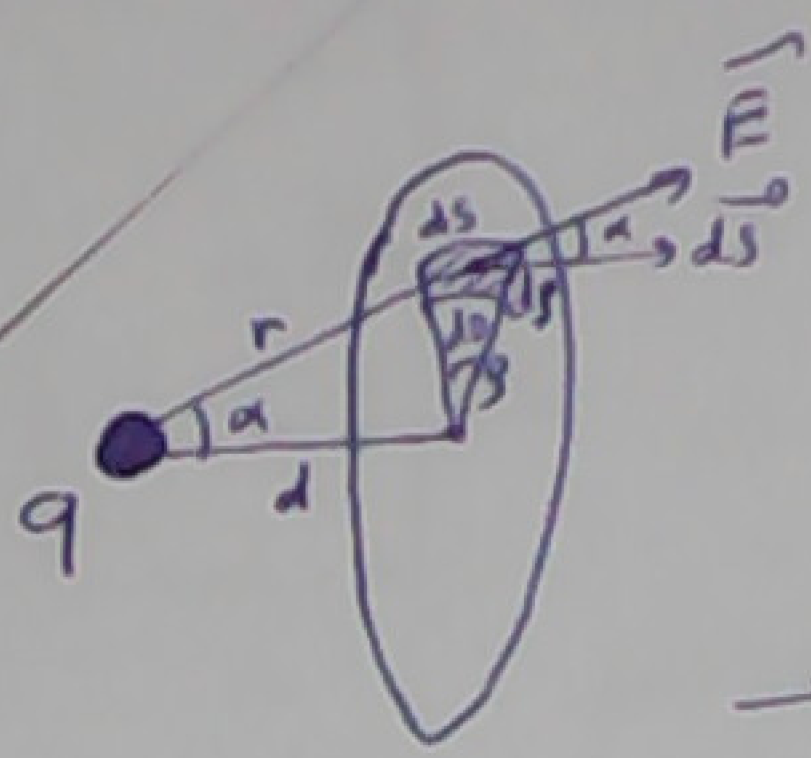
حل مختصر - السلسلة ③ -

التمرين ① :



$$\iint_{\text{سطح القرص}} \vec{E}\,\vec{ds} = \frac{q}{4\varepsilon_0}$$

$$\rightarrow \iint_{\text{سطح القرص}} E\,ds\cos\alpha = \frac{q}{4\varepsilon_0} \longrightarrow (*)$$

حيث :

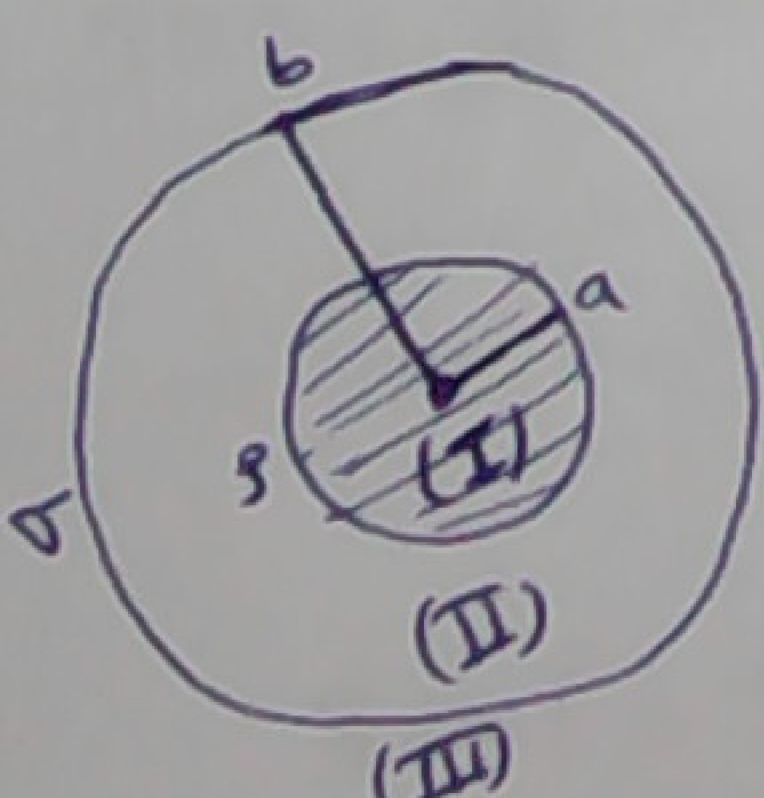
$$\begin{cases} E = k\dfrac{q}{r^2} \\ ds = \rho\,d\rho\,d\theta \\ \cos\alpha = \dfrac{d}{r} \\ r = \sqrt{d^2+\rho^2} \end{cases}$$

