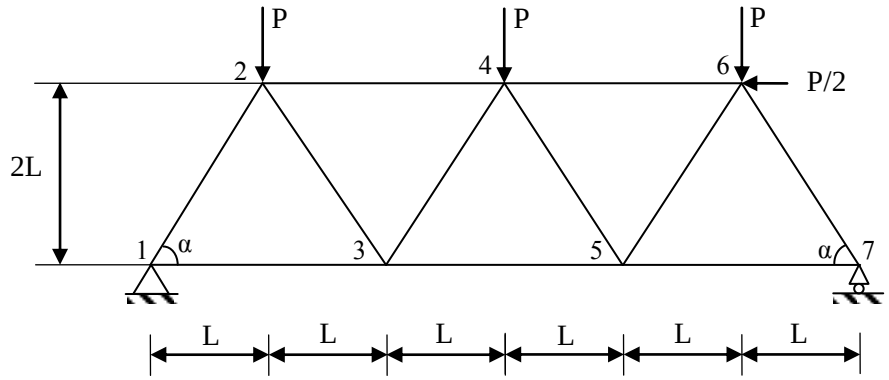
الأنظمة الشبكية المقررة ستاتيكيا
Systèmes en Treillis isostatiques

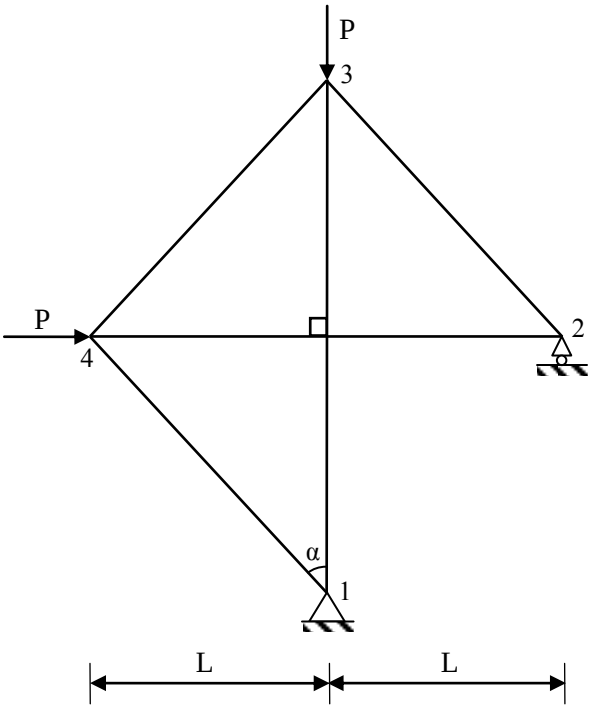
التمرين 01 :

بواسطة طريقة العقد قم بحساب القوى داخل القضبان للجملة الشبكية المبينة في الشكل أسفله.



التمرين 02 :

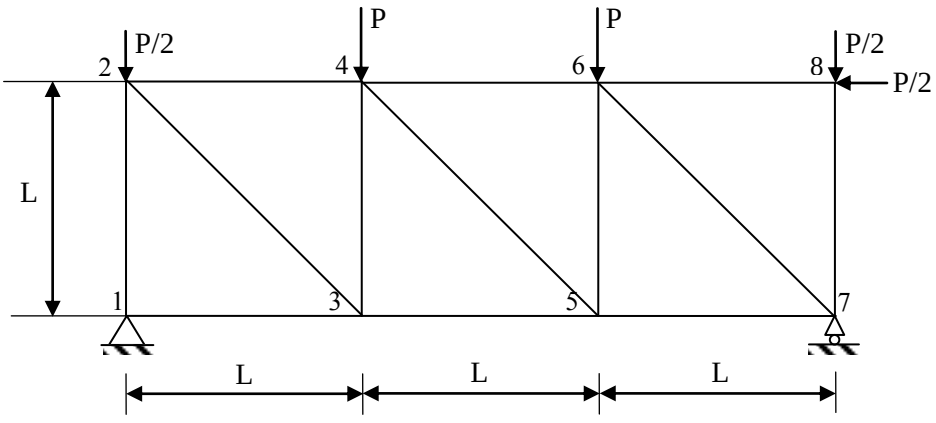
أحسب القوى داخل القضبان للنظام الشبكي المبين في الشكل المقابل.



التمرين 03 :

نعتبر النظام الشبكي المبين في الشكل أسفله.

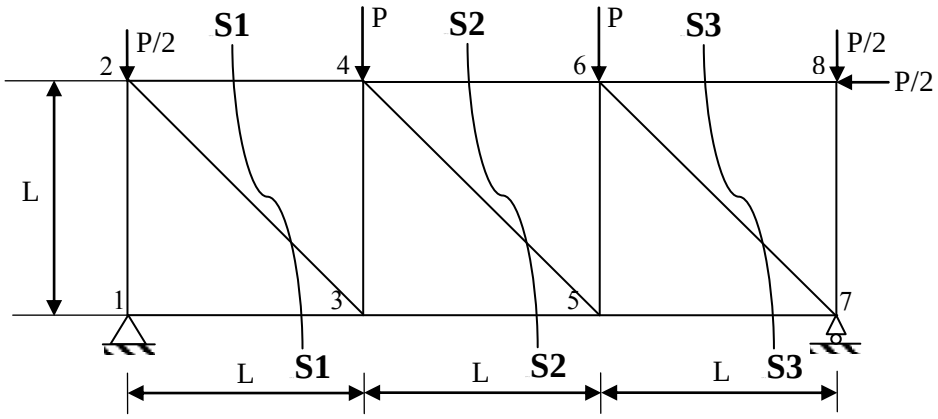
المطلوب حساب القوى الداخلية من أجل كل القضبان.



الأنظمة الشبكية المقررة ستاتيكيا
Systèmes en Treillis isostatiques

التمرين 03 :

حساب القوى الداخلية في المنظومة الشبكية بطريقة المقاطع.



