at the receiver

Its role is to provide useful and exploitable information for the recipient.

Transducer	Information
Speaker	Voice
Screen	Image
command signal	Actuator (valve, pump)

The term transducer should not be confused with that of a decoder which aims to decrypt an encrypted signal into "clear" information.

2 Why Modulate?

Simply, it is a mechanism of gathering two signals together.

The question of modulation arises when:

- we want to pass several information simultaneously in the same transmission channel;
- we want to transmit information at long ranges (distances);
- we want to reduce the noise to which the information is subjected during the transmission.

The modulation consists of adapting the information to be transmitted to a communication channel, but this is not an obligation.

2.1 Base band transmission

Baseband transmission consists of transmitting the signal directly to the medium without frequency transposition.

This can be done for example for digital signals using a so-called baseband modem. This one uses directly physical supports of metallic types (twisted pairs or coaxial cable) or optical fiber.

In most cases, the higher frequency harmonics of some signals cannot be transmitted without an unacceptable alteration of the signal. The harmonics of a signal transmitted on a line are variously attenuated, according to their frequency and the bandwidth of the line. If all the useful harmonics of the signal to be transmitted are in the bandwidth of the line that we wish use, this signal can be applied directly to the input of the line. Hence, it will be transmitted without significant attenuation to the other end.

The major difficulties of transmission process are:

- Sensitivity to parasites (noise);
- High cost transmission especially over optical fiber
- Implementation difficulties
- No way to share a channel directly for multiple sources using the same frequency;
- It is impossible to transmit low frequency signals directly to the air (example: sound with frequencies from 20Hz to 20 kHz or wavelengths of 15 to 15000 km!).

2.2 Transposed band transmission (modulation)

Transposed band transmission, also known as modulation, consists of transmitting the signal of information by making it undergo a prior modification of its spectrum.

The modulation uses two signals:

- A low-frequency modulating signal that contains the information and which may be analog (voice) or digital (computer data);
- A high frequency carrier signal whose one of the parameters (amplitude, frequency, phase) varies according to the evolutions of the modulating signal.

It is not a question here of describing the principles of implementation of these modulations but to give some brief information to understand the principle of modulation. There are different modes of modulation that may consist of carrying out:

- Either by a more or less direct transposition of the spectrum of the message towards the high frequencies (modulation of amplitude, of frequency);
- Either by a radical modification of the signal itself using digital means, in particular sampling (pulse modulation);

The use of modulation allows to

- reduce the wavelength of transmitted signals (example: a frequency of 100 MHz corresponds to a wavelength of 3.00 meters) ;
- reduce the noise during transmission;
- transmit signals over the air for long distance (example: radio) ;
- transmit simultaneously several information on the same physical medium

Its implementation is however:

- more complex: risk of increasing the signal degradation due to equipment;
- more bandwidth consumed than for the original message (transposition to high frequencies).

3 Allocation of radio frequency bands

Frequency allocation is a set of mechanisms that define how radio frequencies are distributed between different users. Thus, the main challenge is to avoid interferences between the transmitters. The International Telecommunication Union (ITU) manages the allocation of radio frequency bands for wireless communication:

- ULF -- 300 Hz à 3 kHz
- VLF -- 3 kHz à 30 kHz
- LF -- 30 kHz à 300 kHz
- MF -- 300 kHz à 3 MHz
- HF -- 3 MHz à 30 MHz
- VHF -- 30 MHz à 300 MHz
- UHF -- 300 MHz à 3 GHz

Some frequency allocations are listed below:

Source	Frequency (MHz)	Typical radiated power (kW)
Longwave BCB (EU)	0.150–0.285	320
AM BCB (EU & J)	0.525–1.605	500
AM BCB (US)	0.530–1.710	50
Amateur	1.8–29.7	0.16 (mobile)
Citizens band	26.9–27.4	0.004
Amateur	28–30	0.2 (mobile)

Land mobile	29–54	0.1
Amateur	50–54	0.2 (mobile)
TV low VHF	54–88	100
Land mobile (EU)	65–85	0.1
FM BCB (J)	76–90	44
FM BCB (US & EU)	88–108	105
Aircraft	108–136	1
Land mobile (EU)	120–160	0.1
Land mobile	132–174	18–100
Land mobile (J)	142–170	
Amateur	144–148	0.2 (mobile)
TV high VHF	174–216	316
Land mobile	216–222	0.2
Amateur	222–225	0.1 (mobile)
Land mobile (J)	335–384	
Amateur	430–450	0.1 (mobile)
TV UHF	470–806	5000
Land mobile	806–947	0.035
Cellular AMPS	806–947	0.003
Amateur	1200–1600	

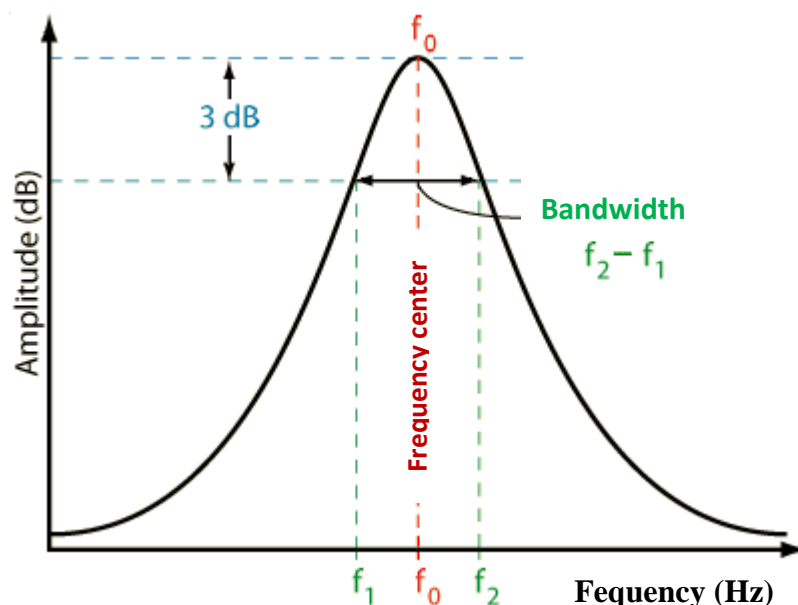
Land mobile GPS		
Cellular PCS	1700–2000	0.003
ISM Bluetooth Wi-Fi	2400–2500	0.0000025

* BCB is an abbreviation for broadcast band, for commercial radio news and music broadcasts.

* International Telecommunication Union or ITU: is the United Nations agency for development specializing in information and communication technologies, located in Geneva (Switzerland). It has 193 member states and 700 sector members and associates. Since 1992, ITU-R replaced the International Radio Consultative Committee or CCIR that has been established in 1927.

4 Bandwidth for a system

A bandwidth of a signal is the range of frequencies in which the significant part of its energy is distributed. But the bandwidth of a system or a physical medium (channel) is the range of frequencies in which the signal's frequencies can pass through.



For a given transmission channel, the capacity of a channel is defined as the maximum theoretical data rate that this channel can support. The channel Capacity C is evaluated using Shannon's law:

$$C = B \log_2(1 + P_S/P_N)$$

B is the bandwidth of the channel, S is the signal power and N is the noise power.

The noise of an electronic device is characterized by the signal / noise ratio in decibel:

$$\text{SNR}_{\text{db}} = 10 \log_{10}(P_S/P_N)$$

As conclusion, the data rate of a noisy channel is limited by the bandwidth of the transmission channel and its signal-to-noise ratio.

5 Behavior of radio waves

There are some simple rules that can be very useful for designing radio waves:

- The longer the wavelength, the farther it will go.
- The longer the wavelength, the more the wave will pass through and around things.
- At shorter wavelength, more data can be transported.

6 Calculate with decibels (dB):

In generally, the decibel (dB) is a logarithmic unit used to express the ratio between two physical quantities, usually amounts of acoustic or electric power. The ratio may be power, sound pressure, voltage or intensity or several other things.

The decibel (dB) is a sub-multiple of the bel, corresponding to one tenth of bel (1/10 of bel). Named on the honor of the inventor Alexander Graham Bell, the Bel is not the most common unit. The decibel (dB) is more commonly used.

The decibel is only a practical method to simplify calculations, and is a unit without dimension, which defines a ratio between two power measurements. It is defined by:

$$X(\text{dB}) = 10 * \log (P_1 / P_0)$$

Where P_1 and P_0 two values to be compared. Generally, in our case, they represent a ratio between two powers.

Examples :

- If $P_1 = 100 \times P_0$, the ratio between the two powers is $100 = 10^2$; which corresponds to 20 dB ;
- If $P_1 = 2 \times P_0$, the ratio is $2 \approx 10^{0,3}$, which corresponds to 3 dB : So, double power is 3 dB.
- If $P_1 = (1/2) \times P_0$, the ratio is $1/2 \approx 10^{-0,3}$, which corresponds to -3 dB : So, half power is -3 dB.

Why are decibels so manageable? Because when we use the trick of applying the logarithm (log), the matter becomes much easier: where instead of raising to the power of n, we simply multiply by n. Also, instead of multiplying values, we will add them. Here are some commonly used values that are important to remember:

0dB = equal power

3 dB = double power

-3dB = half power

10 dB = ten times the power

-10 dB = one-tenth of power

In general way, to obtain the decibel of a power value that does not represent a ratio between two quantities, the decibel equals to ten times the logarithm of the ratio of this power value with respect to a reference power equals 1Watt or 1mWatt, where this depends to the origin unit of a power value.

Reference power:

- For a power value presented in **dBW** → this means that the reference power was 1 W.
- For a power value presented in **dBm** → this means that the reference power was 1 mW.

Example:

- $P=100W \rightarrow X(dB)=10*\log_{10}(\frac{10^2 W}{1W})=20dB.$
- $P=100mW \rightarrow X(dB)=10*\log_{10}(\frac{10^2 mW}{1 mW})=20dBm.$

The End of Chapter I

CH. II

Components of a transmission chain (Oscillators)

II

CHAPTER

oscillator circuits

In this chapter, we will see some oscillators related to transmission chain such as oscillating systems based on RLC and quartz, where we will delay the study of some components like **PLL** and **Super-heterodyne** to the next chapters.

The oscillators are used in different fields like:

- Electromagnetism, electronics
- Acoustic
- Atomic force microscope, intramolecular vibrations
- Seismography
- Tides: resonances according to local configuration of coasts and funds
- Planetary movements: precession; orbital resonances.

RLC, LC Oscillators and Quartz:

1 Description of a coil

A solenoid or coil is obtained by winding an electrical conductor wire on an insulating support. The wire must always be wound in the same direction around the axis of the support.

The coil creates a magnetic field when it is passed by an electric current.

1.1 Behavior of the coil

When a coil is traversed by a variable electric current i , a voltage U_B appears at its terminals.

When the current varies rapidly, this voltage is proportional to the electric current change:

The intensity of the magnetic field B created by a coil traversed by a current is proportional to the intensity of the current. The direction of \vec{B} depends on the direction of the current.

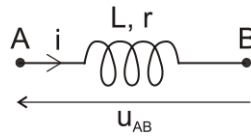
$$u_B \sim \frac{di}{dt}$$

By introducing a coefficient of proportionality and taking the definition of the voltage u_B , it comes:

$$u_B = L \frac{di}{dt}$$

where L is called inductance of the coil and is expressed in henry (H).

The symbol of a coil represents the two characteristic parameters of the coil: its resistance and its inductance, see the following figure



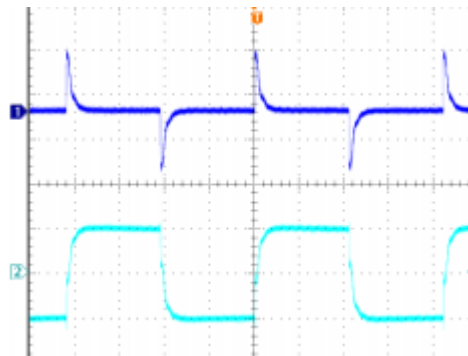
Symbol of the coil

1.2 Modeling the behavior of a coil

For any variation in intensity over time, the voltage is the sum of two terms respectively related to the resistance and inductance of the coil.

$$u_B = r i + L \frac{di}{dt}$$

The oscillograms in the following figure represent the voltage (Electric potential difference) between the coil terminals (Fig 1), for different variable currents (Fig 2).



Voltage across the coil for different currents

1.3 The magnetic energy of the coil (stored energy)

The magnetic energy stored in an inductor L traversed by a current of intensity i is:

$$P_L = U_L i$$

$$dE_L = P_L dt = L \frac{di}{dt} i dt$$

$$E_L = \frac{1}{2} L i^2 + \text{constant}$$

Assuming that the coil is not traversed by a current at the moment 0 → constant=0, then

$$E_L = \frac{1}{2} Li^2$$

2 Description of a capacitor

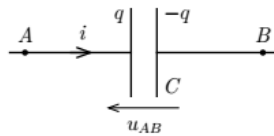
1. A capacitor is formed by two opposite metal surfaces (plates), separated by an insulator (which is dielectric).

2. When a voltage U is applied across a capacitor, charges $Q > 0$ and $-Q < 0$ accumulate on his plates. Charges Q of the capacitor is numerically equal to the product of its capacitance C by the electric voltage U :

$$Q = C U .$$

3. The more the common surface S of the plates is larger and the distance d between the plates is small and the electrical permittivity of the dielectric between the plates is great, the more the capacitance of a capacitor is greater.

The symbol of a capacitor is as follows



The variation of the charge of the capacitor is due to an electric current with an intensity i :

$$i = \frac{dq}{dt}.$$

The electrical energy given to the capacitor

The potential electric energy E_C of a charged capacitor is equal to the electrical work done to charge it.

$$P_C = U_C i$$

$$dE_C = P_C dt = \frac{q}{C} \frac{dq}{dt} dt$$

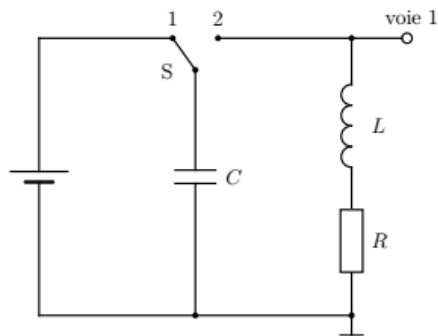
$$E_C = \frac{1}{2} \frac{q^2}{C} + \text{constant}$$

Assuming that the capacitor is already discharged at the moment 0 \rightarrow constant=0, then

$$E_C = \frac{1}{2} \frac{q^2}{C}$$

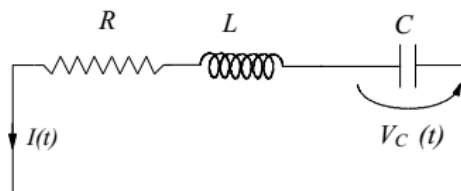
3 RLC Oscillators

The following circuit comprises a DC voltage generator, a capacitor C, a coil and a variable resistance R. When the switcher S is switched to the position 1, the capacitor charges until the voltage at its terminals u_C equals to the electromotive force (e.m.f) generator



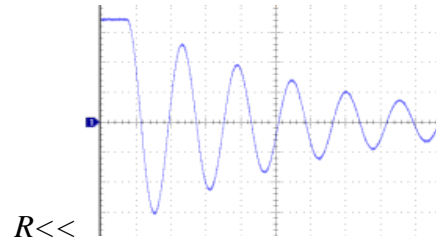
charging of the capacitor

by switching S to the position 2, the generator is isolated, the capacitor is discharged into the coil and in the resistance. The oscilloscope records the u_C voltage during the discharge and the circuit becomes as follows:

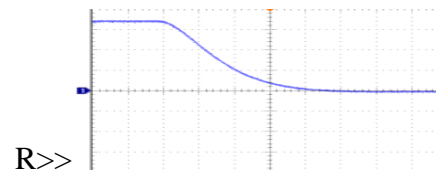


Discharge of a capacitor in a coil (inductance L)

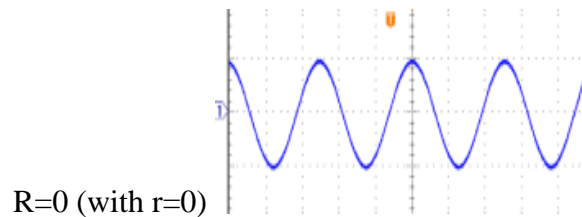
If R is small: The discharge of the capacitor is a damped oscillation and the periodicity is called pseudo-periodic.