بالتعويض في العلاقة (*) نجد :

$$k\,q\,d\int_0^{2\pi} d\theta \int_0^R \frac{\rho}{(d^2+\rho^2)^{3/2}}\,d\rho = \frac{q}{4\varepsilon_0}$$

$$\rightarrow \boxed{R = \sqrt{3}\cdot d} \quad \rightarrow R = 3\sqrt{3}\ \text{cm}$$

التمرين ② :



1 . الحقل الكهربائي : نستعمل نظرية غوص :

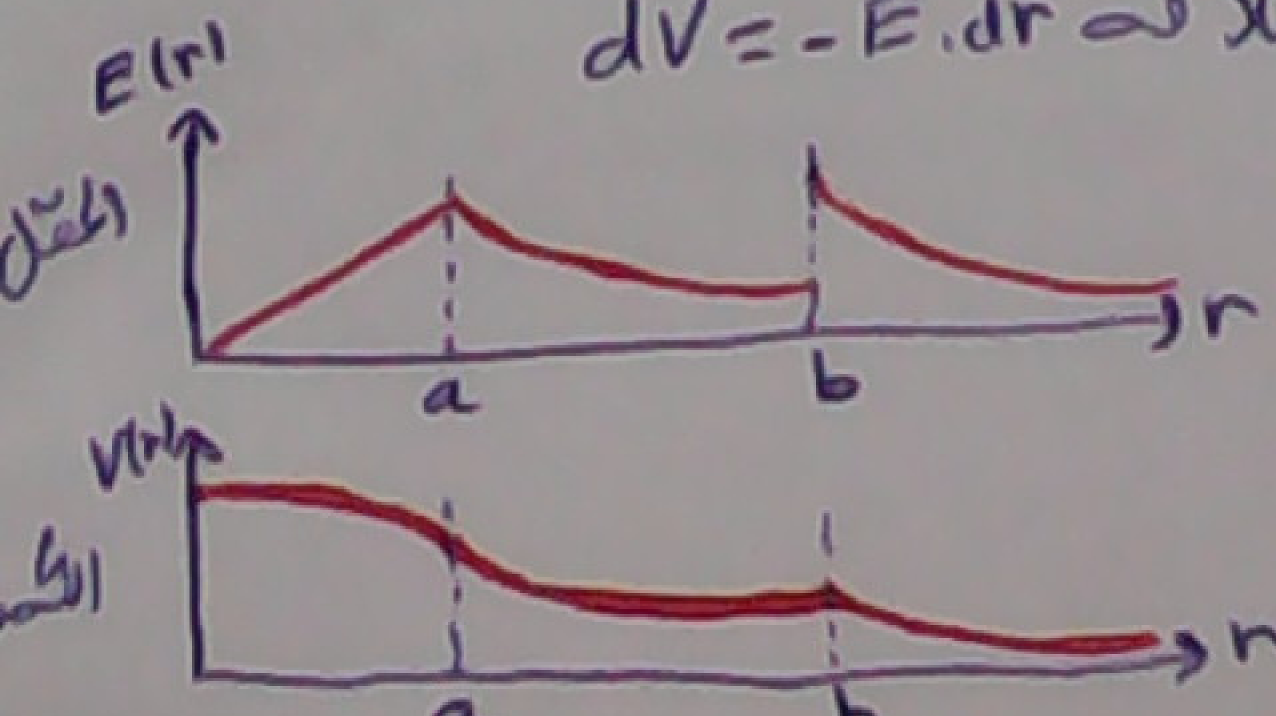
$$\oint\!\!\oint \vec{E}\,\vec{ds} = \frac{\sum q_{int}}{\varepsilon_0} \longrightarrow (*)$$

$r<a$: $\boxed{E = \dfrac{\rho}{3\varepsilon_0}r}$

$a<r<b$: $\boxed{E = \dfrac{\rho a^3}{3\varepsilon_0}\dfrac{1}{r^2}}$

$r>b$: $\boxed{E = \left(\dfrac{\rho a^3}{3\varepsilon_0} + \dfrac{\sigma b^2}{\varepsilon_0}\right)\dfrac{1}{r^2}}$