حساب ردود الافعال :

$$\sum F_x = 0$$

$$H_1 = \frac{P}{2}$$

$$\sum F_y = 0$$

$$V_1 - V_7 = \frac{P}{2} + P + P + \frac{P}{2} = 3P$$

$$\sum M_{/1} = 0$$

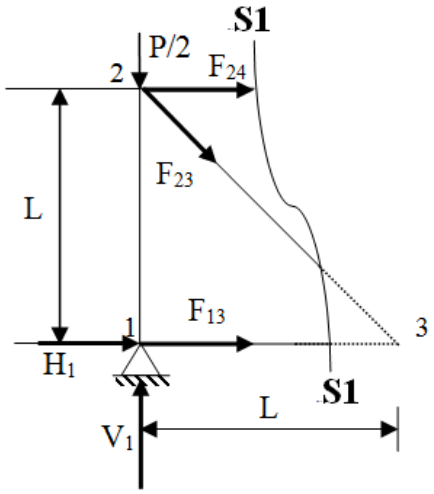
$$V_7.3\ell + \frac{P}{2}.\ell - \frac{P}{2}.3\ell - P.2\ell - P\ell = 0$$

$$3V_7 = -\frac{P}{2} + \frac{3P}{2} + 2P + P$$

$$V_7 = \frac{4P}{3}$$

$$V_1 = \frac{5P}{3}$$

المقطع S1-S1 : (الجزء الأيسر)



$$\sum F_y = 0$$

$$\frac{5}{3}P - \frac{P}{2} - F_{23}.\cos 45^o = 0$$

$$F_{23} = \frac{7P}{6.\cos 45^o} = \frac{7\sqrt{2}}{6}P = 1,6499P$$

$$\sum M_{/2} = 0$$

$$\frac{P}{2}\ell + F_{13}.\ell = 0$$

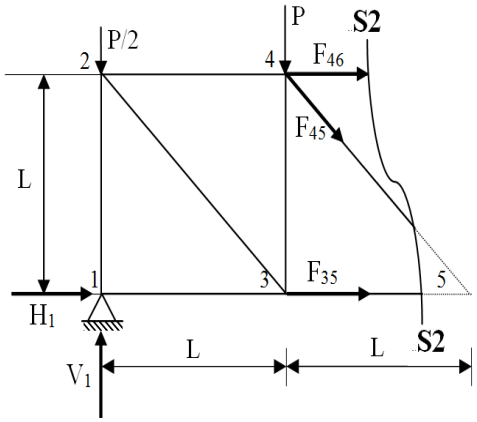
$$F_{13} = \frac{-P}{2}$$

$$\sum M_{/3} = 0$$

$$\frac{5}{3}P.\ell + F_{24}.\ell - \frac{P}{2}\ell = 0$$

$$F_{24} = \frac{-7}{6}P$$

المقطع S2-S2 : (الجزء الأيسر)



$$\sum F_y = 0$$

$$\frac{5}{3}P - \frac{P}{2} - P - F_{45}.\cos 45^o = 0$$

$$F_{45} = \frac{P}{6.\cos 45^o} = \frac{\sqrt{2}}{6}P = 2,2357P$$

$$\sum M_{/4} = 0$$

$$-\frac{5}{3}P.\ell + \frac{P}{2}\ell + \frac{P}{2}\ell + F_{35}.\ell = 0$$

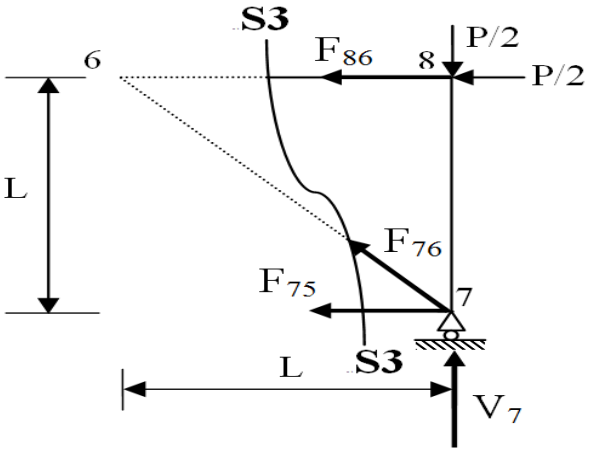
$$F_{35} = \frac{2}{3}P = 0,6667P$$

$$\sum M_{/5} = 0$$

$$F_{46}.\ell + \frac{5}{3}P.2\ell - \frac{P}{2}.2\ell - P.\ell + = 0$$

$$F_{46} = -\frac{4}{3}P = -1,3333P$$

المقطع S3-S3 : (الجزء الأيمن)



$$\sum F_y = 0$$

$$\frac{4}{3}P - \frac{P}{2} + F_{76}.\cos 45^o = 0$$

$$F_{76} = -\frac{5P}{6.\cos 45^o} = -1,1785P$$

$$\sum M_{/7} = 0$$

$$F_{86}.\ell + \frac{P}{2}.\ell = 0$$

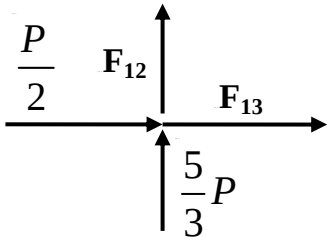
$$F_{86} = -0,5P$$

$$\sum M_{/6} = 0$$

$$\frac{4}{3}P.\ell - \frac{P}{2}.\ell - F_{75}.\ell = 0$$

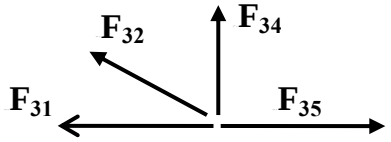
$$F_{75} = \frac{5P}{6} = 0,8333P$$

العقدة 01 :



$$F_{12} = -\frac{5}{3}P$$

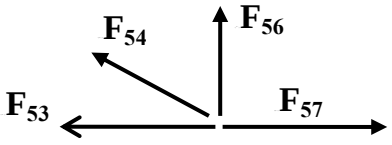
العقدة 03 :



$$F_{34} = -F_{32}.\cos 45^o$$

$$F_{34} = -\frac{7}{6}P$$

العقدة 05 :



$$F_{56} = -F_{54}.\cos 45^o$$

$$F_{56} = -\frac{P}{6}$$

العقدة 08 :