If R is big : The voltage decreases without changing the sign, where the discharge of the capacitor is overdamped oscillation and the periodicity is called aperiodic.



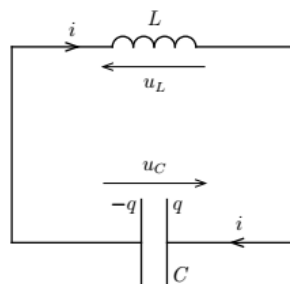
If $R=0$ (with $r=0$) : The discharge of the capacitor is undamped oscillation and the periodicity is called periodic.



4 Free oscillators

4.1 Undamped oscillating discharge of the LC circuit

In this case we will assume the ideal case where the total resistance of the circuit R and r equal to zero, by applying the mesh law in order to find the differential equation which controls the periodic oscillations of the LC circuit.



Oscillating LC Circuit

By applying mesh law, we get: $U_C + U_L = 0$

By taking $r = 0$ and applying the law of **Energy conservation** we get: $E_C + E_L = 0$

Energy conservation:

The law of conservation of energy states that in any isolated system, energy is not destroyed or created from nothing, but can be converted or transformed from one form to another.

$$u_L = L \frac{di}{dt} \quad \text{et} \quad u_C = \frac{q}{C}$$

The equation may be given by:

$$L \frac{di}{dt} + \frac{q}{C} = 0.$$

We know that

$$i = \frac{dq}{dt}.$$

$$\frac{di}{dt} = \frac{d^2q}{dt^2} = \ddot{q}$$

Then, the differential equation for the LC circuit becomes:

$$\ddot{q} = -\frac{1}{LC} q$$

4.2 Solution of the differential equation

The solution of the differential equation is a function of time;

Due to experimental results that provide a sinusoidal oscillation shape, we suggest a sinusoidal solution of the form:

$$q = Q_m \cos(\omega_0 t + \varphi)$$

Let's check that this function is a solution for the differential equation of the circuit.

Taking the first Derivative compared to t :

$$\dot{q} = -Q_m \sin(\omega_0 t + \varphi)$$

$$q'' = -Qm \omega_0^2 \cos(\omega_0 t + \varphi) = -\omega_0^2 Qm \cos(\omega_0 t + \varphi) = -\omega_0^2 q$$

We find that the differential equation is verified by the sinusoidal function provided that:

$$\omega^2 = \frac{1}{LC}$$

What makes the proper frequency of the oscillations equal to:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

4.3 Graphical representations

Consider the example of a capacitor that has been charged to the voltage U_0 . At the moment $t=0$, we assume $\varphi=0$ when the coil is connected to this capacitor, the initial voltage across the capacitor is U_0 and its initial charge $Q_0 = C.U_0$ and we have at the moment $t = 0$:

$$U_0 = Um \cos(\varphi).$$

The intensity of the current must be zero initially because the current cannot be established instantaneously in the coil:

$$0 = -Im \sin(\varphi)$$

As a result from these two conditions is $\varphi = 0$ and $Um = U_0$. The time equations for the different electrical quantities are written:

$$U_C = Um \cos(\omega_0 t) = U_0 \cos(\omega_0 t)$$

$$U_L = -Um \cos(\omega_0 t) = -U_0 \cos(\omega_0 t)$$

$$i = -Im \sin(\omega_0 t) = -Q_0 \omega_0 \sin(\omega_0 t)$$

The electrical energy of the capacitor at the moment t is given by:

$$E_C = 1/(2C) (U_C)^2 = 1/(2C) U_0^2 \cos^2(\omega_0 t). \text{ where } U_C = q.C$$

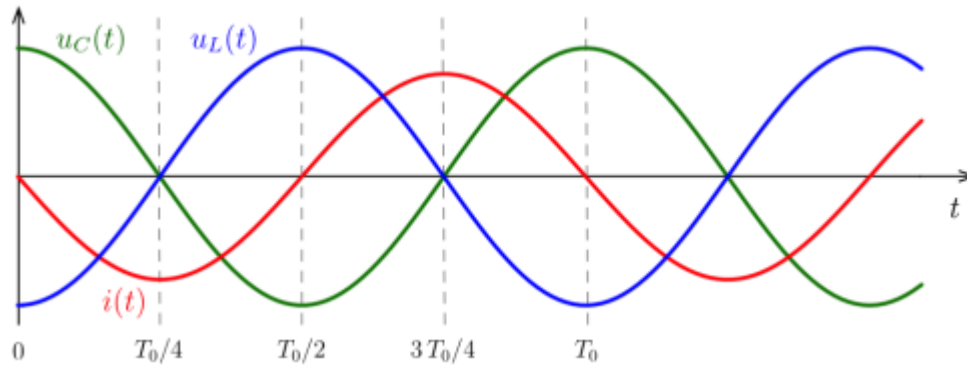
The expression for the magnetic energy of the coil becomes:

$$E_L = 1/2 L i^2 = 1/2 L Q_0^2 \omega_0^2 \sin^2(\omega_0 t).$$

So the electromagnetic energy E equals the sum of E_C and E_L .

The following figure shows the graphical representation of $u_C(t)$, $u_L(t)$ et $i(t)$. We can see that $u_C(t)$ and $i(t)$ have a phase difference of $\pi/2$.

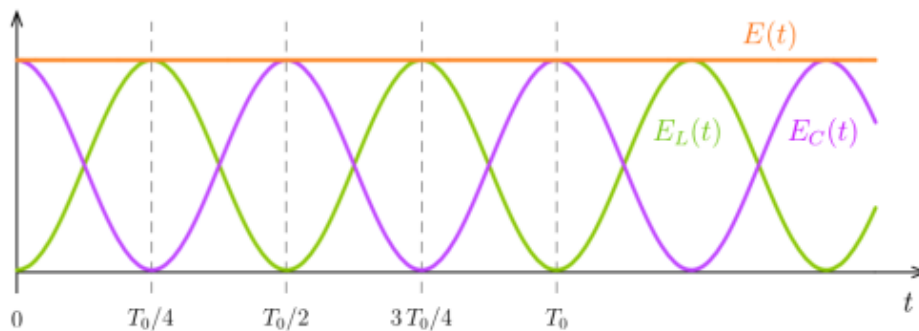
If we take $q(t) = Q_m \cos(\omega_0 t + \phi) \rightarrow U_C = U_0 \cos(\omega_0 t + \phi)$ où $U_0 = Q_m/C$



Graphical representation of $u_C(t)$, $u_L(t)$ et $i(t)$

4.4 Conservation of the energy transferred between the capacitor and the coil

The fact that $U_L = -U_C$ and the electromagnetic energy E equals the sum of E_C et de E_L , it can be judged that the energy is constant and conserved. See the following figure



Graphical representation of $E(t)$, $E_C(t)$ et $E_L(t)$

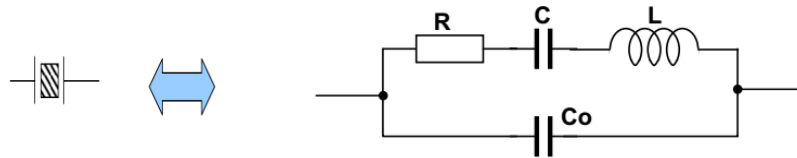
Thus the energy in an LC circuit with negligible internal resistance is exchanged between the capacitor and the coil making an undamped periodic oscillation.

5 Forced Oscillations: **Search more to learn more**

6 Quartz oscillators

Quartz is a mineral composed of crystallized silica and present the nature in its pure state (sometimes with a semitransparent appearance in the rock crystal) or as constituent of granites, sandstones or pure sands, it can be colorless or colored by impurities. This mineral has the property of being transparent to ultraviolet, which is not the case of ordinary glass. In electronics, its piezoelectric properties are used.

So some anisotropic materials (quartz, tourmaline, ...) have the property of being electrically polarized when compressed: this effect is called **PIEZOELECTRIC** effect. This effect is also reversible, that is to say that a crystal of such a material undergoes deformations when an electric field is applied.



Quartz symbol and its corresponding in electrical modeling

The End of Chapter II

CH. III

AM Modulation and Demodulation

III AM Modulation and Demodulation

CHAPTER

In this chapter, we will see a study about AM modulation and demodulation, focusing mainly on DSB-LC modulation type.

The transmission of a baseband signal can put a problem for the following reasons:

- if the signal consists of low frequencies, there is a risk of noise superposition related to the electric components and coexisting systems,
- it is difficult to have a good adaptation of the antenna size towards the frequencies transmitted at low frequency; the transmission is optimal when the dimensions of the antenna have the same order as the wavelength,
- for some frequencies, it is impossible to have a transmission in the medium with good conditions,
- it is not possible to simultaneously transmit several signals

In order to resolve the baseband signal problems, we use a carrier that will be adapted to the transmission medium. In this chapter, this carrier will be modulated in amplitude.

We will distinguish several types of AM modulations:

- Double SideBand – with Large Carrier DSB-LC
- Double SideBand - with Suppressed Carrier DSB-SC
- Single SideBand - with Suppressed Carrier SSB-SC

1 AM Modulation

1.1 Double SideBand - Large Carrier DSB-LC or (Full Carrier (FC))

It is known that the Amplitude Modulation is a modulation where the amplitude of a carrier signal whose high frequency is varied linearly with a low frequency signal (message).

In view that in our case the modulated signal (AM) has double side band with carrier, then it is easier to be demodulated with a simple circuit such as a diode envelope demodulator, this type of modulation is usually preferred rather than with suppressed carrier.

1.1.1 Principle

The modulating signal (high frequency signal) is called: $u_s(t) = U_s \cos(2\pi f_s t)$

The carrier signal (high frequency signal) is called: $u_c(t) = U_c \cos(2\pi f_c t)$

While the AM modulation is based on varying the amplitude of the carrier signal according to the modulating signal changes, the AM signal can be given by this equation

$$U_{am}(t) = [a.U_s \cos(2\pi f_s t) + b] \cos(2\pi f_c t)$$

Where, a and b are parameters of linear function

$$U_{am}(t) = [U_s \cos(2\pi f_s t) + b/a] a \cos(2\pi f_c t)$$

We put, $(b/a)=U_o$, $a=U_c$, $U_o.U_c=k$

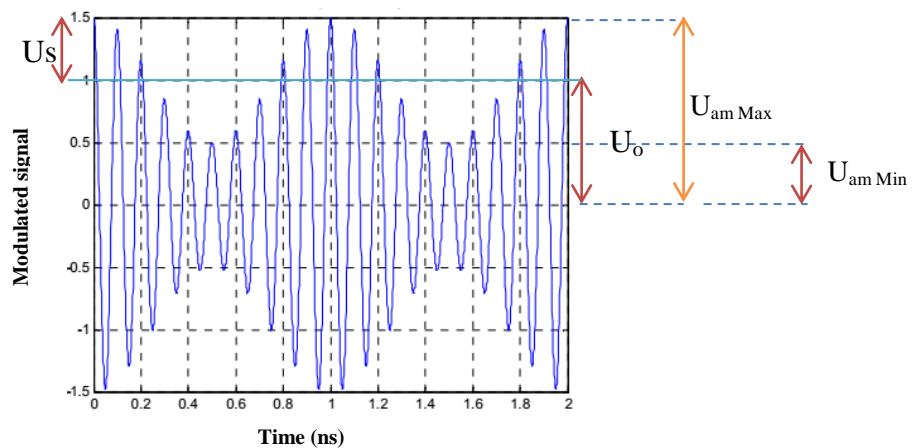
$$U_{am}(t) = [(U_s/ U_o) \cos(2\pi f_s t) + 1] k \cos(2\pi f_c t)$$

We define $m = U_s/ U_o$ as the modulation index, then

$$U_{am}(t) = k [m \cos(2\pi f_s t) + 1] \cos(2\pi f_c t)$$

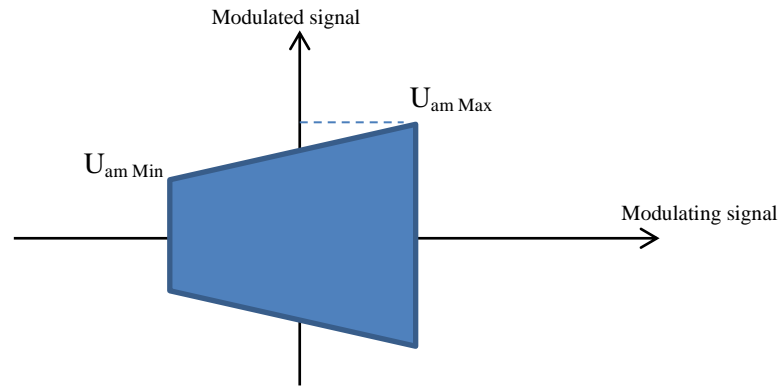
1.1.2 Graphical presentation in time domain and trapezoidal display of AM signal

AM signal can be presented in time domain as follows



To study the linearity of the amplitude modulation, we often visualize on the oscilloscope the modulation trapezoid, where we can judge the on the quality of modulation

It is obtained by switching to the oscilloscope to X-Y mode and connecting the modulating signal with the X channel and the modulated signal with the Y channel. The modulation index “m” can be measured directly from the trapezoidal display, where $m = (U_{am \max} - U_{am \min}) / (U_{am \max} + U_{am \min})$

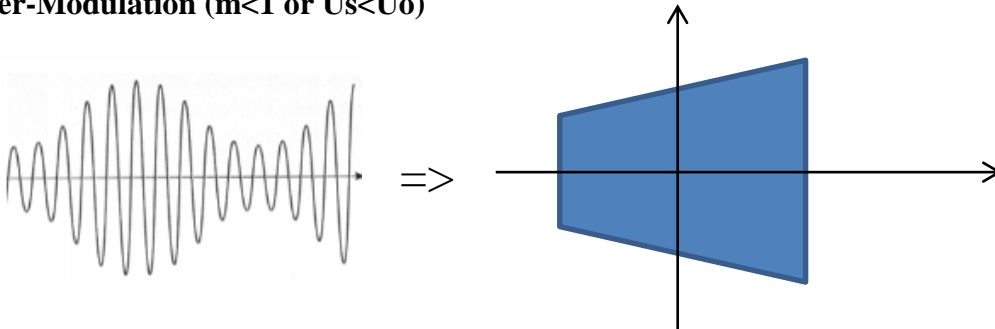


Trapezoidal display obtained from X-Y mode

The judgment about the modulation quality can be given by a percentage of $m\%$ where.

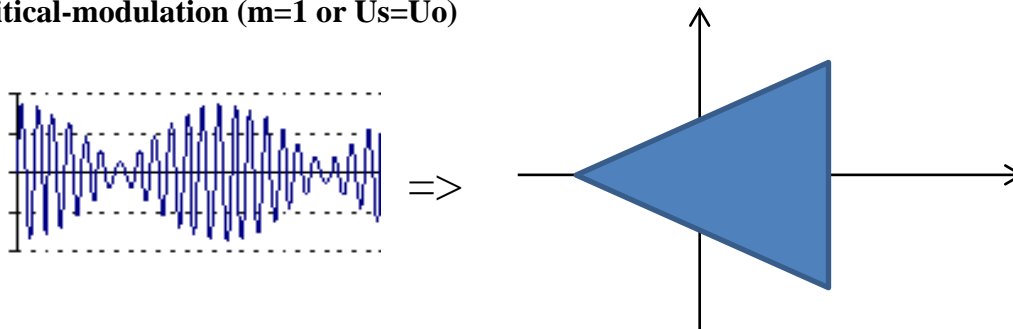
1.1.3 The modulated signal types (for only DSB-LC):

1- Under-Modulation ($m < 1$ or $U_s < U_o$)



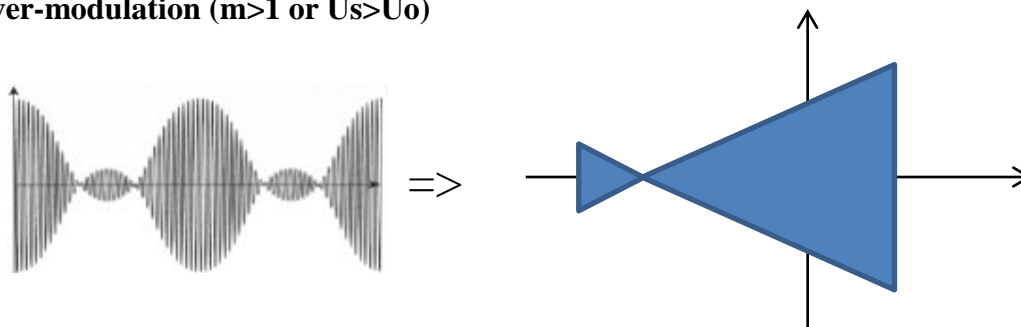
This is the preferred modulation, where the typical modulation is obtained when $m=0.5$

2- Critical-modulation ($m=1$ or $U_s=U_o$)



This is a critical case, where we try to avoid it as possible as we can

3- Over-modulation ($m > 1$ or $U_s > U_o$)



This modulation is unacceptable

1.1.4 Spectrum of the modulated signal

From the signal $U_{am}(t) = k[m \cos(2\pi f_s t) + 1] \cos(2\pi f_c t)$, its spectrum is determined by calculating the Fourier transform of $U_{am}(t)$.