- الكمون الكهربائي : نستعمل العلاقة $dV = -E.dr$

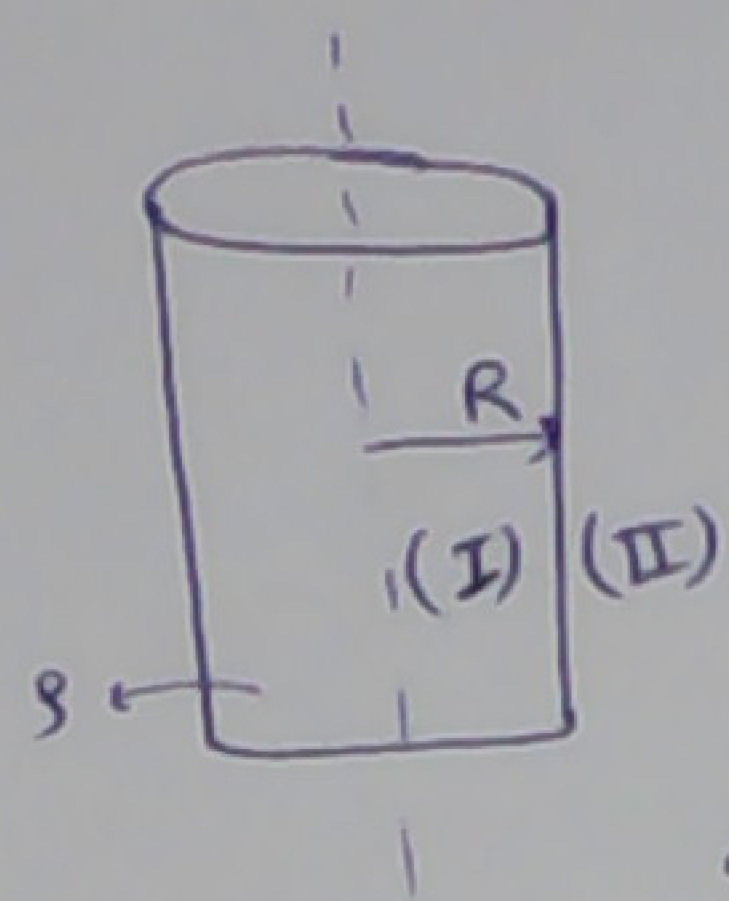


$r>b$: $\boxed{V = \left(\dfrac{\rho a^3}{3\varepsilon_0} + \dfrac{\sigma b^2}{\varepsilon_0}\right)\dfrac{1}{r}}$

$a<r<b$: $\boxed{V = \dfrac{\rho a^3}{3\varepsilon_0}\dfrac{1}{r} + \dfrac{\sigma b}{\varepsilon_0}}$

$r<a$: $\boxed{V = \dfrac{-\rho}{6\varepsilon_0}r^2 + \dfrac{\rho a^2}{2\varepsilon_0} + \dfrac{\sigma b}{\varepsilon_0}}$

التمرين ③ :



1) الحقل الكهربائي: $\oint \vec{E}\,d\vec{S} = \frac{\sum q_{int}}{\varepsilon_0}$

$r<R$: $\boxed{E = \frac{\rho r}{2\varepsilon_0}}$

$r>R$: $\boxed{E = \frac{\rho R^2}{2\varepsilon_0}\cdot\frac{1}{r}}$

② الكمون الكهربائي: نستعمل العلاقة $dV = -E\,dr$

المنطقة $r<R$: $\boxed{V = \frac{-\rho}{4\varepsilon_0} r^2}$

المنطقة $r>R$: $\boxed{V = \frac{\rho R^2}{2\varepsilon_0}\left(\frac{-1}{2} + \ln\left(\frac{R}{r}\right)\right)}$

③

$$\oint \vec{E}\,d\vec{S} = \frac{\iiint \rho\,dv}{\varepsilon_0} \rightarrow E\,2\pi r H = \frac{1}{\varepsilon_0}\int_0^H\int_0^{2\pi}\int_0^r \rho_0 \frac{r}{R}\, r\,dr\,d\theta\,dz$$

$$\rightarrow \boxed{E = \frac{\rho_0}{3\varepsilon_0 R} r^2}$$

التمرين ④ :

1)