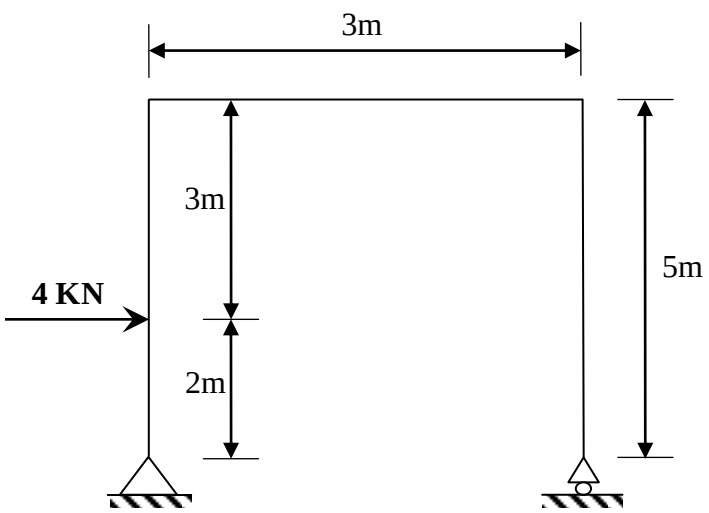
$$\frac{P}{2} + F_{87} = 0$$

$$F_{87} = -\frac{P}{2}$$

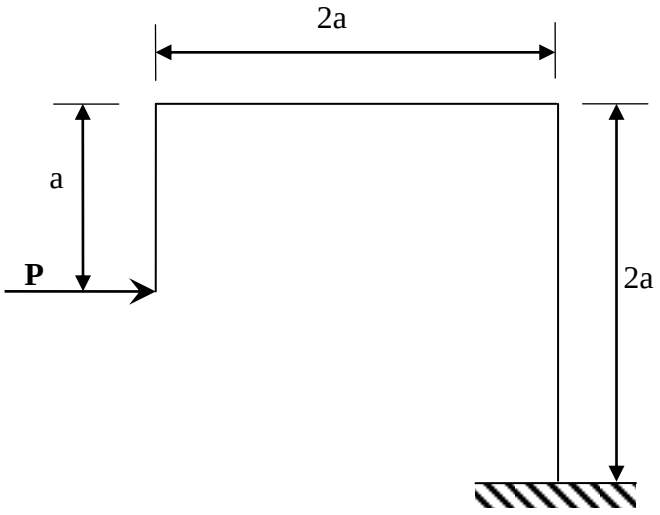
Portiques isostatiques
الهياكل المقررة ستاتيكيا

أدرس الهياكل المتعددة القضبان و المقررة ستاتيكيا المبينة في الأشكال التالية:

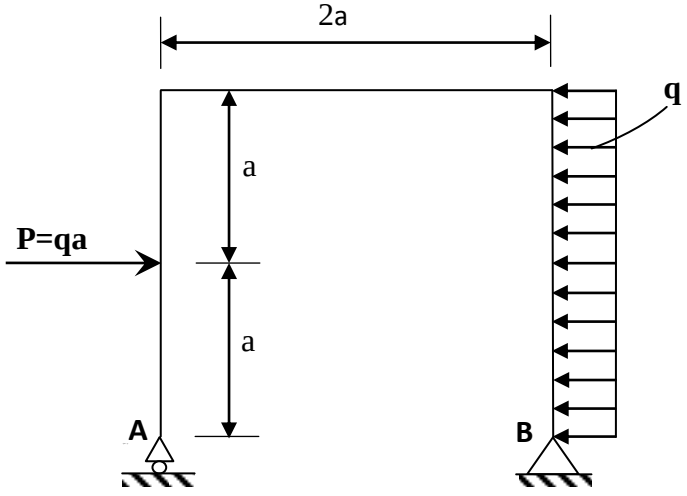
a



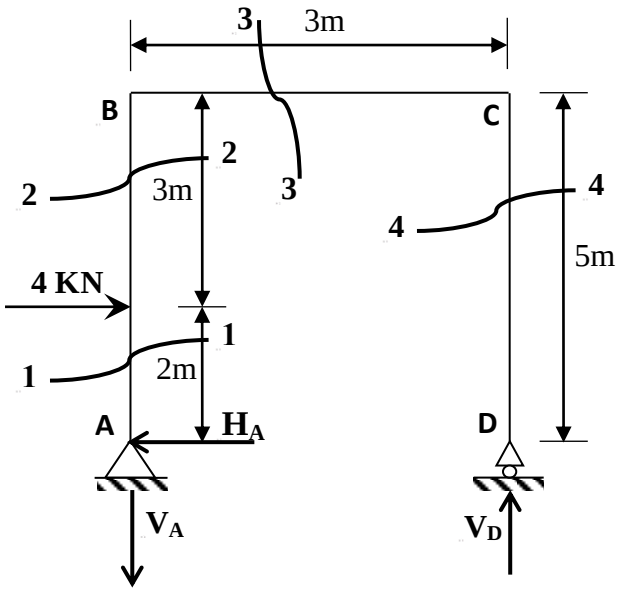
b



c



1- دراسة الهيكل (a):



حساب ردود الافعال :

$$\sum F_x = 0$$
$$H_A = 4KN$$
$$\sum F_y = 0$$
$$V_D - V_A = 0$$
$$\sum M_{/A} = 0$$
$$V_D.3 - 4.2 = 0 \rightarrow V_D = \frac{8}{3}KN$$
$$et \quad V_A = \frac{8}{3}KN$$

المقطع 1-1 : $0 \le x \le 2m$ (الجزء الأيسر)

$$\sum F_x = 0$$
$$N_1 = \frac{8}{3}KN$$
$$\sum F_y = 0$$
$$T_1 - 4 = 0 \rightarrow T_1 = 4KN$$
$$\sum M_{/z} = 0$$
$$M_1(x) - 4x = 0 \rightarrow M_1(x) = 4x$$
$$\begin{cases} M_1(0) = 0 \\ M_1(2) = 8KNm \end{cases}$$

المقطع 2-2 : $2 \le x \le 5m$ (الجزء الأيسر)

$$\sum F_x = 0$$
$$N_2 = \frac{8}{3}KN$$
$$\sum F_y = 0$$
$$T_2 + 4 - 4 = 0 \rightarrow T_2 = 0$$
$$\sum M_{/z} = 0$$
$$M_2(x) - 4x + 4(x-2) = 0 \rightarrow M_2 = 8KNm$$

المقطع 3-3 : $0 \le x \le 3m$ (الجزء الأيسر)

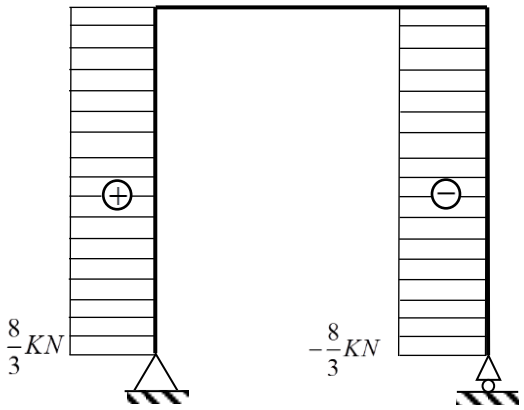
$$\sum F_x = 0$$
$$N_3 + 4 - 4 = 0 \rightarrow N_3 = 0 \rightarrow$$
$$\sum F_y = 0$$
$$T_3 + \frac{8}{3} = 0 \rightarrow T_3 = \frac{-8}{3}KN$$
$$\sum M_{/z} = 0$$
$$M_3(x) + V_A.x - 4.5 + 4.3 = 0$$
$$M_3(x) = -\frac{8}{3}x + 8$$
$$\begin{cases} M_3(0) = 8KNm \\ M_3(3) = 0 \end{cases}$$