We know that $\cos A \cos B = (\frac{1}{2})\cos(A + B) + (\frac{1}{2})\cos(A - B)$

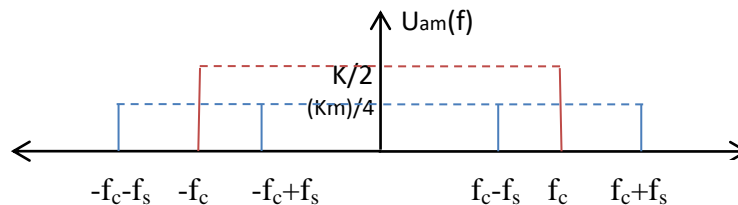
Hence, after doing some operations we will get

$$U_{am}(t) = (km)/2 \cos(2\pi(f_c - f_s)t) + (km)/2 \cos(2\pi(f_c + f_s)t) + k \cos(2\pi f_c t)$$

Thus,

$$U_{am}(f) \iff \mathcal{F}(U_{am}(t))$$

$$U_{am}(f) = (k/2)[\delta(f - f_c) + \delta(f + f_c)] + (km)/4 [\delta(f - (f_c - f_s)) + \delta(f + (f_c - f_s)) + \delta(f - (f_c + f_s)) + \delta(f + (f_c + f_s))]$$

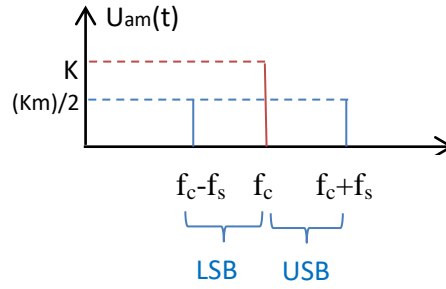


While we have that $\cos(2\pi ft) = \frac{1}{2} \cdot (e^{j2\pi ft} + e^{-j2\pi ft})$, we conclude that

- Two frequency components present the same amplitude
- The negative frequencies are not existed practically, they are only mathematical, so although we have 4 components, we actually have only 2 components

Then, the modulated signal becomes

$$U_{am}(f) = k\delta(f - f_c) + (km)/2 [\delta(f - (f_c - f_s)) + \delta(f - (f_c + f_s))]$$



USB: Upper Side Band

LSB: Lower Side Band

Bandwidth is twice of original bandwidth: $B=2f_s \Leftrightarrow (USF-LSF)$

LSF: Lower Side Frequency

USF: Upper Side Frequency

1.1.5 Transported power by $U_{am}(t)$

The total transported power= Power (Carrier signal) + Power (Upper Side Frequency)+ Power (Lower Side Frequency)

$$P_{tot}=P_c + P_{LSF} + P_{USF}$$

The transported power is given by squaring the effective amplitude value and dividing by a resistance value normalised to 1 ($R=1\Omega$).

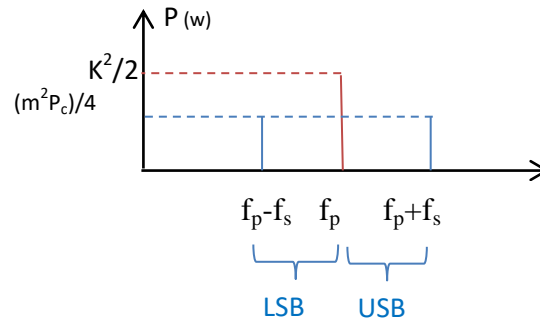
$$P_c = \frac{K^2}{2R} = \frac{K^2}{2}$$

$$P_{LSF} = P_{USF} = \frac{(km)^2}{8R} = \frac{(km)^2}{8} = \frac{m^2}{4} \left(\frac{k^2}{2} \right) = \frac{m^2}{4} (P_c)$$

$$P_{tot} = P_c + 2 \left[\frac{m^2}{4} (P_c) \right]$$

$$P_{tot} = P_c \left[1 + \frac{m^2}{2} \right] = K^2 \left[\frac{1}{2} + \frac{m^2}{4} \right]$$

Hence, the Power Spectrum of DSBFC signal is presented by the following figure



1.2 Double SideBand - Suppressed Carrier DSB-SC

In this type we won't consider the carrier signal that has no information, we know from DSB-LC:

$$U_{am}(t) = k[m\cos(2\pi f_s t) + 1]\cos(2\pi f_c t) \Leftrightarrow km\cos(2\pi f_s t) \cos(2\pi f_c t) + k \cos(2\pi f_c t)$$

Hence, the modulated signal for DSB-SC is the product of

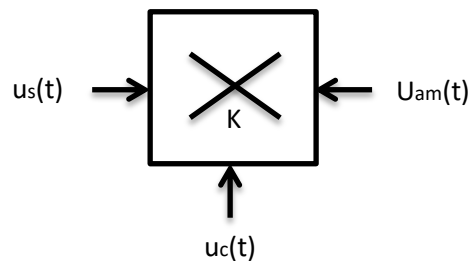
$$U_{am}(t) = u_s(t)u_c(t) = KU_s \cos(2\pi f_s t)U_c \cos(2\pi f_c t)$$

Where, in DSB-SC, there is no modulation index m , K is the multiplier factor.

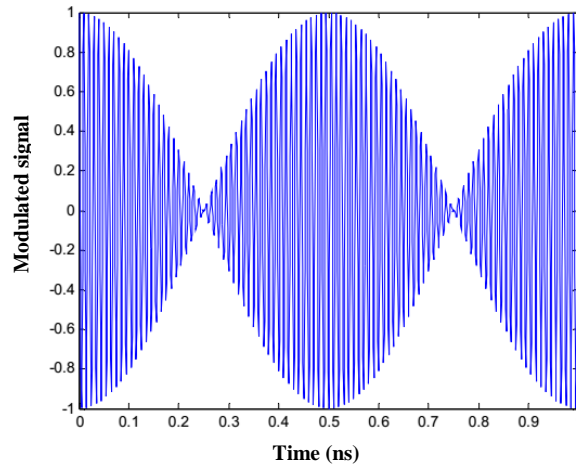
The amplitude of the carrier is then multiplied directly by the modulating signal.

The modulating signal (high frequency signal) is called: $u_s(t) = U_s \cos(2\pi f_s t)$

The carrier signal (high frequency signal) is called: $u_c(t) = U_c \cos(2\pi f_c t)$



The modulated signal form for DSB-SC is presented in the following figure



It is shown that the external envelope of the carrier signal presents the modulating signal and internal one presents the carrier frequency.

From the signal $U_{am}(t) = u_s(t)u_c(t) = KU_s \cos(2\pi f_s t)U_c \cos(2\pi f_c t)$, its spectrum is determined by calculating the Fourier transform of $U_{am}(t)$.

We know that $\cos A \cos B = (\frac{1}{2})\cos(A + B) + (\frac{1}{2})\cos(A - B)$

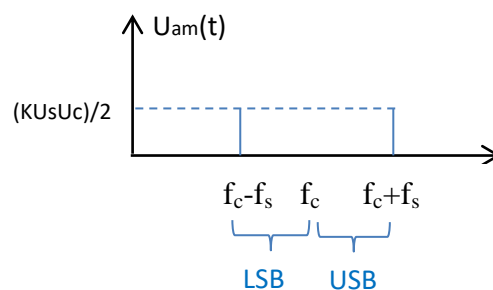
Hence, after doing some operations we will get

$$U_{am}(t) = (kU_s U_c) / 2 [\cos(2\pi (f_c - f_s)t) + \cos(2\pi (f_c + f_s)t)]$$

Thus,

$$U_{am}(f) \iff \mathcal{F}(U_{am}(t))$$

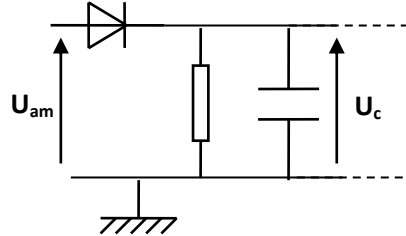
$$U_{am}(f) = (kU_s U_c) / 2 [\delta(f - (f_c - f_s)) + \delta(f - (f_c + f_s))]$$



2 AM Demodulation

2.1 Envelop detector (for DSB-LC)

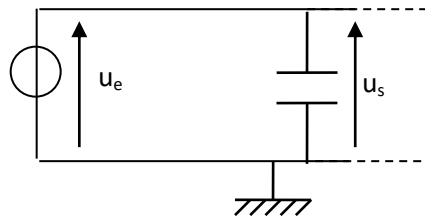
The envelope detector consists of a diode and parallel RC circuit.



During the study of this part of the circuit, we should not see the stuff as a succession of a rectification stage and a filter, but we should study the effect of each element within the general framework.

According to the the diode role, we get the following cases:

- When the input voltage U_{am} is higher than U_c , the diode is polarised directly and the capacitor charges without time constant (the assembly is considered as an ideal voltage generator):



The End of Chapter III

CH. IV

Angular modulation and demodulation (FM and PM)

IV

CHAPTER

Angular Modulation and Demodulation (FM and PM)

The angular modulation consists of frequency modulation (FM) or phase modulation (PM), where in order to modulate information signal; it comes to modulate the angle or the frequency of the carrier signal rather than the amplitude.

1 Frequency modulation (FM)

If we consider a modulated signal whose sinusoidal form as follows

$$S(t) = A \cdot \cos(\theta_i(t))$$

We can say that the instantaneous frequency f_i is controlled by the modulating signal $m(t)$ around a carrier signal f_c

$$f_i = f_c + K_f \cdot m(t)$$

Where K_f is the sensitivity factor of the modulator and it is expressed in $\text{Hz} \cdot \text{V}^{-1}$.

So, the instantaneous phase is given by

$$\omega_i = 2\pi \cdot f_c + 2\pi \cdot K_f \cdot m(t)$$

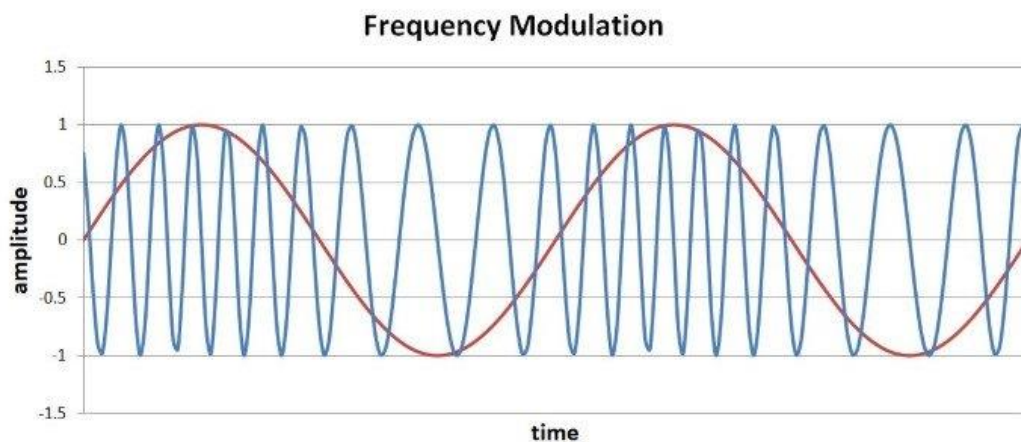
The instantaneous phase of the modulated signal is given by the following expression

$$\theta_i = \int \omega_i dt = 2\pi \cdot f_c t + 2\pi \cdot K_f \cdot \int m(t) dt$$

Hence, the modulated signal is expressed by

$$S(t) = A \cos(2\pi \cdot f_c t + 2\pi \cdot K_f \cdot \int m(t) dt)$$

FM signal shape in time domain [36]



— : the modulated signal $S(t)$

— : the modulating signal $m(t)$

2 Phase modulation (PM)

We concluded from the last expressions that phase θ is proportional to the modulating signal.

$$\theta_i = 2\pi f_c t + 2\pi K_f \cdot \int m(t) dt$$

The expression of the modulated signal can be written as follows

$$S(t) = A \cdot \cos(2\pi f_c t + K_\phi \cdot m(t))$$

When $m(t)$ is sinusoidal $m(t) = V_m \cdot \cos(\omega_m t)$, the quantity $\Delta\phi = V_m \cdot K_\phi$ is called the excursion in phase.

3 Frequency modulation with a sinusoidal signal**3.1 Sinusoidal modulating signal**

Let's assume that the modulating signal is of the form: $m(t) = V_m \cdot \cos(\omega_m t)$

and the instantaneous frequency is given by: $f_i = f_c + K_f \cdot V_m \cdot \cos(\omega_m t)$.

So, the instantaneous frequency varies in the following interval: $f_c - \Delta f \leq f_i \leq f_c + \Delta f$

$\Delta f = K_f \cdot V_m$ is called the frequency excursion.

The modulated signal is given by the following expression:

$$S(t) = A \cdot \cos\left(2\pi f_c t + \frac{\Delta f}{f_m} \sin(\omega_m t)\right)$$

$\beta = \frac{\Delta f}{f_m}$ is called the modulation index.

3.2 Narrowband modulation

In the case where β is low ($\beta < 1$), it comes to narrowband modulation. So, if we have

$$S(t) = A \cos(\omega_c t + \beta \sin(\omega_m t))$$

$$S(t) = A \cos(\omega_c t) \cos(\beta \sin(\omega_m t)) - A \sin(\omega_c t) \sin(\beta \sin(\omega_m t))$$

By taking into account ($\beta < 1$), we can write

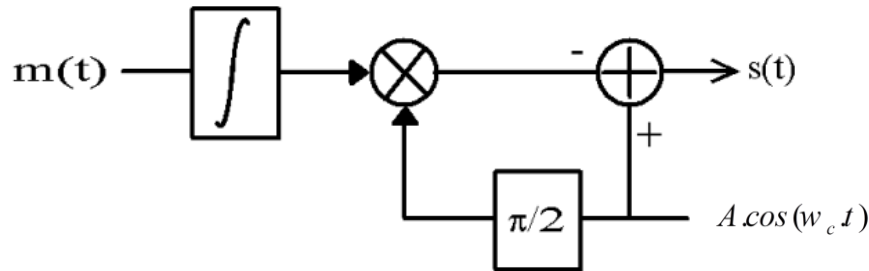
$$\cos(\beta \sin(\omega_m t)) \approx 1$$

$$\sin(\beta \sin(\omega_m t)) \approx \beta \sin(\omega_m t)$$

$$S(t) = A \cos(w_c t) - A \sin(w_c t) \beta \sin(w_m t)$$

$$S(t) = A \cos(w_c t) - \frac{A\beta}{2} [\cos(w_c - w_m)t - \cos(w_c + w_m)t]$$

The spectrum is identical to that of a DSB-LC, but there is phase opposition between the upper line and the lower line. [36]



3.3 Calculation of the spectrum of the frequency modulated signal

$$S(t) = A \cos(w_c t + \beta \sin(w_m t)) = A \cdot \text{Re} \{ \exp(j(w_c t + \beta \sin(w_m t))) \}$$

$$S(t) = A \cdot \text{Re} \{ \exp(jw_c t) \} \cdot \text{Re} \{ \exp(j\beta \sin(w_m t)) \}$$

We are first interested in the 2nd term of the exponential. Let's put $\theta = w_m t$ and calculate $y = e^{j\beta \sin \theta}$

We know that

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \Rightarrow j \cdot \sin \theta = \frac{e^{j\theta}}{2} - \frac{e^{-j\theta}}{2}$$

$$y = e^{\frac{\beta}{2} e^{j\theta}} e^{-\frac{\beta}{2} e^{-j\theta}}$$

When we replace the exponential by its limited development, we get

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$$

Hence,

$$y = \left[1 + \frac{\beta}{2} e^{j\theta} + \frac{\beta^2}{4 \cdot 2!} e^{2j\theta} + \frac{\beta^3}{8 \cdot 3!} e^{3j\theta} \dots \right] \cdot \left[1 - \frac{\beta}{2} e^{-j\theta} + \frac{\beta^2}{4 \cdot 2!} e^{-2j\theta} - \frac{\beta^3}{8 \cdot 3!} e^{-3j\theta} \dots \right]$$

$$y = \left[1 - \frac{\beta^2}{4} + \frac{\beta^4}{2^4 \cdot (2!)^2} \dots \right] \cdot e^0 + \left[\frac{\beta}{2} - \frac{\beta^3}{2^3 \cdot 2!} + \frac{\beta^5}{2^5 \cdot 2! \cdot 3!} \dots \right] e^{j\theta} \dots + \left[-\frac{\beta}{2} + \frac{\beta^3}{2^3 \cdot 2!} - \frac{\beta^5}{2^5 \cdot 2! \cdot 3!} \dots \right] e^{-j\theta} \dots$$

The 1st term in square brackets is $J_0(\beta)$

The 2nd term in square brackets is $J_1(\beta)$

The 3rd term in square brackets is $J_{-1}(\beta)$

3.3.1 Bessel functions and properties

The terms $J_n(\beta)$ are the Bessel functions of the 1st kind of order n and they are obtained from the following expression

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{\left(\frac{\beta}{2}\right)^{n+2k}}{(n+k)!}$$

These are some important properties of Bessel functions

1. $J_{-n}(\beta) = (-1)^n J_n(\beta)$

2. For low values of β we have:

- $J_0(\beta) \cong 1$
- $J_1(\beta) \cong \frac{\beta}{2}$
- $J_n(\beta) \cong 0$ for $n > 2$.

3. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

We continue our calculus and basing on what we have seen, we can write Bessel's identity as follows

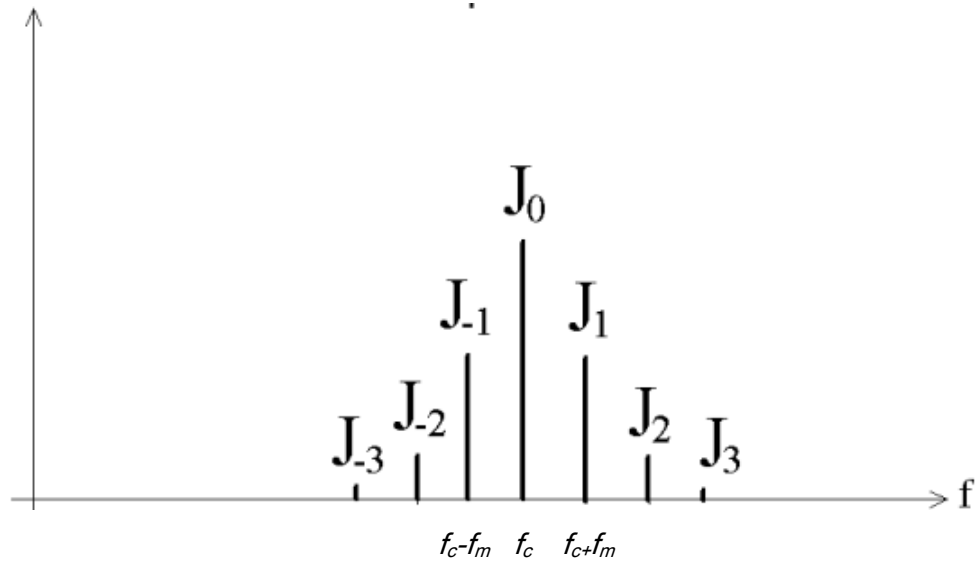
$$y = \exp(j\beta \sin(w_m t)) = \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \exp(j.n.w_m t)$$

Now, let's go back to the expression $S(t)$, where it can be given by [36]

$$S(t) = A \cdot \text{Re}\{ \exp(jw_c t) \} \cdot \text{Re}\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \exp(j.n.w_m t) \right\}$$

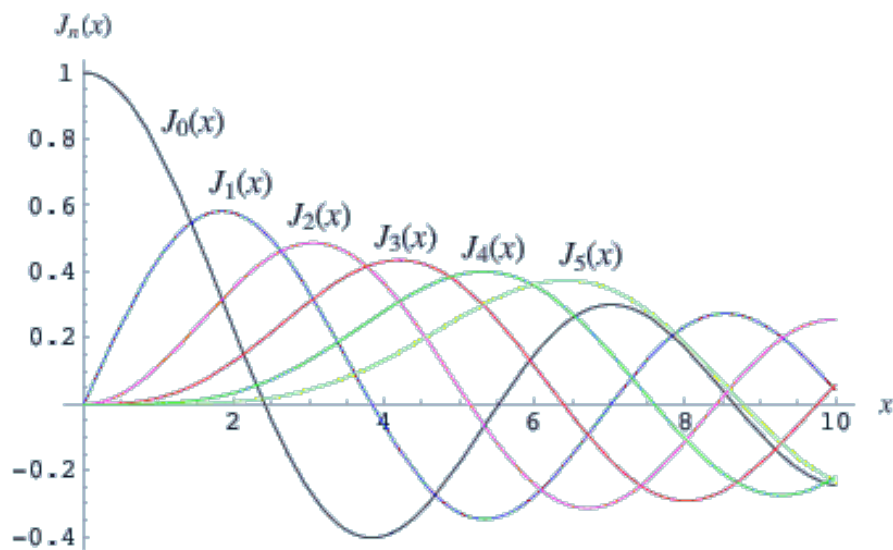
$$S(t) = A \cdot \begin{pmatrix} J_0(\beta) \cos(w_c t) + \\ J_1(\beta) [\cos((w_c - w_m)t) - \cos((w_c + w_m)t)] + \\ J_2(\beta) [\cos((w_c - w_m)t) - \cos((w_c + w_m)t)] + \\ J_3(\beta) [\cos((w_c - w_m)t) - \cos((w_c + w_m)t)] + \\ \vdots \end{pmatrix}$$

We obtain a spectrum of symmetrical harmonics with respect to f_p and with a difference of f_m between each harmonic, we took $\beta=1$ as an example where $|J_n| = |J_{-n}|$



3.3.2 Bessel functions of 1st kind

The following Figure shows the shape of Bessel functions of the 1st kind versus the modulation index β .



Bessel coefficients of 1st kind is given by the following table

Modulation Index	Sidebands (Pairs)																
	Carrier	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th
0.00	1.00	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
0.25	0.98	0.12	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
0.5	0.94	0.24	0.03	--	--	--	--	--	--	--	--	--	--	--	--	--	--
1.0	0.77	0.44	0.11	0.02	--	--	--	--	--	--	--	--	--	--	--	--	--
1.5	0.51	0.56	0.23	0.06	0.01	--	--	--	--	--	--	--	--	--	--	--	--
2.0	0.22	0.58	0.35	0.13	0.03	--	--	--	--	--	--	--	--	--	--	--	--
2.5	-0.06	0.50	0.45	0.22	0.07	0.02	--	--	--	--	--	--	--	--	--	--	--
3.0	-0.20	0.34	0.49	0.31	0.13	0.04	0.01	--	--	--	--	--	--	--	--	--	--
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	--	--	--	--	--	--	--	--	--
5.0	-0.18	-0.33	0.06	0.36	0.39	0.26	0.13	0.05	0.02		--	--	--	--	--	--	--
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	--	--	--	--	--	--	--
7.0	0.30	0.00	-0.30	-0.17	0.16	0.35	0.34	0.23	0.13	0.08	0.02	--	--	--	--	--	--
8.0	0.17	0.23	-0.11	-0.29	-0.10	0.19	0.34	0.32	0.22	0.13	0.06	0	--	--	--	--	--
9.0	-0.09	0.24	0.14	-0.18	-0.27	-0.06	0.20	0.33	0.30	0.21	0.12	0.1	0.03	0.01	--	--	--
10.0	-0.25	0.04	0.25	0.06	-0.22	-0.23	-0.01	0.22	0.31	0.29	0.20	0.1	0.06	0.03	0.01	--	--
12.0	-0.05	-0.22	-0.08	0.20	0.18	-0.07	-0.24	-0.17	0.05	0.23	0.30	0.3	0.20	0.12	0.07	0.03	0.01
15.0	-0.01	0.21	0.04	0.19	-0.12	0.13	0.21	0.03	-0.17	-0.22	-0.09	0.10	0.24	0.20	0.25	0.18	0.12

3.3.3 Power of modulated signal

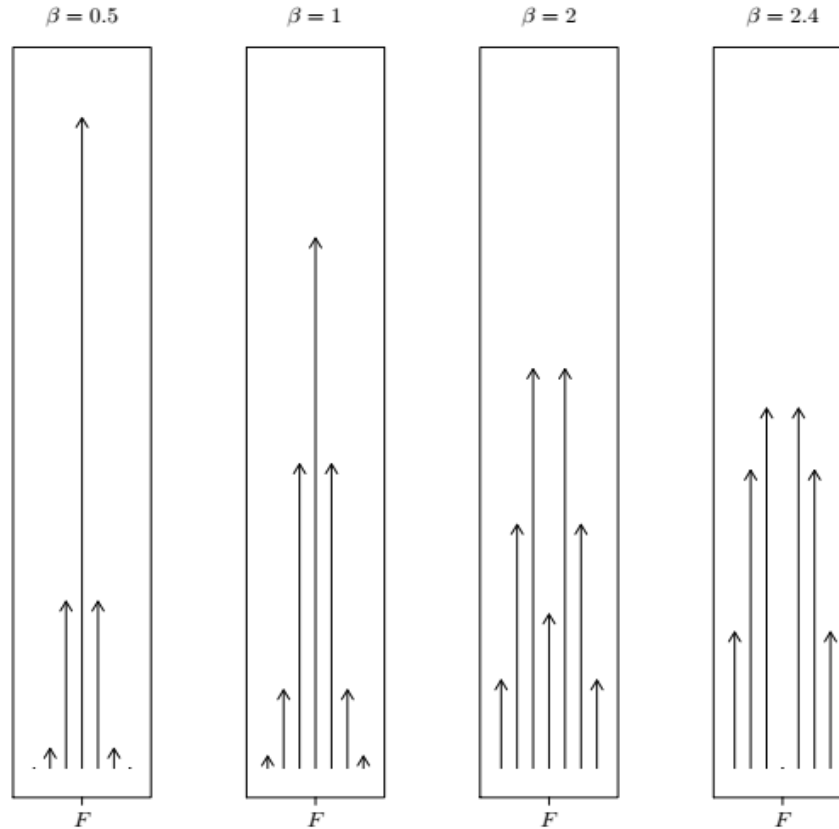
We must know that whatever the modulation index, the power transported P is constant, where we have

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

$$P = s^2 = A^2 \cdot \left(\sum_{n=-\infty}^{\infty} J_n^2(\beta) \cdot \cos^2((\omega_c + n \cdot \omega_m) \cdot t) \right) = \frac{A^2}{2}$$

3.3.4 Spread spectrum VS modulation index

The following figure gives the shape of the spectrum of the modulated signal versus the modulation index β [32].



The bandwidth of the modulated signal is proportional to β . We deduce that the Bandwidth **B** necessary for the transmission of a FM signal with an index β and a frequency f_m can be given by

$$B=2(\beta+1).f_m$$

The bandwidth B is called Carson Band, which ensures almost 98% of the FM signal's power.

4 Frequency demodulation

There are many ways to demodulate FM signals, however we discuss here the widely used techniques

4.1 Frequency discriminator (slope detector)

The principle of any frequency demodulator is to provide a signal whose amplitude is a linear function of the instantaneous frequency of the input wave (FM signal).

From this observation, it's enough to use a device having a linear frequency /voltage transfer characteristic.

The modulated signal $s(t)$ has the following expression

$$S(t) = A \cos(2\pi.f_c t + 2\pi.K_f \cdot \int m(t) dt)$$

When we derivate it, we get

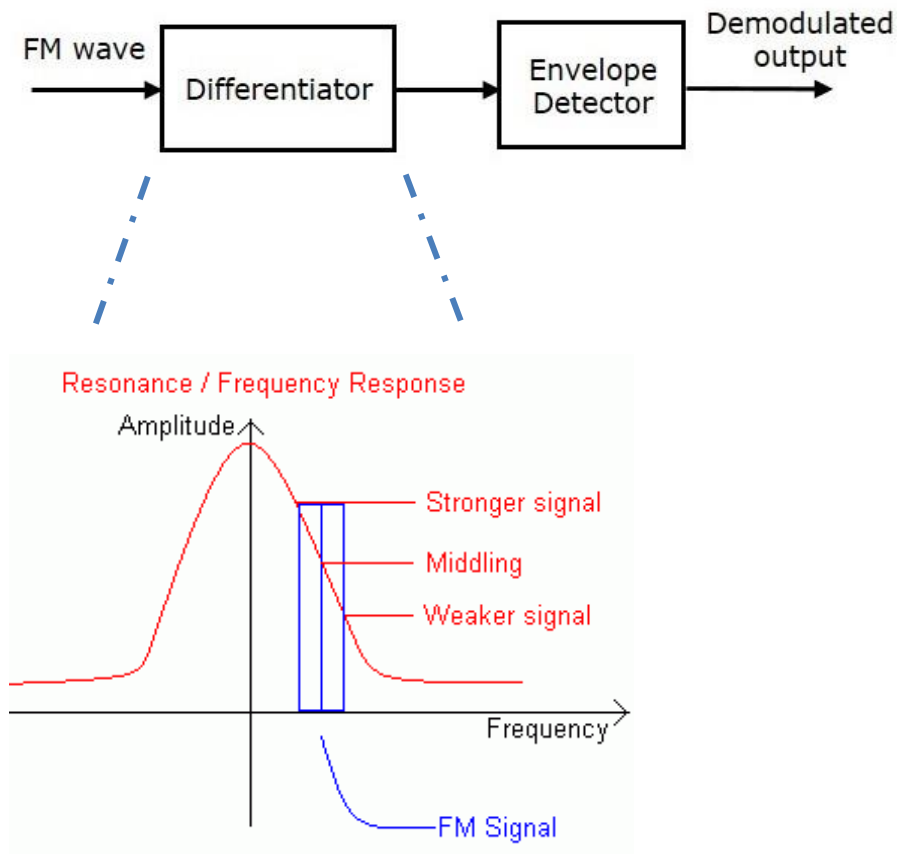
$$\frac{dS(t)}{dt} = A \cdot [2\pi f_c + 2\pi K_f \cdot m(t)] \sin(2\pi f_c t + 2\pi K_f \cdot \int m(t) dt)$$

After removing the envelope, the signal becomes

$$\frac{dS(t)}{dt} = A \cdot [2\pi f_c + 2\pi K_f \cdot m(t)]$$

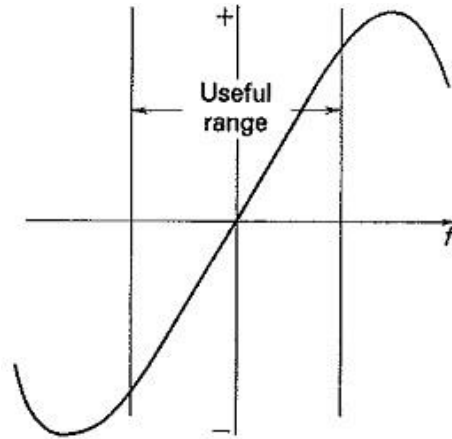
To derivate the signal, we use a filter that varies the amplitude as a function of the frequency.

To establish the derivative filter (differentiator), in practice we use the linear part of the transfer function of a resonant circuit.

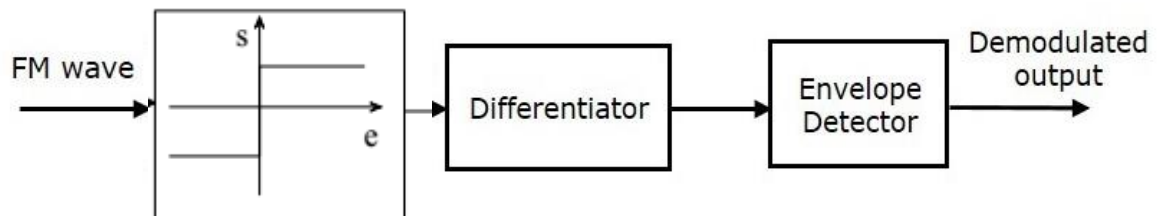


The filter used must have a linear response as a function of frequency over the entire range of frequency modulation.