المقطع 4-4 : $0 \le x \le 5m$ (الجزء الأيمن)

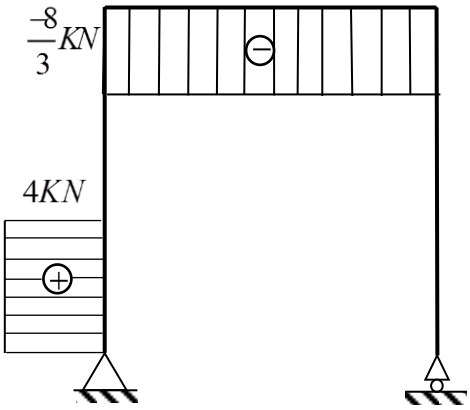
$$\sum F_x = 0$$
$$N_4 = -\frac{8}{3}KN$$
$$\sum F_y = 0$$
$$T_4 = 0$$
$$\sum M_{/z} = 0$$
$$M_4 = 0$$

رسم مخططات M ,T, N :

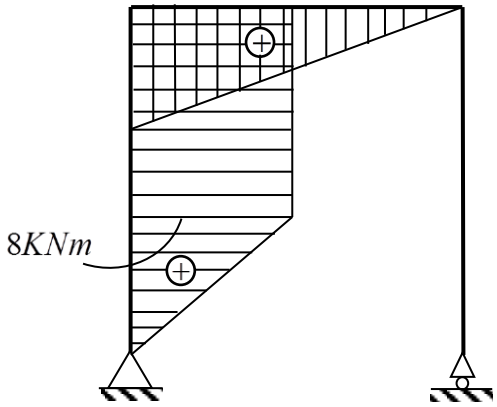
مخطط N :



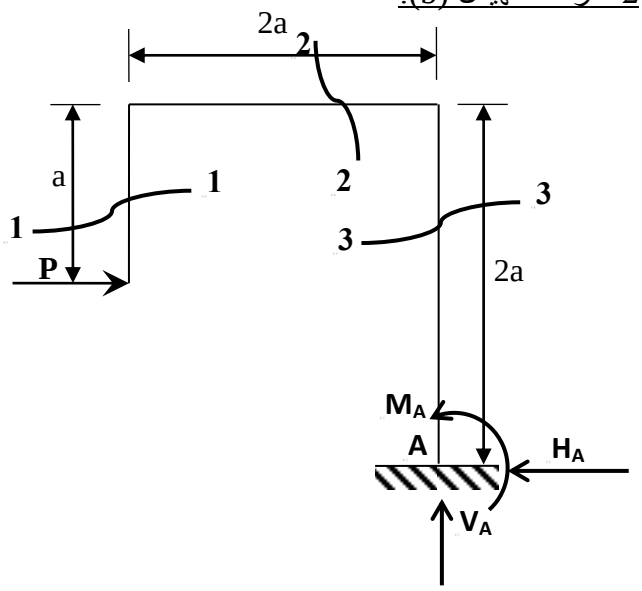
مخطط T:



مخطط M:



2- دراسة الهيكل (b):



حساب ردود الافعال :

$$\sum F_x = 0$$
$$H_A - P = 0 \rightarrow H_A = P$$
$$\sum F_y = 0$$
$$V_A = 0$$
$$\sum M_{/A} = 0$$
$$M_A - Pa = 0 \rightarrow M_A = Pa$$

المقطع 1-1 : $0 \le x \le a$ (الجزء الأيسر)

$$\sum F_x = 0$$
$$N_1 = 0$$
$$\sum F_y = 0$$
$$T_1 + P = 0 \rightarrow T_1 = -P$$
$$\sum M_{/z} = 0$$
$$M_1(x) + Px = 0 \rightarrow M_1(x) = -Px$$
$$\begin{cases} M_1(0) = 0 \\ M_1(a) = -Pa \end{cases}$$

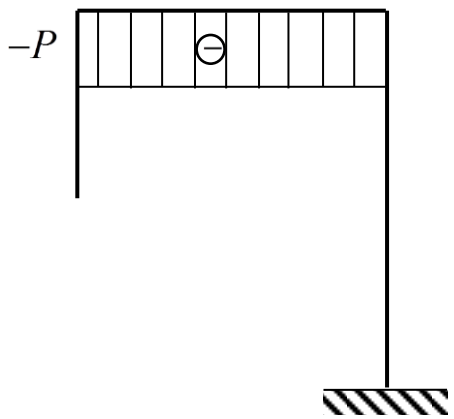
المقطع 2-2 : $0 \le x \le 2a$ (الجزء الأيسر)

$$\sum F_x = 0$$
$$N_2 + P = 0 \rightarrow N_2 = -P$$
$$\sum F_y = 0$$
$$T_2 = 0$$
$$\sum M_{/z} = 0$$
$$M_2 + Pa = 0 \rightarrow M_2 = -Pa$$

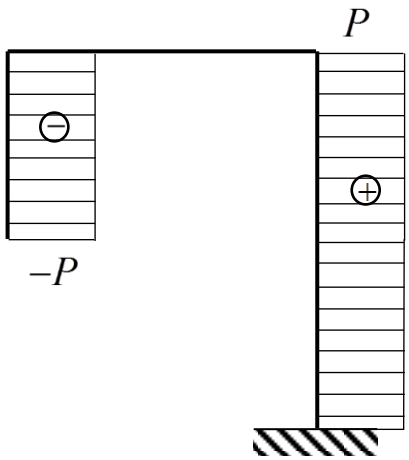
المقطع 3-3 : $0 \le x \le 2a$ (الجزء الأيمن)

$$\sum F_x = 0$$
$$N_3 = 0$$
$$\sum F_y = 0$$
$$T_3 - P = 0 \rightarrow T_3 = P$$
$$\sum M_{/z} = 0$$
$$M_3(x) - M_A + P(2a - x) = 0$$
$$M_3(x) = -P(2a - x) + Pa$$
$$\begin{cases} M_3(0) = -Pa \\ M_3(a) = Pa \end{cases}$$

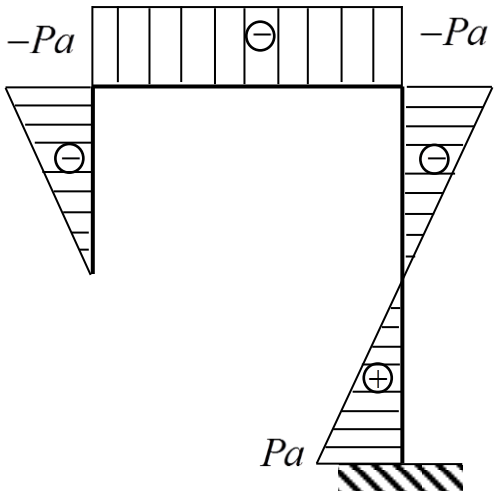
مخطط N :



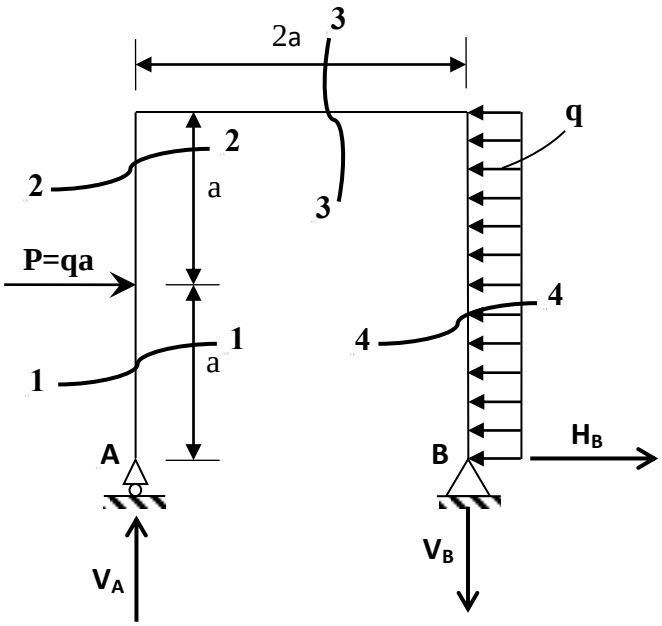
مخطط T :



مخطط M :



3- دراسة الهيكل (c):



حساب ردود الافعال :

$$\sum F_x = 0$$

$$H_B + qa - q.2a = 0$$

$$H_B = qa$$

$$\sum F_y = 0$$

$$V_A - V_B = 0$$

$$\sum M_{/B} = 0$$

$$V_A.2a + q.a.a - q.2a.a = 0$$

$$2aV_A - qa^2 = 0$$

$$V_A = \frac{qa}{2}$$

ومنه:

$$V_B = \frac{qa}{2}$$

المقطع 1-1 : $0 \le x \le a$ (الجزء الأيسر)

$$\sum F_x = 0$$

$$N_1 + V_A = 0 \rightarrow N_1 = -\frac{qa}{2}$$

$$\sum F_y = 0$$

$$T_1 = 0$$

$$\sum M_{/z} = 0$$

$$M_1 = 0$$

المقطع 2-2 : $a \le x \le 2a$ (الجزء الأيسر)

$$\sum F_x = 0$$

$$N_2 + V_A = 0 \rightarrow N_2 = -\frac{qa}{2}$$

$$\sum F_y = 0$$

$$T_2 + qa = 0 \rightarrow T_2 = -qa$$

$$\sum M_{/z} = 0$$

$$M_2(x) + qa(x - a) = 0$$

$$M_2(x) = -qa(x - a)$$

$$\begin{cases} M_2(a) = 0 \\ M_2(2a) = -qa^2 \end{cases}$$

المقطع 3-3 : $0 \le x \le 2a$ (الجزء الأيسر)

$$\sum F_x = 0$$

$$N_3 + qa = 0 \rightarrow N_3 = -qa$$

$$\sum F_y = 0$$

$$T_3 - V_A = 0 \rightarrow T_3 = \frac{qa}{2}$$

$$\sum M_{/z} = 0$$

$$M_3(x) + qa.a - \frac{qa}{2}x = 0$$

$$M_3(x) = \frac{qa}{2}x - qa^2$$

$$\begin{cases} M_3(a) = -qa^2 \\ M_3(2a) = 0 \end{cases}$$

المقطع 4-4 : $a \le x \le 2a$ (الجزء الأيمن)

$$\sum F_x = 0$$

$$N_4 + V_B = 0 \rightarrow N_4 = \frac{qa}{2}$$

$$\sum F_y = 0$$

$$T_4(x) - q(2a - x) + H_B = 0$$

$$T_4(x) = q(2a - x) - qa$$

$$\begin{cases} T_4(0) = qa \\ T_4(2a) = -qa \end{cases}$$

$$\sum M_{/z} = 0$$

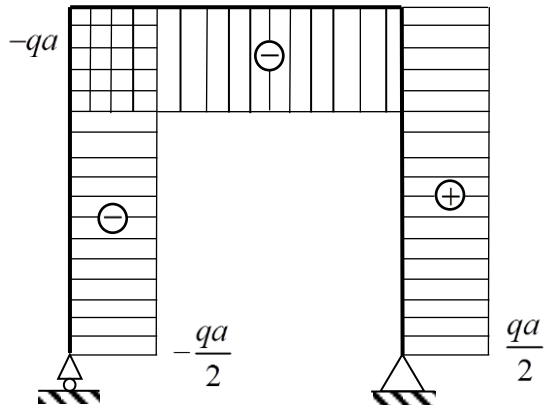
$$M_4(x) + \frac{q(2a-x)^2}{2} - H_B(2a - x) = 0$$

$$M_4(x) = \frac{-q}{2}(2a - x)^2 + qa(2a - x)$$

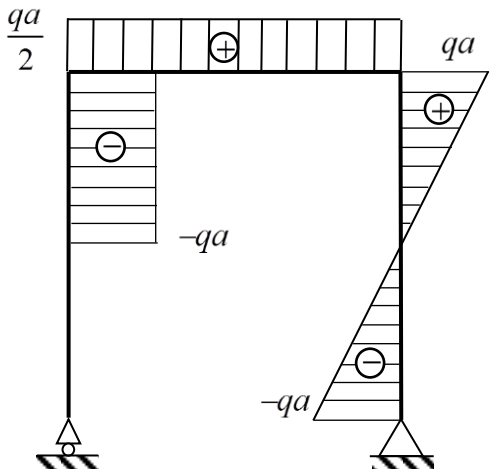
$$\begin{cases} M_4(a) = 0 \\ M_4(2a) = 0 \\ M_4(a) = \frac{qa^2}{2} \end{cases}$$

رسم مخططات M ,T, N :