The discriminator has a limited range of linearity. This can be improved by combining two “head-to-tail” resonant circuits (called **Foster-Seely discriminator** or **Balanced discriminator**) as shown in the following figure:



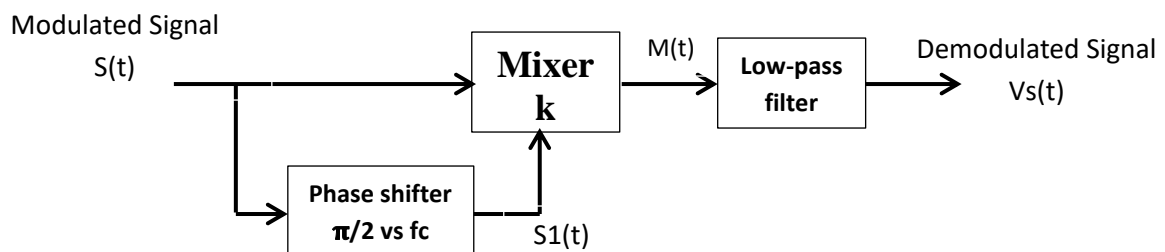
This type of demodulator being sensitive to variations in signal amplitude, the use of a limiter clipping the signal is necessary.



This type of circuit has been used for FM reception for a long time, but is now obsolete (integration impossible, tedious adjustments, etc.).

4.2 Quadrature FM demodulator (or coincidence detector)

The quadrature discriminator is the most used solution in frequency demodulation. Such a device can be produced inexpensively in integrated form. It's a circuit that shifts the phase of the modulated signal by an amount proportional to its instantaneous frequency and uses a phase detector (multiplier), thus to convert the phase variation into an amplitude variation and find the modulating signal. [32, 37]



We know that [36]

$$S(t) = A \cdot \cos(\omega_c t + \phi(t))$$

Where, $\phi(t) = 2\pi.K_f \cdot \int m(t)dt = \beta \sin(\omega_m t)$

After a phase shift of θ , it becomes

$$S_1(t) = A \cdot \cos(\omega_c t + \phi(t) + \theta(t))$$

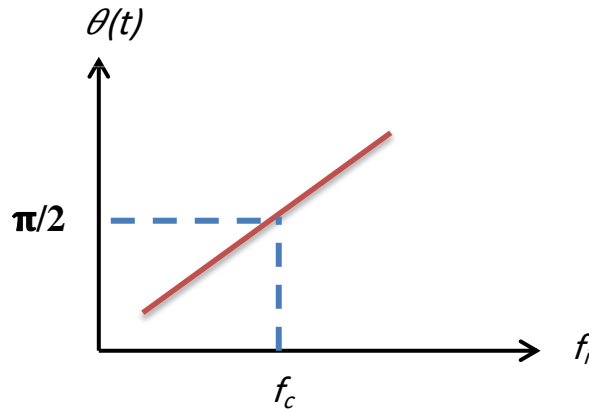
After mixing them through the multiplier, the voltage is

$$M(t) = S(t)S_1(t) = k \frac{A^2}{2} [\cos \theta(t) + \cos(2\omega_c t + 2\phi(t) + \theta(t))]$$

After the low pass filter, we get

$$V_s(t) = k \frac{A^2}{2} \cos \theta(t)$$

The following Figure shows that the phase shifter provides a phase shift $\theta(t)$ which depends linearly on the instantaneous frequency of the FM signal $S(t)$.



We manage to have a phase shifter with the following mathematical characteristic

$$\theta(f) = \frac{\pi}{2} + a.(f_i - f_c)$$

Where the brief function is $\theta(t) = a.f_i$, however for objective and mathematical reasons we added $\pi/2$ then we subtracted it by $a.f_c$ because $\pi/2$ is assumed to be equal to $a.f_c$

$$V_s = k \frac{A^2}{2} \cos \left(\frac{\pi}{2} + a.(f_i - f_c) \right) = -k \frac{A^2}{2} \sin(a.(f_i - f_c))$$

When, the phase shifter is adjusted so that the phase variation around $\pi/2$ is small and the difference between f_i and f_c is small, the expression becomes

$$V_s = -k \frac{A^2}{2} a.(f_i - f_c)$$

By substituting the instantaneous frequency f_i which is equal to: $f_i = f_c + K_f.m(t)$

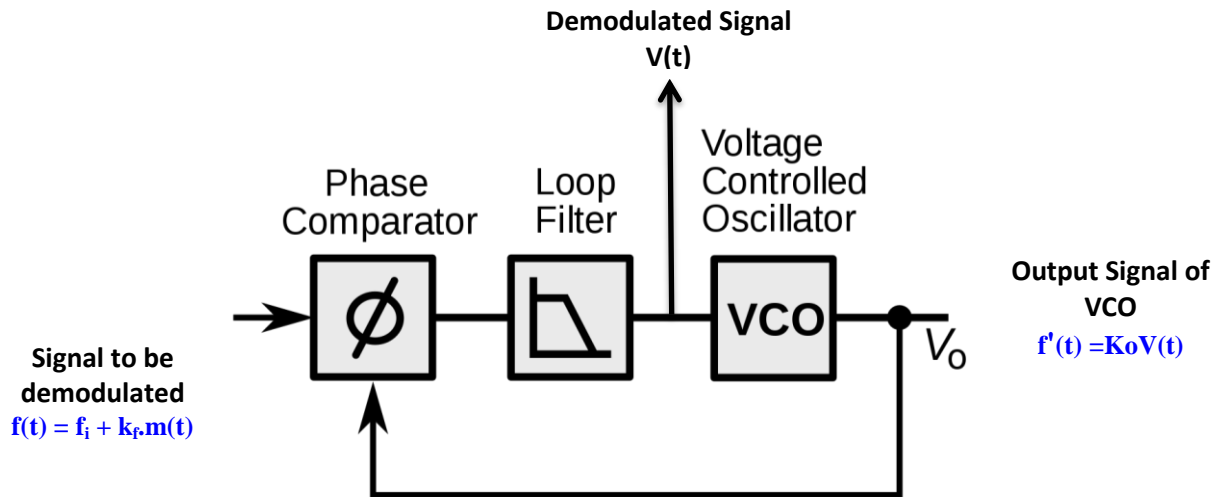
So the final demodulated FM signal is

$$V_s(t) = -k \frac{A^2}{2} . a . K_f . m(t)$$

Where, the output signal $V_s(t)$ is proportional to the modulating signal $m(t)$.

4.3 FM receiver using PLL (Phase-Locked Loop)

The FM signal can also be demodulated by a Phase-Locked Loop demodulator



The process is summarized in following points:

- When the PLL is working and after some loops, the VCO synchronizes with the signal given at the input

The VCO therefore provides at its output a signal whose a frequency equals to that of the input signal: $f(t) = f'(t) = f_i + k_f.m(t)$

- If the frequency $f'(t)$ varies, this means that the voltage $v(t)$ at the input of the VCO varies

Assuming that the VCO is linear and characterized by its slope K_o , we have: $f'(t) = K_o V(t)$

- We can easily deduce the expression of the signal $v(t)$:

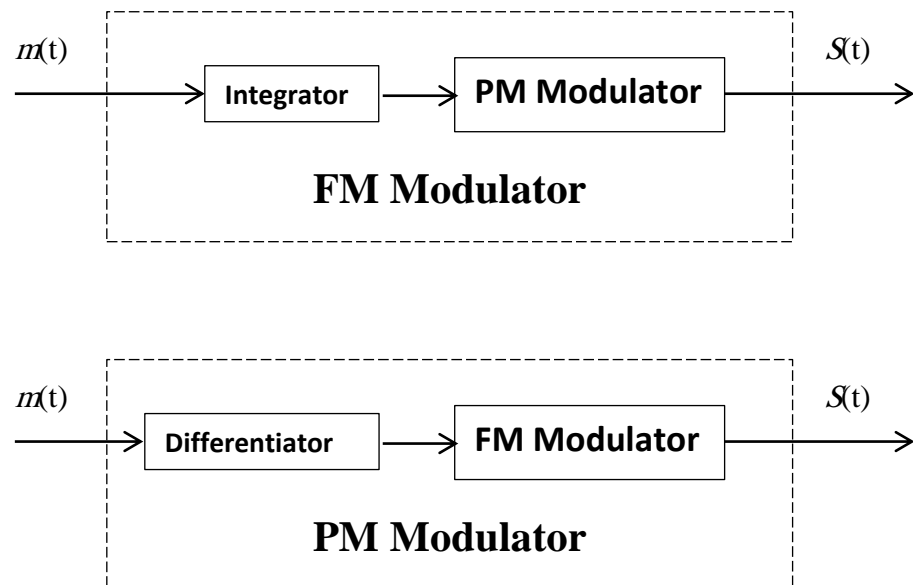
$$v(t) = f'(t)/K_o = f_i / K_o + k_f.m(t) / K_o = V_o + A.m(t)$$

- The control voltage of the VCO comprises two components: a DC component V_o which is easy to eliminate using a capacitor put in series, and a variable voltage proportional to the modulating signal that we are looking for.

Finally, it is important to point out that the PLL demodulator is better than the quadrature discriminator when the signal to be demodulated is very noisy and requires too much precision . Therefore, it is used to detect weak signals coming from distant transmitters like the satellites for example

5 Conversion between FM and PM

When we look at the relation between the phase and the frequency modulation, we can remark that the two modulations are linked. Indeed, we can build a frequency modulator using a phase modulator, and a frequency demodulator as well using a phase demodulator and vice-versa.



The End of Chapter IV

CH. V

Performance of the
different modulations in
the presence of noise

V

CHAPTER

Performance of the different modulations in the presence of noise

The communication process becomes quite challenging because of the unwanted signals in the communications system. These undesirable signals, usually named as *noise*, it is a random signal which interferes with the message signals. The receiver input, in general, consists of message signal and noise. In this chapter we will calculate the Signal to Noise Ratios and Figure of Merits of different modulated waves, which are demodulated at the receiver.

In digital communication systems, we determine the system performance quite uniquely through calculating the probability of error. However, because of the continuous nature of the analog modulations, it is difficult to adopt this method. For that, we determine the system performance of analog modulation systems through the **signal-to-noise ratio (SNR)** at the **receiver input and output**.

1 Signal to Noise Ratio (SNR)

Signal-to-noise ratio is a measure used in science and engineering so as to compares the level of a desired signal to the level of related noise. It is often expressed in decibels.

2 Additive white Gaussian noise (AWGN)

AWGN noise is an elementary model of noise used in information theory to mimic random processes that occur in nature, it is uncorrelated with the useful signal. The higher the SNR's value, the greater is the quality of the captured signal at the receiver output. The type of noise has the following characteristics:

Additive: because it is added to the intrinsic noise of the information system;

White: its power is uniform over the entire frequency bandwidth of the system; it is similar to the colored noise which has uniform emissions over all frequencies in the visible spectrum.

Gaussian: it has a normal distribution in the time domain with a zero mean

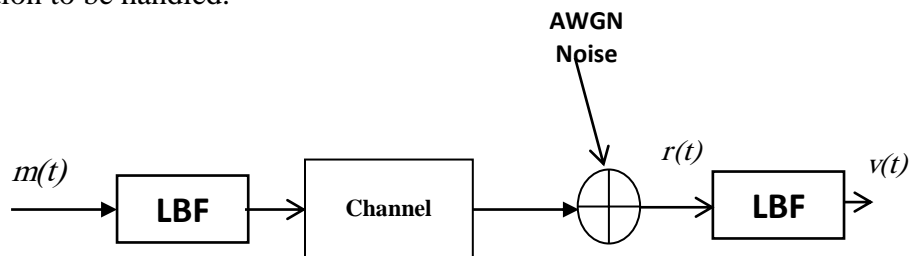
3 Noise sources

Many natural sources generate broadband noise: (thermal vibrations of atoms, thermal noise in conductors issued from moving the electrons through it). The central limit theorem of probability theory shows that a sum of many stochastic processes tends to have a Gaussian distribution (normal distribution) [43].

Source of Noise	What Causes It	How to Prevent It
Line Outages	Storms, Accidents	
White Noise	Movement of electrons	Increase signal strength
Impulse Noise	Lightning or sudden increase of electricity	Shield wires
Cross-Talk	Wires too close together	Isolate or shield wires
Echo	Poor connections	Fix/isolate connections
Intermodulation Noise	Signals from several circuits combine	Move or shield wires
Jitter	Signals change phase	Tune equipment
Harmonic distortion	Circuits change phase	Tune equipment

4 SNR in baseband communication

Baseband communication means a transmission technique in which the signal is sent directly over the channel without modulation and without using a carrier signal. Thus, there is no carrier in demodulation to be handled.



In order to analyse the noise in a baseband system, we assume that the receiver contains only of an ideal low pass filter with the bandwidth W .

The average power of noise at the output of the receiver, for AWGN input, is

$$N_0 = \int_{-W}^W \frac{no}{2} df = noW$$

If we represent the received power by P_r , the baseband SNR is

$$SNR = \int_{-W}^W \frac{no}{2} df = \frac{P_r}{noW}$$

5 SNR in communication with carrier signal

The SNR at different points is calculated using the following methods:

1/ Pre-detection SNR via Input SNR (we will not discuss it here):

Input SNR = (SNR)_I = Average power of the modulated signal / Average power of noise at receiver **input**

2/ Reference SNR via Channel SNR:

Channel SNR = (SNR)_r = Average power of the modulated signal at receiver **input** / Average power of noise in message bandwidth at receiver **input**

3/ Post-detection SNR via Output SNR:

Output SNR = (SNR)_O = Average power of the message at receiver **output** / Average power of noise at receiver **output**

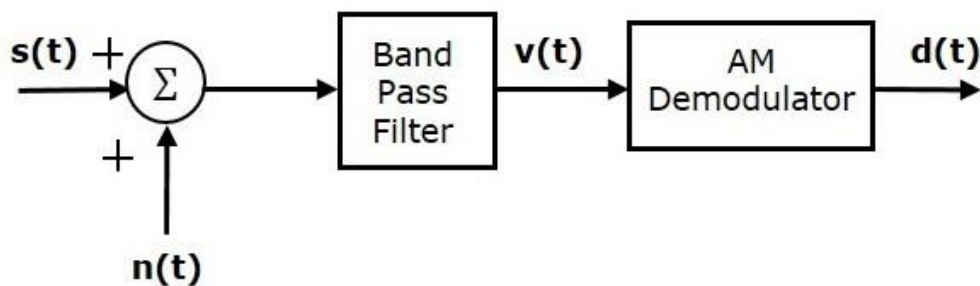
5.1 Figure of Merit

The ratio between output SNR and input SNR is termed as **Figure of Merit (F)**. It describes the performance of the various modulation-demodulation schemes, where larger the value of F, better is the noise performance of the system.

$$F = (SNR)_O / (SNR)_R$$

5.2 SNR in AM System (DSB-LC)

Let us consider the following receiver model for a sinusoidal signal in AM system [36-42].



As a recall, the AM modulation signal is given by

$$s(t) = U_{am}(t) = k[m \cdot u_s(t) + 1] \cos(2\pi f_c t)$$

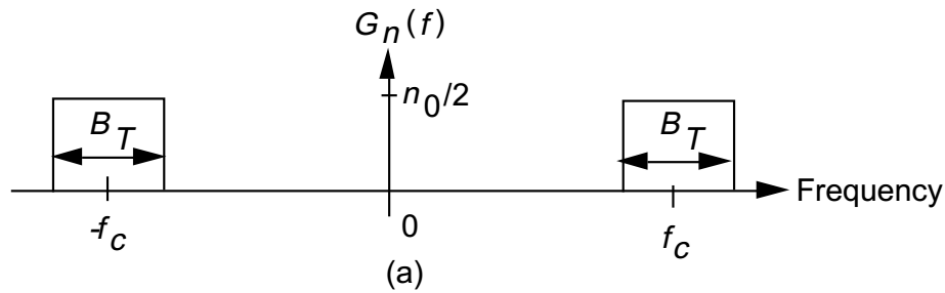
After doing some operations we got

$$s(t) = (km)/2 \cos(2\pi(f_c - f_s)t) + (km)/2 \cos(2\pi(f_c + f_s)t) + k \cos(2\pi f_c t)$$

We have seen that the average power transported is

$$P_{i,AM} = P_c \left[1 + \frac{m^2}{2} \right] = K^2 \left[\frac{1}{2} + \frac{m^2}{4} \right]$$

Let us put $Gn(f)=Gx(f)+Gy(f)$ as the power spectral density (PSD) of the narrowband noise $n(t)$ at the AM demodulator input.