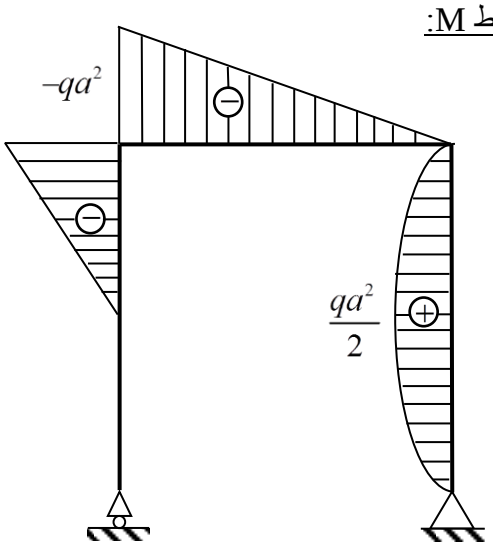
مخطط N :



مخطط T:



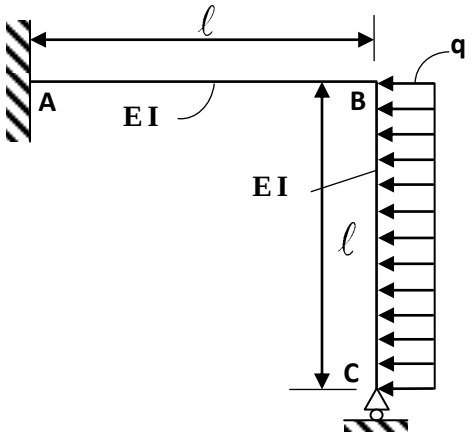
مخطط M:



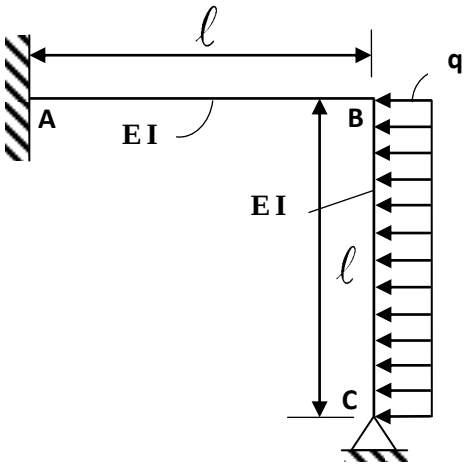
Systèmes hyperstatiques
الأنظمة الغير مقررة ستاتيكيا

أرسم مخططات عزوم الإنحناء M و قوى القص T للهياكل الغير مقررة ستاتيكيا وذلك بالإستعانة بطريقة القوى.

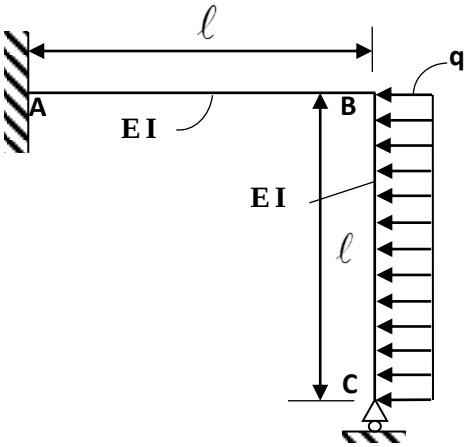
a



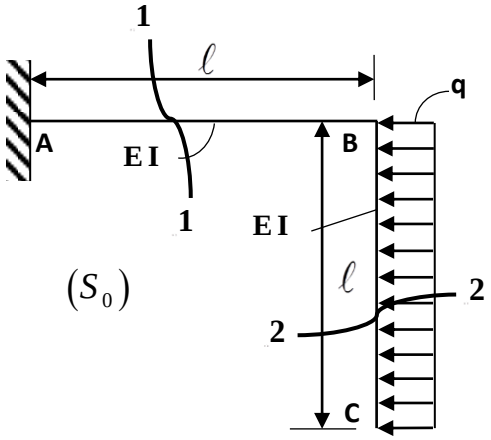
b



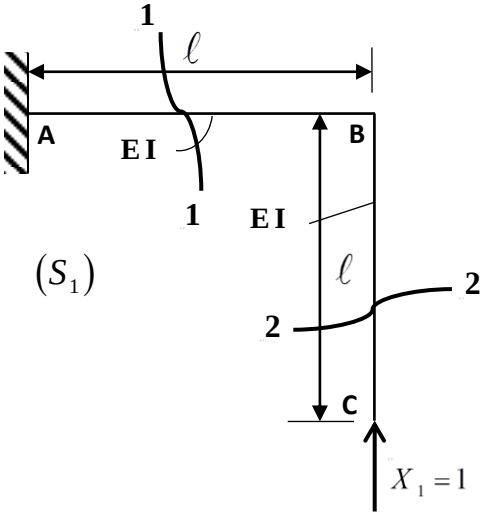
1- دراسة الهيكل (a):



≡



+



+ $\Delta_1 = 0$

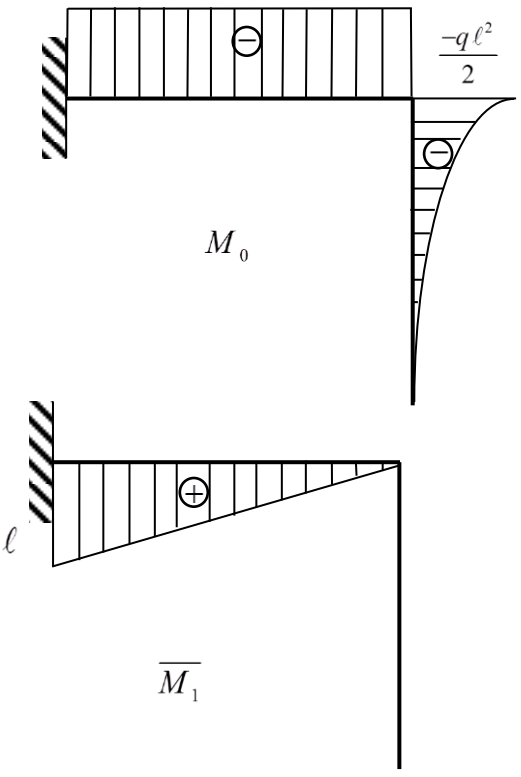
المقطع 1-1 : $0 \le x \le \ell$ (الجزء الأيمن)

$$M_0(x) = -\frac{q\ell^2}{2}$$
$$T_0(x) = 0$$
$$\overline{M_1}(x) = (\ell - x)$$
$$\overline{T_1}(x) = -1$$

المقطع 2-2: $0 \le x \le \ell$ (الجزء الأيمن)

$$M_0(x) = -\frac{q(\ell - x)^2}{2}$$
$$T_0(x) = q(\ell - x)$$
$$\overline{M_1}(x) = 0$$
$$\overline{T_1}(x) = 0$$

مخططات M_0 و $\overline{M_1}$:



$$\delta_{11}X_1 = -\Delta_{10}$$

$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{2}\ell.\ell.\frac{2}{3}\ell\right] = \frac{\ell^3}{3EI}$$

$$\Delta_{10} = \frac{1}{EI}\left[(\frac{-q\ell^2}{2}).\ell.\frac{\ell}{2}\right] = \frac{-q\ell^4}{4EI}$$

$$\frac{\ell^3}{3EI}X_1 = \frac{q\ell^4}{4EI} \rightarrow X_1 = \frac{3}{4}q\ell$$

المقطع 1-1 :

$$M(x) = M_0(x) + X_1\overline{M_1}(x)$$

$$= \frac{-q\ell^2}{2} + \frac{3}{4}q\ell.(\ell - x)$$

$$\begin{cases} M(0) = \frac{q\ell^2}{4} \\ M(\ell) = -\frac{q\ell^2}{2} \end{cases}$$

$$T(x) = T_0(x) + X_1\overline{T_1}(x)$$

$$= -\frac{3}{4}q\ell$$

المقطع 2-2 :

$$M(x) = M_0(x) + X_1\overline{M_1}(x)$$

$$= \frac{-q(\ell - x)^2}{2}$$

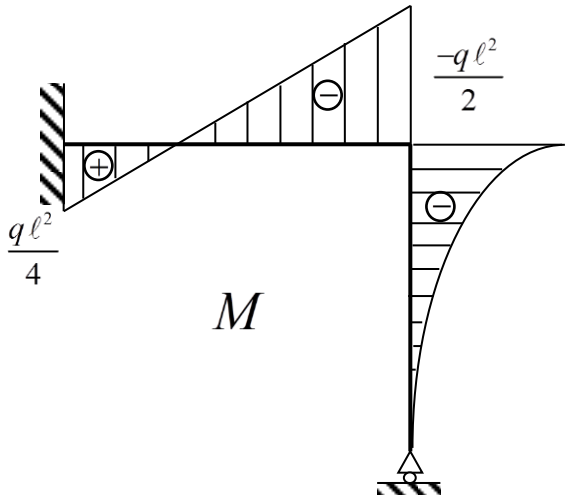
$$\begin{cases} M(0) = \frac{-q\ell^2}{2} \\ M(\ell) = 0 \end{cases}$$

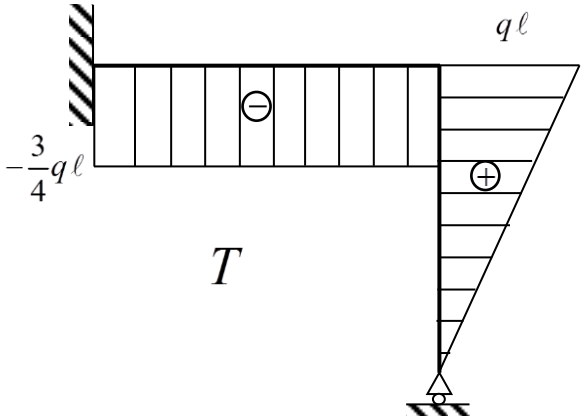
$$T(x) = T_0(x) + X_1\overline{T_1}(x)$$

$$= q(\ell - x)$$

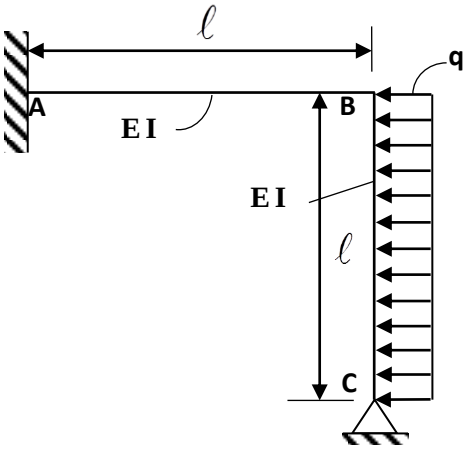
$$\begin{cases} T(0) = q\ell \\ T(\ell) = 0 \end{cases}$$

رسم مخططات M و T :





2- دراسة الهيكل (b):



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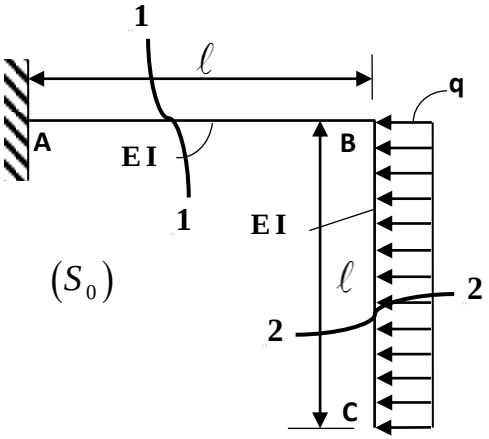
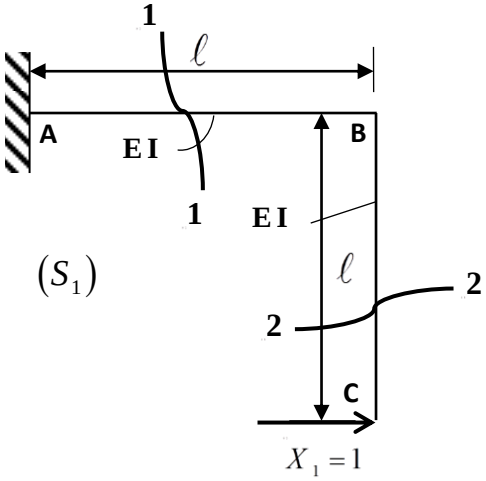


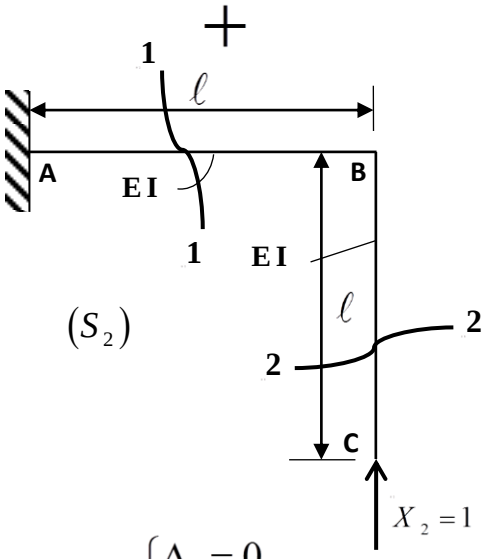
(S_0)

+



(S_1)