The average power of AWGN noise in the two band sides is

$$\begin{aligned} N_{i,AM} &= \int_{-\infty}^{\infty} Gn(f) df \\ &= \int_{-fc-B}^{-fc+B} \frac{n_0}{2} df + \int_{fc-B}^{fc+B} \frac{n_0}{2} df \\ &= n_0 B + n_0 B \\ N_{i,AM} &= 2Bn_0 \end{aligned}$$

Where **B** is the message bandwidth

When we substitute, the above values in **channel SNR** formula

(SNR)_{R,AM}= Average power of the modulated signal at receiver **input** / Average power of noise in message bandwidth at receiver **input**

$$\Rightarrow (SNR)_{R,AM} = \frac{K^2 \left[\frac{1}{2} + \frac{m^2}{4} \right]}{2Bn_0}$$

As presented in the above figure, where the noise is mixed with AM signal in the channel and filtered through the band pass filter of W band. The $v(t)$ is applied at the AM demodulator input.

$$\begin{aligned} v(t) &= s(t) + n(t) \\ n(t) &= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \end{aligned}$$

Where $x(t)$ and $y(t)$ are in-phase and quadrature components of the noise.

$$v(t) = k[m.u_s(t) + 1] \cos(2\pi f_c t) + [x(t) \cos(2\pi f_c t) - y(t) \sin(2\pi f_c t)]$$

$$v(t) = [kmu_s(t) + k + x(t)] \cos(2\pi f_c t) - y(t) \sin(2\pi f_c t)$$

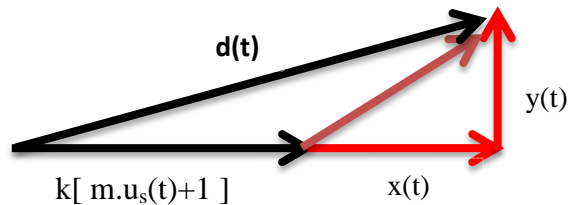
The output $d(t)$ of AM demodulator presents the envelope of the above signal.

$$d(t) = \sqrt{[kmu_s(t) + k + x(t)]^2 + y(t)^2}$$

If the SNR at the receiver input is sufficiently large, we can approximate

$$\Rightarrow d(t) \approx kmu_s(t) + k + x(t)$$

Where the DC voltage (k) term issued from the transmitted carrier is blocked.



Phasor diagram to analyze the envelope detector

Hence, the average power of the demodulated signal (where the carrier power is excluded) is given by

$$P_{o,AM} = \left(\frac{km}{\sqrt{2}} \right)^2 = \frac{(km)^2}{2}$$

Regarding $N_{o,AM}$ we know that

$$\begin{aligned} G_x(f) &= G_y(f) \\ &= 2 G_n(f + f_c) \\ &= n_0 \end{aligned}$$

The average power of the noise at the output is $N_{i,AM} = \int_{-\infty}^{\infty} G_n(f) df$

$$N_{i,AM} = \int_{-B}^B G_x(f) df$$

$$N_{o,AM} = 2Bn_0$$

When we substitute, the above values in **output SNR** formula

Output SNR = $(\text{SNR})_o$ = Average power of the message at receiver **output** / Average power of noise at receiver **output**

$$\Rightarrow (SNR)_{o,AM} = \frac{K^2 m^2}{4Bn_0}$$

Now let us substitute all values in **Figure of merit** related to AM receiver formula.

$$F = (SNR)_o / (SNR)_R$$

$$F = \left(\frac{K^2 m^2}{4Bn_0} \right) / \left(\frac{K^2 \left[\frac{1}{2} + \frac{m^2}{4} \right]}{2Bn_0} \right) = \frac{m^2}{1 + \frac{m^2}{2}}$$

1- In the case of Under-Modulation ($m < 1$)

The Figure of merit F for AM receiver is less 0.66, where this will be the best choice for F

2- In the case of Critical-modulation ($m = 1$)

The Figure of merit F for AM receiver equals to 0.66; however this will be the critical choice for F.

3- In the case of Over-modulation ($m > 1$)

The Figure of merit F for AM receiver is more than 0.66, where this is the critical choice for F

5.3 SNR in FM System

As a recall we have [39- 41]

$$S(t) = A \cdot \cos(\theta(t))$$

The instantaneous frequency f_i is

$$f_i(t) = f_c + K_f \cdot m(t)$$

Where K_f is the sensitivity factor of the modulator and it is expressed in $\text{Hz} \cdot \text{V}^{-1}$.

The instantaneous phase is

$$\omega = 2\pi \cdot f_c + 2\pi \cdot K_f \cdot m(t)$$

The instantaneous phase of the modulated signal is

$$\theta(t) = \int \omega dt = 2\pi f_c t + 2\pi K_f \cdot \int m(t) dt$$

$$\theta(t) = 2\pi f_c t + \phi(t)$$

Hence, the modulated signal is expressed by

$$S(t) = A \cdot \cos(2\pi f_c t + 2\pi K_f \cdot \int m(t) dt)$$

The modulating signal is assumed to be in this form: $m(t) = V_m \cdot \cos(\omega_m t)$

$\Delta f = K_f \cdot V_m$ is the frequency excursion.

The modulated signal is given by

$$S(t) = A \cdot \cos\left(2\pi f_c t + \frac{\Delta f}{f_m} \sin(w_m t)\right)$$

$\beta = \frac{\Delta f}{f_m}$ is the modulation index.

We know that $\phi(t) = 2\pi K_f \cdot \int m(t) dt = \beta \sin(w_m t)$ (*)

The FM signal power in the demodulator input is

$$P_i = \frac{A^2}{2}$$

From (*) we derivate both sides with respect to time and solving for $m(t)$, we conclude that $m(t)$ can be written by

$$m(t) = \frac{\beta f_m}{K_f} \cos(w_m t)$$

The message signal power in the demodulator output is

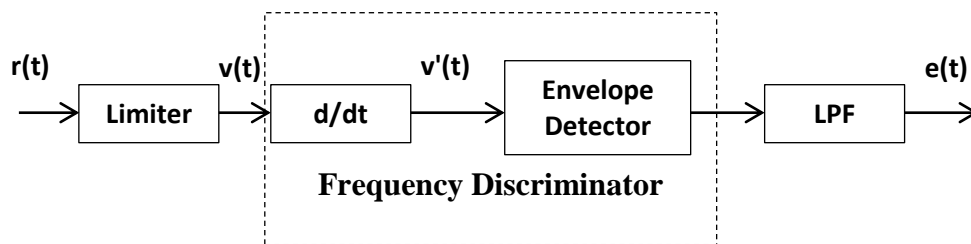
$$S_o = \frac{1}{2} \left(\frac{\beta B}{K_f} \right)^2$$

Where $B = fm$ which is the modulating signal's bandwidth.

Now, we will find the noise power at the output N_o .

The FM system is nonlinear modulation, which means the superposition doesn't exist. However, it can be shown that for high SNR, the noise output is approximately independent of the message; this is because all non-linear systems in smooth variation are locally linear. For that, we only consider the carrier signal and noise signals.

In order to simplify the noise analysis in FM demodulator, let us assume $m(t)=0$ and consider the frequency discriminator as shown in the following Figure:



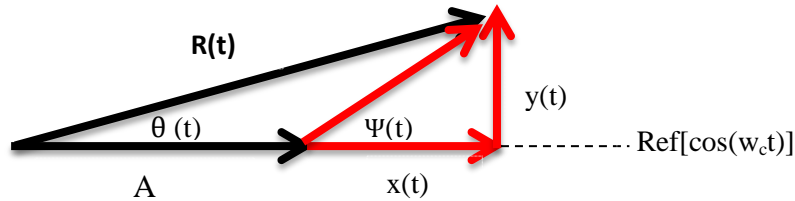
The input signal at the limiter is

$$r(t) = A \cos(2\pi f_c t) + n(t)$$

$$r(t) = [x(t) + A] \cos(2\pi f_c t) - y(t) \sin(2\pi f_c t)$$

$$\begin{aligned}
 &= \operatorname{Re} \left\{ \sqrt{[x(t) + A]^2 + y(t)^2} e^{j\theta(t)} \right\} \\
 &= \sqrt{[x(t) + A]^2 + y(t)^2} \cos[2\pi f_c t + \psi(t)] \\
 r(t) &= R(t) \cos \theta(t)
 \end{aligned}$$

Where $\theta(t) = \tan^{-1} \frac{y(t)}{x(t) + A}$



Phasor diagram to analyze the envelope detector for FM demodulator

The signal $v(t)$ at the output of the limiter is

$$v(t) = K \cos \theta(t)$$

Putting $K=1$ and making the derivative of $v(t)$, the output of the differentiator is

$$v'(t) = - \left[2\pi f_c + \frac{d\psi(t)}{dt} \right] \sin[2\pi f_c t + \psi(t)]$$

So, after passing the envelope detector, the signal at the output of the low-pass filter $e(t)$ becomes

$$e(t) = 2\pi f_c + \frac{d\psi(t)}{dt}$$

Where f_c here represents a power quantity (DC voltage) not a frequency value, therefore it can be removed using a blocking capacitor.

While the variation of $\theta(t)$ is directly related by $\psi(t)$ than we can write

$$\frac{d\psi(t)}{dt} = \dot{\psi}(t) = \frac{\frac{dy(t)}{dt} [x(t) + A] - \frac{dx(t)}{dt} y(t)}{y(t)^2 + [x(t) + A]^2}$$

In the case of large SNR at the input of the FM demodulator, we can write $\dot{\psi}(t)$ as follows

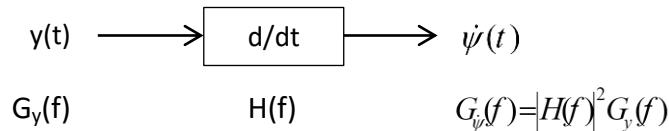
$$\dot{\psi}(t) = \frac{1}{A} \frac{d\psi(t)}{dt}$$

It is well seen that the noise phase $\psi(t)$ is represented by the quadrature component $y(t)$ after passing by the differentiator which has the following transfer function:

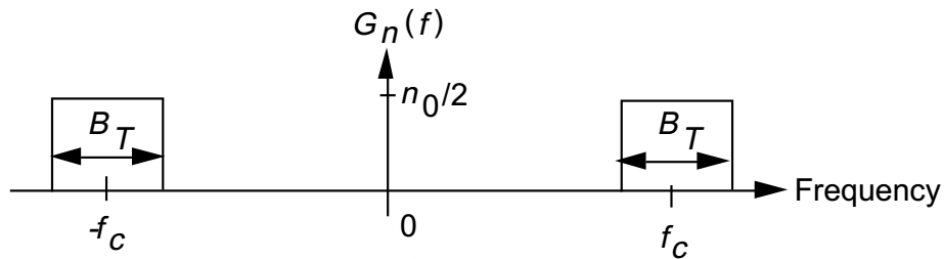
$$H(f) = \frac{2\pi f}{A} j$$

Where $F\left[\frac{d\psi(t)}{dt}\right] = H(f)Y(f)$

The transfer function related to a differentiator in an FM receiver given by the following figure:



Let us put $G_n(f) = G_x(f) + G_y(f)$ as the power spectral density (PSD) of the narrowband noise $n(t)$ as shown in the following figure:



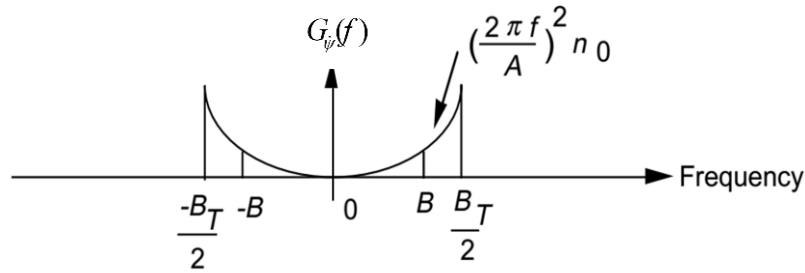
Where the average noise power at the differentiator input is

$$N_i = 2n_0 B T$$

The noise power spectral density at the output of the differentiator is given by

$$\begin{aligned} G_{\psi}(f) &= |H(f)|^2 G_y(f) \\ &= \left[\frac{2\pi f}{A} \right]^2 G_y(f) \\ &= 2 \left[\frac{2\pi f}{A} \right]^2 G_n(f + f_c) \\ &= \left[\frac{2\pi f}{A} \right]^2 n_0 \end{aligned}$$

The following figure shows the PSD of Narrowband noise at the output



Consequently, the average noise at the output of LPF with a bandwidth B is

$$N_o = \int_{-B}^B G_{\phi}(f) df = \frac{2(2\pi)^2 n_0}{3A^2} B^3$$

Finally, the output signal-to-noise ratio SNRo is

$$\frac{S_o}{N_o} = \frac{1}{2} \left(\frac{\beta B}{Kf} \right)^2 \frac{3A^2}{2(2\pi)^2 n_0 B^3}$$

$$\frac{S_o}{N_o} = \frac{\beta^2 3A^2}{4(2)K_f^2 \pi^2 (2n_0 B)} = \frac{\beta^2 3}{4K_f^2 \pi^2} \frac{S_i}{N_i}$$

Where $\frac{S_i}{N_i} = \frac{A^2/2}{2n_0 B}$

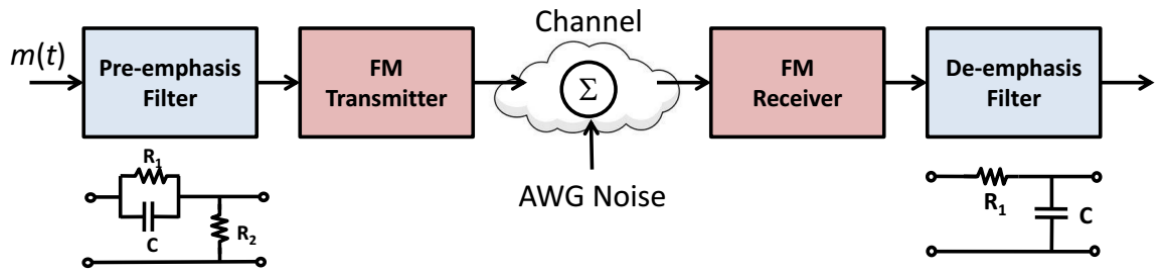
This implies that the **Figure of merit (F)** related to FM demodulator is

$$F = (SNR)_O / (SNR)_R$$

$$F = \frac{3\beta^2}{4K_f^2 \pi^2}$$

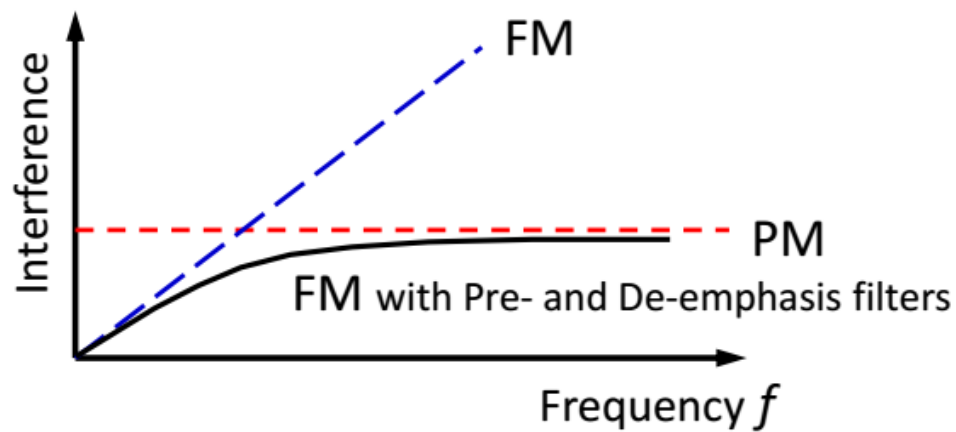
6 Pre-Emphasis and De-Emphasis in FM system

The AWGN noise is considered as interference in FM system, where it is uniform over the entire bandwidth. In order to improve SNR in FM transmission in some cases like the voice and the music that have more energy at lower frequencies, it is better to “**emphasise**” their upper frequencies through “**Pre-emphasis Filter**”. Nevertheless, at the receiver, the HF emphasis must be removed by inserting a “**De-emphasis Filter**”.