+



(S_2)

$$+ \begin{cases} \Delta_1 = 0 \\ \Delta_2 = 0 \end{cases}$$

المقطع 1-1 : $0 \le x \le \ell$ (الجزء الأيمن)

$$M_0(x) = -\frac{q\ell^2}{2}$$

$$T_0(x) = 0$$

$$\overline{M_1}(x) = \ell$$

$$\overline{T_1}(x) = 0$$

$$\overline{M_2}(x) = (\ell - x)$$

$$\overline{T_2}(x) = -1$$

المقطع 2-2 : $0 \le x \le \ell$ (الجزء الأيمن)

$$M_0(x) = -\frac{q(\ell - x)^2}{2}$$

$$T_0(x) = q(\ell - x)$$

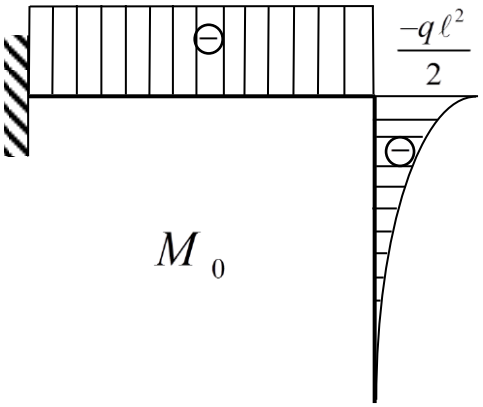
$$\overline{M_1}(x) = (\ell - x)$$

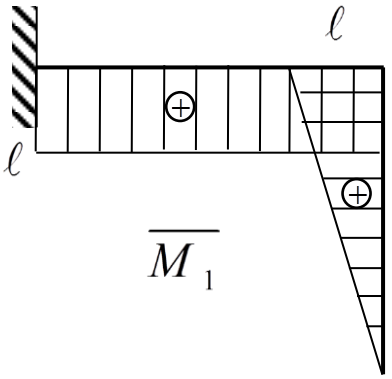
$$\overline{T_1}(x) = -1$$

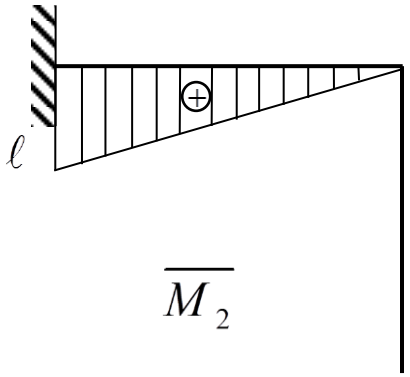
$$\overline{M_2}(x) = 0$$

$$\overline{T_2}(x) = 0$$

مخططات M_0 , $\overline{M_1}$ و $\overline{M_2}$:







$$\begin{cases} \delta_{11}.X_1 + \delta_{12}.X_2 = -\Delta_{10} \\ \delta_{21}.X_1 + \delta_{22}.X_2 = -\Delta_{20} \end{cases}$$

$$\delta_{11} = \frac{1}{EI}\left[\ell^2.\ell + \frac{1}{2}\ell^2.\frac{2}{3}\ell\right] = \frac{4\ell^3}{3EI}$$

$$\delta_{22} = \frac{1}{EI}\left[\frac{1}{2}\ell^2.\frac{2}{3}.\ell\right] = \frac{\ell^3}{3EI}$$

$$\delta_{12} = \delta_{21} = \frac{1}{EI}\left[\ell^2.\frac{\ell}{2} + \frac{1}{2}\ell^2.0\right] = \frac{\ell^3}{2EI}$$

$$\Delta_{10} = \frac{1}{EI}\left[\left(\frac{-q\ell^2}{2}\right).\ell.\ell + \frac{1}{3}.\left(\frac{-q\ell^2}{2}\right).\ell.\frac{3}{4}\ell\right]$$

$$\Delta_{10} = \frac{1}{EI}\left[\frac{-q\ell^4}{2} - \frac{q\ell^4}{8}\right] = -\frac{5q\ell^4}{8EI}$$

$$\Delta_{20} = \frac{1}{EI}\left[\left(\frac{-q\ell^2}{2}\right).\ell.\frac{\ell}{2}\right] = \frac{-q\ell^4}{4EI}$$

$$\begin{cases} \left(\frac{4\ell^3}{3EI}\right).X_1 + \left(\frac{\ell^3}{2EI}\right).X_2 = \frac{5q\ell^4}{8EI} \\ \left(\frac{\ell^3}{2EI}\right).X_1 + \left(\frac{\ell^3}{3EI}\right).X_2 = \frac{q\ell^4}{4EI} \end{cases}$$

$$\begin{cases} \left(\frac{4}{3}\right).X_1 + \left(\frac{1}{2}\right).X_2 = \frac{5}{8}q\ell \\ \left(\frac{1}{2}\right).X_1 + \left(\frac{1}{3}\right).X_2 = \frac{q\ell}{4} \end{cases}$$

$$X_1 = \frac{3}{7}q\ell \quad , \quad X_2 = \frac{3}{28}q\ell$$

المقطع 1-1 : $0 \le x \le \ell$

$$M(x) = M_0(x) + X_1\overline{M_1}(x) + X_2\overline{M_2}(x)$$

$$= \frac{-q\ell^2}{2} + \frac{3}{7}q\ell^2 + \frac{3}{28}q\ell.(\ell - x)$$

$$\begin{cases} M(0) = \frac{q\ell^2}{28} \\ M(\ell) = -\frac{q\ell^2}{14} \end{cases}$$

$$T(x) = T_0(x) + X_1\overline{T_1}(x) + X_2\overline{T_2}(x)$$

$$T(x) = -\frac{3}{28}q\ell$$

المقطع 2-2 : $0 \le x \le \ell$

$$M(x) = M_0(x) + X_1\overline{M_1}(x) + X_2\overline{M_2}(x)$$

$$= \frac{-q(\ell - x)^2}{2} + \frac{3}{7}q\ell.(\ell - x)$$

$$\begin{cases} M(0) = -\dfrac{q\ell^2}{14} \\ M(\ell) = 0 \end{cases}$$

$$T(x) = T_0(x) + X_1\overline{T_1}(x) + X_2\overline{T_2}(x)$$

$$T(x) = q(\ell - x) - \frac{3}{7}q\ell$$

$$\begin{cases} T(0) = \dfrac{4}{7}q\ell \\ T(\ell) = -\dfrac{3}{7}q\ell \end{cases}$$

$$T(x) = 0 \rightarrow x = \frac{4}{7}\ell$$

$$M_{\max}(x = \frac{4}{7}\ell) = -\frac{q}{2}.\frac{9}{49}\ell^2 + q\ell^2.\frac{9}{49}$$

$$M_{\max} = \frac{9}{98}q\ell^2$$

رسم مخططات M و T :