7 Comparisons between angular modulation and AM modulation

Angular modulation	AM modulation
<ul style="list-style-type: none"> The amplitude of carrier remains constant during the modulation The phase or frequency of the carrier change with modulation The phase or frequency of the carrier change according to the strength of the modulating signal The value of modulation index cannot be more than 1 AM modulation is less sensitive to AWGN noise, however the phase modulation is more sensitive to phase noise. 	<ul style="list-style-type: none"> The amplitude of carrier changes with modulation The carrier frequency remains constant with modulation The carrier amplitude changes according to the strength of the modulating signal The value of modulation index cannot be more than 1 AM modulation is more sensitive to AWGN noise and less sensitive to phase noise.



The End of Chapter V

CH. VI

Superheterodyne Receivers

VI

CHAPTER

Superheterodyne Receivers

In radio frequency systems, a heterodyne receiver is a receiver designed on the principle of frequency mixing, or heterodyning, to convert the received signal into a lower intermediate frequency which is easier to be exploited rather than the received frequency. In general, all radio receivers and television work on superheterodyne principle.

The word "superheterodyne" is made up of Greek, hetero: "different" and dyne: "power". The term "Heterodyne" refers to a beating produced by two radio carrier signals applied on a receiver.

1 Direct Amplification Receiver

Until the 1930s, the radio reception was made up by direct amplified receivers. This latter receiver works on the principle of the following figure. However, this structure was then avoided in favour of a more complex structure which is the superheterodyne Receiver [47]. The aim of this course is to understand the advantages of **superheterodyne receivers** compared with **direct amplification structures**.

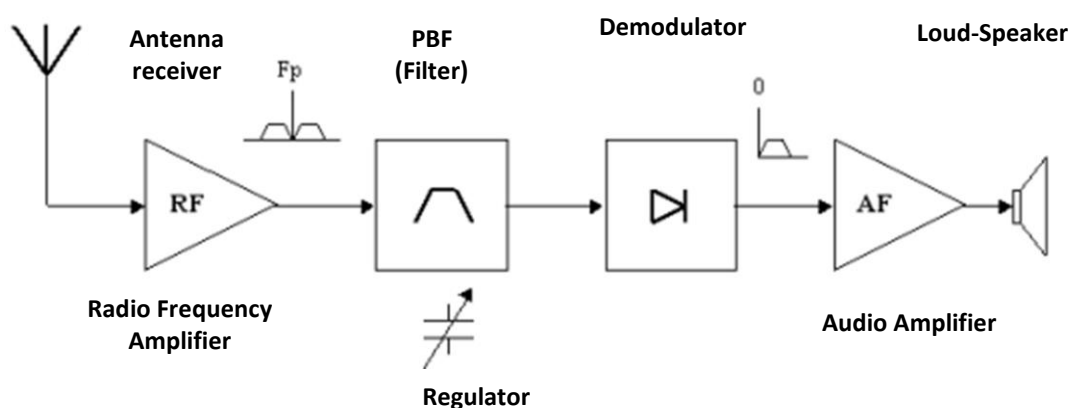


Diagram of a direct amplification receiver

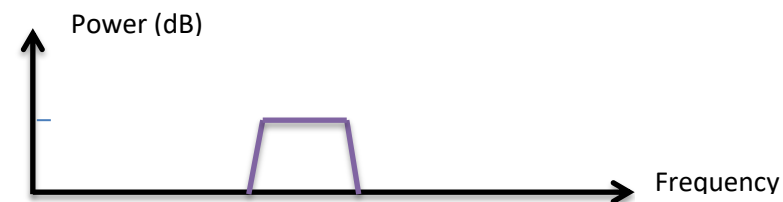
The power spectral density PSD of before and after the direct amplification can be described by the following figures



PSD of the signal at the received antenna



PSD of the signal after the direct amplification



PSD of the signal after the Pass-Band Filter (PBF)



PSD of the signal after the demodulator



PSD of the signal after the Audio Amplifier (AF)

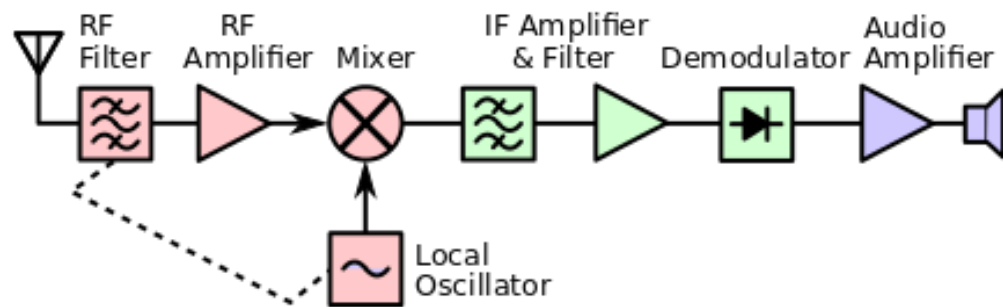
2 Superheterodyne Receivers

Based on what we have said in the head of this chapter, any receiver especially the radars are used to detect weak signals received by the antenna, then to amplify them enough to extract the useful information. Thus, this device must be able to extract signals that presents weak magnitude than the signal transmitted by the radar and to amplify them by a factor of 20 to 30 million times. This is a hard task at the original frequency of the radar carrier signal, and this is why receivers use a process called "SUPERHETERODYNE" which transforms the received signal at an intermediate frequency before amplifying it.

2.1 Minimum Detectable Signal (MDS)

The minimum detectable signal for a receiver is an important characteristic because it determines the maximum range of the radar. For a typical receiver, its sensitivity is around 10^{-13} Watts (–100 dBm), which is called the minimum detectable signal MDS.

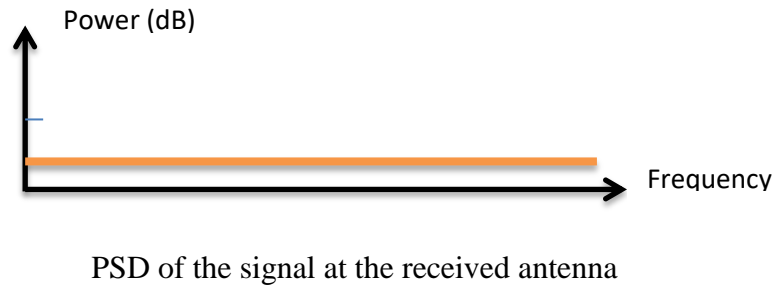
All receivers are built with an MDS which corresponds to the conditions of use. The MDS must not higher than necessary as this would limit the reception bandwidth and require processing of weak signals that are not significant. Nevertheless, the higher the minimum threshold of MDS, the lower the rate of false alarms in target detection.



Synoptic diagram for Superheterodyne Receivers

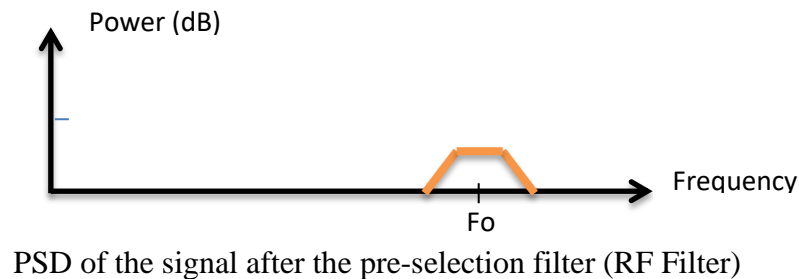
The above figure shows the diagram of the components of a superheterodyne receiver. The radio frequency (RF) signal coming from the antenna passes through a filter which allows only the range of desired frequencies. The signal then passes through a mixer which adds to it a local signal produced by a stable local oscillator. The two signals enter in beat at an intermediate frequency (IF) which is the frequency difference between the two signals, so it is the process of frequency change, called "heterodyne". The signal coming from the local oscillator LO is automatically adjusted so as to always have the same frequency difference with that of the RF signal, which gives a constant IF over the entire frequency range of the receiver. The IF signal is then sent to an amplifier which

increases its intensity. Finally, it goes into a detector to be demodulated and having the final signal of low radio frequency [48].



2.2 Pre-selection Filter

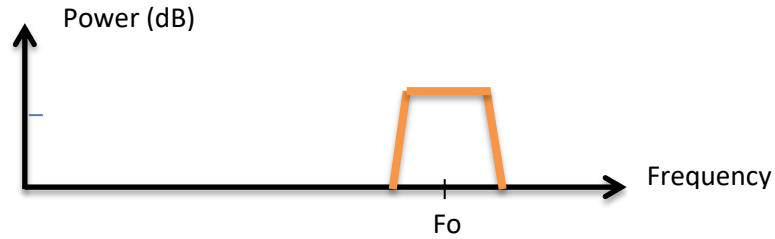
After receiving the signal containing the frequency we want to tune, where it is surrounded by other undesired frequencies. An antenna filter (pre-selection filter) placed before the amplifier, eliminates unwanted frequencies in order to prevent possible signals of high amplitude from saturating the high frequency (HF) amplifier. The limits of this filter are chosen in order to eliminate any spurious or fake image frequency. The receiver bandwidth should not be greater than the intermediate frequency (IF).



2.3 RF Amplifier (Low Noise Amplifier, LNA)

The RF amplifier of high frequency provides a first amplification. It is designed to obtain the best possible signal-to-noise ratio through amplifying the signal without amplifying a significant noise. Thus it introduces the least amount of internal noise for better reception.

The old receivers did not use this kind of filters and pre-amplifiers, where it sends the received signal directly to the mixer (multiplier). This presented some disadvantages, like receiving signals with spurious frequencies from different sources.



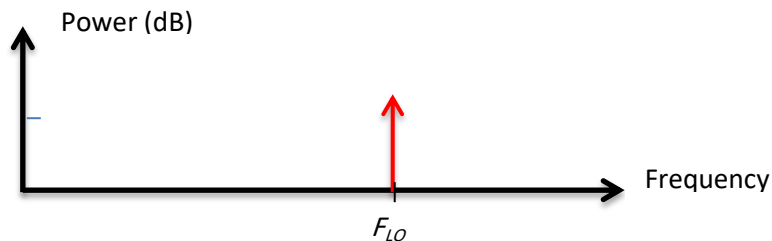
PSD of the signal after the RF Amplifier (Pre-amplifier)

The frequencies of the pre-selection filter and the local oscillator must be modified at the same time so as to keep the resulted frequency band that contains the intermediate frequency IF.

2.4 Mixer and Local Oscillator

The frequencies of the pre-selection filter and the local oscillator must be modified at the same time so as to keep the resulted frequency band constant, where that band contains the intermediate frequency IF [47]. Consequently, an Automatic Frequency Control (AFC) must be inserted in the local oscillator to ensure the said task.

In this way if the difference between F_o and F_{LO} is constant, the subsequent operations will be carried out at the intermediate frequency FI.



PSD of Local Oscillator

The mixer or the multiplier combines the pre-amplified signal of F_o and the signal produced by the local oscillator F_{LO} . These two signals beat at double intermediate frequencies (IFs) by frequency change.

$$F_{FI} = F_{LO} - F_o$$

$$F_{FI} = F_o - F_{LO}$$

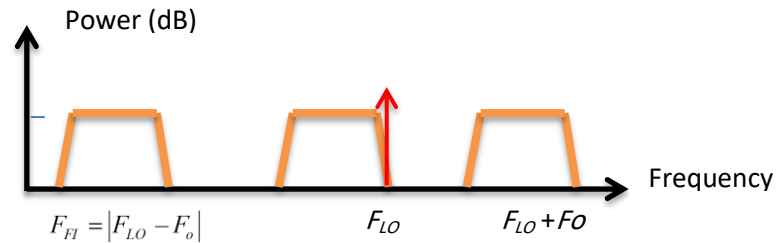
The output spectrum is therefore composed of two side bands, positive and negative, identical. The measurement can only be done on the absolute amplitude of the signal, in general on the following difference

$$F_{FI} = |F_{LO} - F_o|$$

The mixer allows carrying out a frequency transposition as follows

$$\cos(2\pi f_1) \cdot \cos(2\pi f_2) = \frac{1}{2} \cos(2\pi(f_1 + f_2)) + \frac{1}{2} \cos(2\pi(f_1 - f_2))$$

However, the mixer is a non-linear component like the multiplier, so the input frequencies might be found at the output without any mathematical logic, hence the following power spectrum:



PSD of the signal at the mixer output

2.5 IF Filter

The intermediate frequency filter allows keeping only the desired frequency of the mixer output signal. It is designed to have one or more narrow bandwidths which do not affect the signal power [48].



PSD of the signal at the IF Filter output

2.6 IF Amplifier

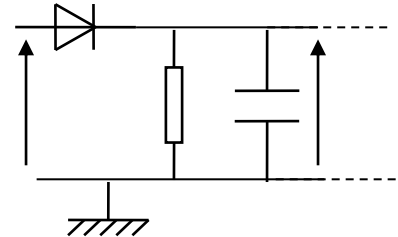
After the transposition to IF, the signal goes through a series of amplifiers to allow exploiting it. This intermediate frequency amplification step must be adapted in order to be able to vary the bandwidth and the gain of the receiver. The receiver bandwidth is defined by that of these amplifiers and the gain must be variable in order to obtain a constant voltage at the output for all parts of the signal. Thus, the Automatic Gain Control (AGC) is a method that allows automatically controlling the gain in the amplification stages of a radio receiver so that the output signal remains relatively insensitive to variations in reception intensity [47, 48].



PSD of the signal at the IF Amplifier

2.7 Demodulator

The simplest demodulator is a diode circuit which is an envelope detector such as in AM, where it is possible to have a detector which extracts other types of modulation.



PSD of the signal at the output of the demodulator

2.8 AF Amplifier

The audio frequency amplifier receives the signal from the detector and increases its power so that it is possible to stimulate the membrane of the loudspeaker.



PSD of the signal at the output of the AF Amplifier

4 Problem of Image Frequency

This problem may occur when we fall in a case where we have another F_o' in the signal bandwidth such as $F_{FI} = |F_{LO} - F_o'|$ which gives the same F_{IF} but it is considered as fake IF.

Consequently, the unwanted frequency F_o' , called the **IMAGE FREQUENCY** and must be outside the bandwidth of the IF filter.

5 Applications of Superheterodyne Receivers

- Obviously in the Radio:

AM modulation: FI = 455kHz

FM modulation: FI = 10.7 MHz

- Television for the sound, IF = 39.2 MHz and for the image, IF = 32.7 MHz

- RADAR receiver

- Bioacoustics: in zoology where it is possible to transpose the ultrasounds emitted by some animals into the audible frequency range to identify species more easily

- Radio astronomy (high frequency resolution) for interferometry in order to measure vibrations, displacement.

- Optical superheterodyne detection: transpose a lower frequency part of the spectrum [47].

The End of Chapter VI

CH. VII

Phase-Locked Loop (PLL)

VII

CHAPTER

Phase-Locked Loop (PLL)

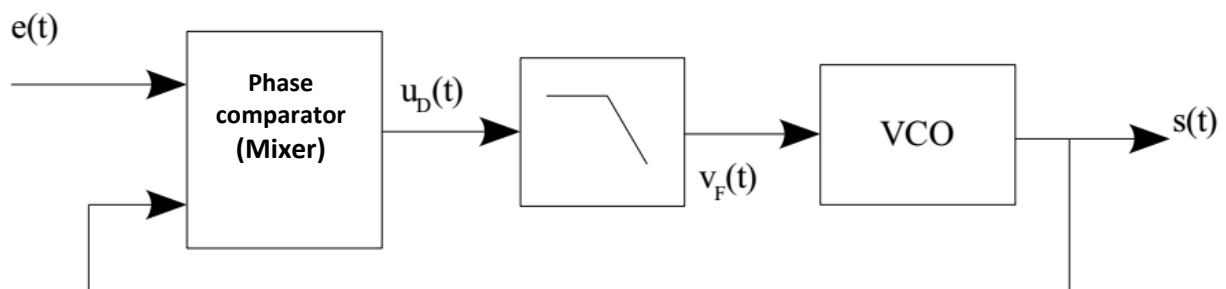
Phase-Locked Loops or PLL were invented by De Bellescize in 1932 to make synchronous detection. The implementation of this principle was delicate taking into account the means of the time, which explains why its generalization had to await the progress of technology.

After the appearance of integrated circuits, the use of phase locked loops expanded considerably in all fields of communications. We can distinguish two types of applications:

The phase locked loop is used in many domains such as oscillators, modulators, demodulators, band pass filter, clock signal recovery circuits, synthesizers, and the list goes on.

The object of such a device is to synchronise the signal of a frequency oscillator with a reference input signal, the synchronisation being ensured by an auto-tracking of the phase of the signals.

PLL is therefore a looped system which produces a variable voltage $s(t)$ whose phase is controlled by that of the variable voltage applied at the input $e(t)$. In other words, the input frequency of a reference signal is tracked by the frequency of the VCO (Voltage Controlled Oscillator) in a range around of the free running frequency F_0 . The PLL consists of a VCO, a low-pass filter, and a phase comparator (it is just a multiplier) [44].



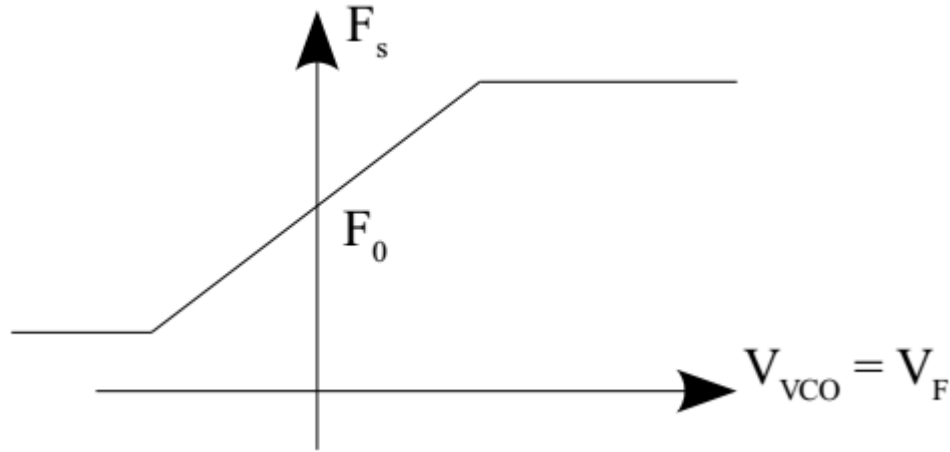
1 PLL with a fixed frequency (sinusoidal signal)

Let's assume that the VCO has the following voltage-frequency characteristic $F_s = f_{ct}(V_f)$ where $F_s(t) = F_0 + K_{VCO} \cdot V_f(t)$

F_0 is called the free running frequency of the VCO.

K_{VCO} is the gain of VCO.

The form of the relationship between F_0 and K_{VCO} depends on the type of VCO used in the integrated circuit and varies from VCO to another [44].



We consider that the multiplier has the characteristic

$$u_D(t) = G_M e(t) s(t)$$

G_M is a gain related to the multiplier

The Input voltage is $e(t) = \hat{E} \cdot \cos(2\pi F_e t + \varphi_e)$

The output voltage is $s(t) = \hat{S} \cdot \cos(2\pi F_s t + \varphi_s)$

At the output of the multiplier there are two spectral components, one reflects the sum of two frequencies and the other one reflects the difference between the two frequencies that are inserted at the input of the multiplier. The low-pass filter removes the sum component (high frequencies), this is its role. We will therefore only be interested in the difference of frequencies.

For the seek of convenience, and in order to understand the coming examples, let's take

$F_0 = 1\text{MHz}$, $G_M = 0.1 \text{ V}^{-1}$,

F_c (filter cutoff frequency, first order) = 30kHz

1.1 PLL Unlocked

At first time, let us put that the signal $e(t)$ has a frequency of 100 kHz. Assume that at this instant the VCO oscillates at its free running frequency 1Mhz. The multiplier (phase detector) output

therefore includes two frequencies (1.1 MHz, and 900 kHz) which will be filtered, so the input voltage of the VCO is $V_f(t) = 0V$.

In this case, we say that the PLL is unlocked. Where no control can be done, the input and output frequencies have nothing to do [44].

1.2 PLL Locked

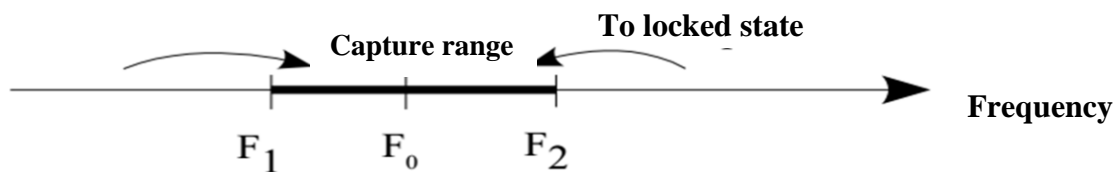
We assume now that the input signal has a frequency of 1.010 MHz. If the VCO is on its normal free frequency 1Mhz, At the output of the mixer (another name for a multiplier), we will have 2.01 Mhz (filtered) and 10 kHz. The input voltage $V_f(t)$ of the VCO is no longer zero.

The output frequency temporarily oscillates around the free running frequency, so as to finally freezing (capture) at the input frequency, 1.010 Mhz. The frequency difference (the frequency beat) becomes zero. The input voltage $V_f(t)$ of the VCO is then continuous voltage, it corresponds to the input frequency where $F_e = F_s$. The PLL is then locked.

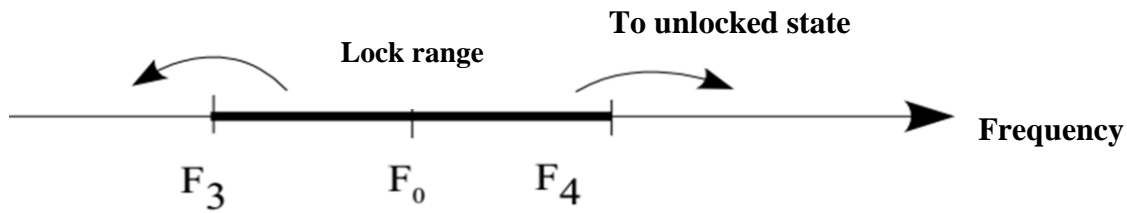
In this case, we say that the PLL is locked, where the PLL circuit is looped [44].

1.3 Capture range and hold (lock) range

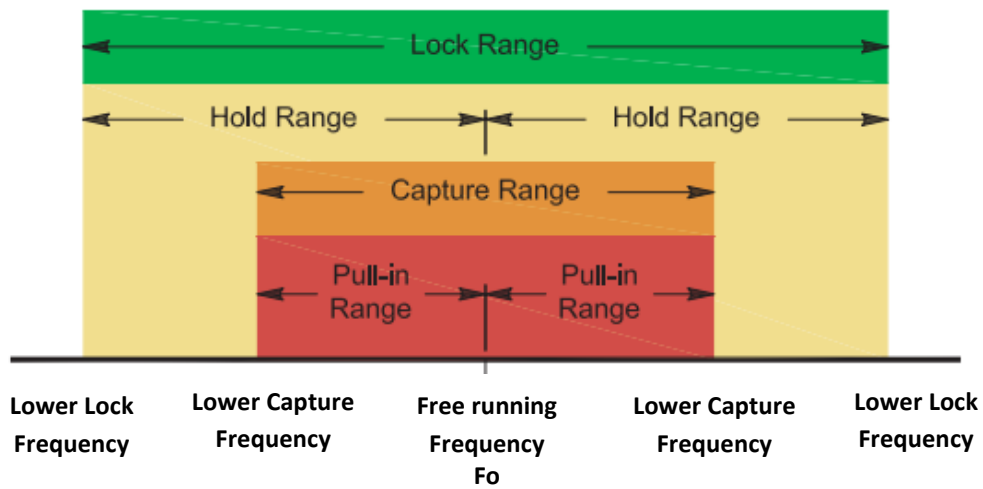
The capture range is the range between two frequencies (F_1 , F_2) for which the PLL moves from an unlocked state to a locked state, where F_1 is lower than the free running frequency, and F_2 is higher than it. The parts of the capture range (above and below F_0) are named the pull-in range. We note that the pull-in ranges are not necessarily symmetrical.



Now, when the PLL is locked (PLL works between F_1 and F_2), thus the input frequency related to $e(t)$ may vary, the output frequency related to $s(t)$ will follow it. However, if the variation is too big (variation for which the difference between f_e and f_s gets out the filter bandwidth), in this case the PLL will be unlocked [44].



The **hold range** is therefore the frequency range for which the PLL is locked but with the limits of the two frequencies (F3 and F4) leading to the unlock state of the PLL. Obviously, the lock range is greater than or equal to the capture range.



Capture and Lock range for the PLL [45]

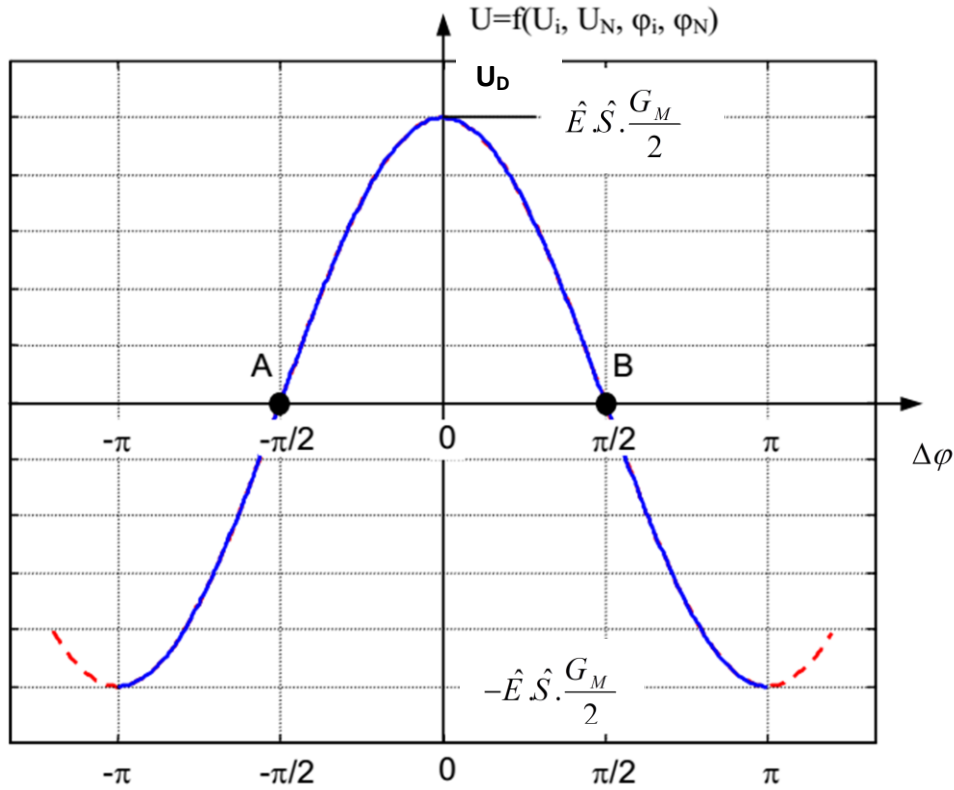
1.4 Estimation of the hold (lock) range

The unlocking of the PLL is due to a saturation of either the VCO or the multiplier (called mixer or phase comparator). In the case of VCO, the holding or the locking range corresponds to the operating range of the VCO.

In the case where the phase comparator is saturated (the most frequent case), this is a little more complicated. When the PLL is locked, we saw that the voltage of the VCO (V_{vco}) is continuous (Dc voltage), where the filter eliminates the component at $F_e + F_s$. As it is only the frequency beat which influences the VCO, we will only consider the following:

$$u_D(t) \approx V_f(t) = \hat{E} \hat{S} \cdot \frac{G_M}{2} \cdot \cos(2\pi F_e t + \varphi_e - 2\pi F_s t - \varphi_s)$$

$$u_D(t) \approx V_f(t) = \hat{E} \hat{S} \cdot \frac{G_M}{2} \cdot \cos(\varphi_e - \varphi_s) = \hat{E} \hat{S} \cdot \frac{G_M}{2} \cdot \cos(\Delta\varphi)$$



The characteristic the phase comparator [46]

Consequently, the hold range is

$$\left[F_o - \hat{E} \hat{S} \cdot \frac{G_M}{2} K_{vco}, F_o + \hat{E} \hat{S} \cdot \frac{G_M}{2} K_{vco} \right]$$

It is shown that the characteristic of the multiplier presents a cosine function which becomes zero

$$V_f(t)=0 \text{ for } \Delta\varphi = + / - \frac{\pi}{2} .$$

When the loop is locked and the phase difference between the two input and return signals is $+ / - \frac{\pi}{2}$

If the loop is of positive sign, the lock point will be at A ($-\pi / 2$), because at this point, a positive variation of the phase shift causes a positive variation of the error voltage $U_d(t)$. On the other hand, if the loop is of negative sign, the lock point will be at B ($\pi / 2$). One of these points is therefore necessarily stable and the other unstable; the choice will be made automatically.

The operation of this type of phase detector is only linear if we work around (point A or B).

Let's take the right side of the curve, if the VCO's control signal $V_f(t)$ is proportional to the cosine of the phase comparator, it is equal to zero when the phase difference is $\pi / 2$. It clear that the maximum values are when the input and the output signals are in phase 0 or out of phase π , between

these values, the PLL can keep the input signal and VCO signal locked together. At this stage, when the phase difference gets closer to 0 or π , the input frequency moves farther and farther from F_0 , so the control action can keep the VCO frequency at the same input frequency.

When the phase difference falls in the case of 0 or π , any additional change will let the control signal $V_f(t)$ move back towards $\pi / 2$ and the VCO signal away from the input signal. Hence, the PLL is no longer locked [44-46].

The range of values of input frequencies at which the PLL is locked with a phase difference of 0 and π is named the **lock range** of the PLL, where the lock range above and below F_0 are named the **hold ranges** of the PLL. The lock range is not necessarily centred on F_0 .

1.5 Capture range estimation

It is difficult to estimate the capture range of a PLL because the capturing is a complex non-linear phenomenon.

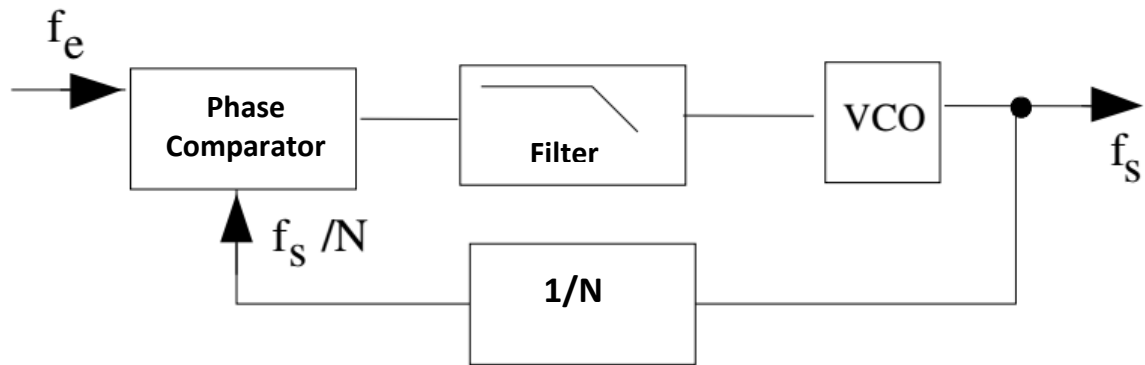
However, if we imagine a cardinal selective filter (ideal, infinite slope); it is clear that if the frequency beat enters the filter bandwidth, the PLL locks, otherwise it does not catch. The limit is therefore directly fixed by the bandwidth of the filter. In other words, the capture range will be $[F_0 - F_c, F_0 + F_c]$. We will therefore base on the results of an ideal filter, while knowing that the range will necessarily be greater for a real filter [44].

2 PLL in dynamic regime (variable frequency) **Search more to learn more**

3 Frequency synthesizers

The utility of synthesizing or generating a frequency in the radio domain is to select the station to listen to. For this, a so-called heterodyne structure is used in which the frequency of the local oscillator indirectly determines the desired station. Thus, the local oscillator is a frequency synthesizer.

When we inset a frequency divider, la PLL can easily perform the frequency multiplication function. Hence, when the loop is locked $f_e = f_s / N$ which implies that $f_s = N f_e$. This multiplier is used as a basic element in the production of high quality frequency synthesizers using programmable dividers and a quartz as a reference frequency [44].



If the user has access to the value of N , so he can control as he wants the value of f_s and therefore develop the frequency of his choice.

The End of Chapter VII

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