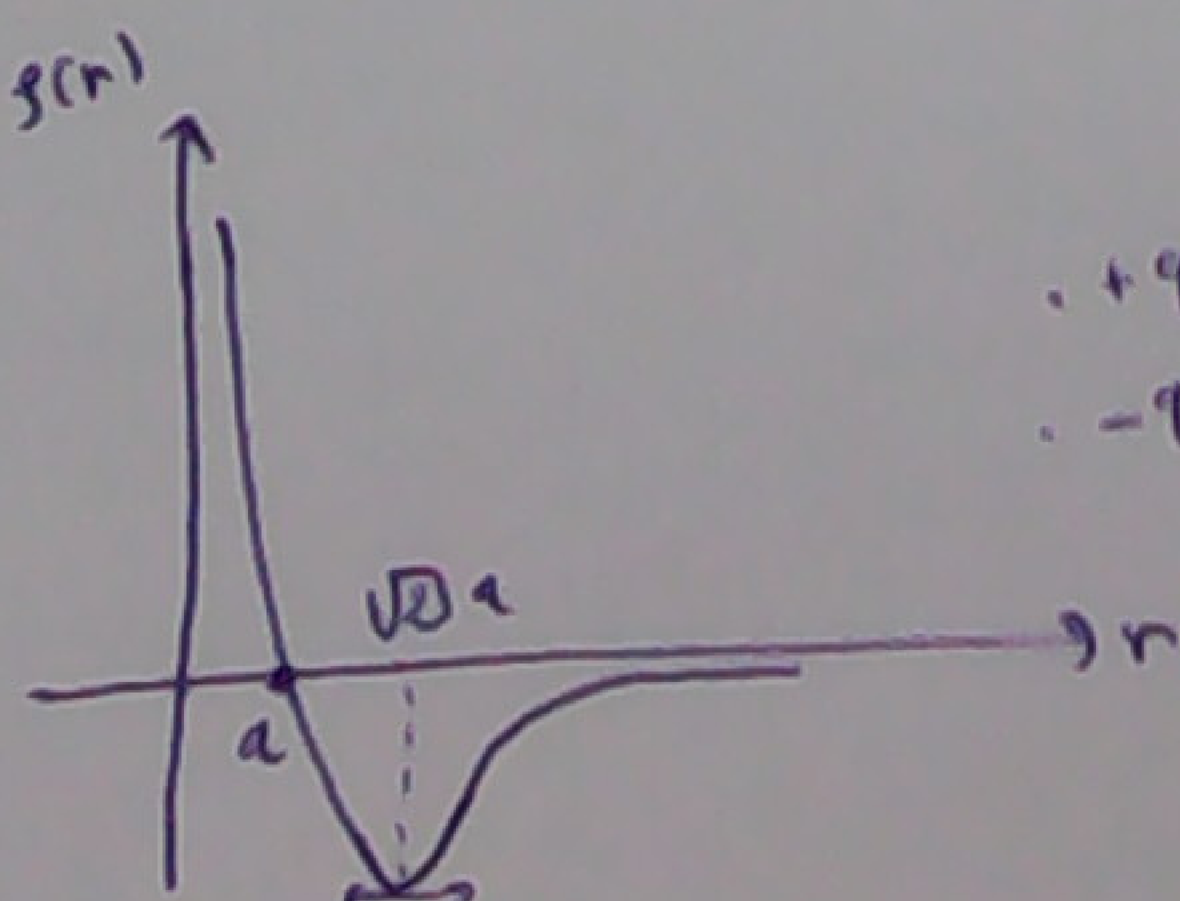
$$q(r) = \iiint_v \rho\,dv = \int_0^{2\pi}\int_0^{\pi}\int_0^r \frac{q_0}{4\pi a}\frac{1}{r^2}\left(1-\frac{r}{a}\right)e^{-\frac{r}{a}}\left(r^2\sin\theta\,dr\,d\theta\,d\varphi\right)$$

$$\rightarrow \boxed{q(r) = \frac{q_0}{a}\cdot r\cdot e^{-\frac{r}{a}}}$$

الحقل الكهربائي: نستعمل نظرية غوص:

$$\oint \vec{E}\,d\vec{S} = \frac{\sum q_{int}}{\varepsilon_0} \rightarrow \boxed{E = \frac{q_0}{4\pi\varepsilon_0 a}\,\frac{e^{-\frac{r}{a}}}{r}}$$

2) بيان الدالة $\rho(r)$:

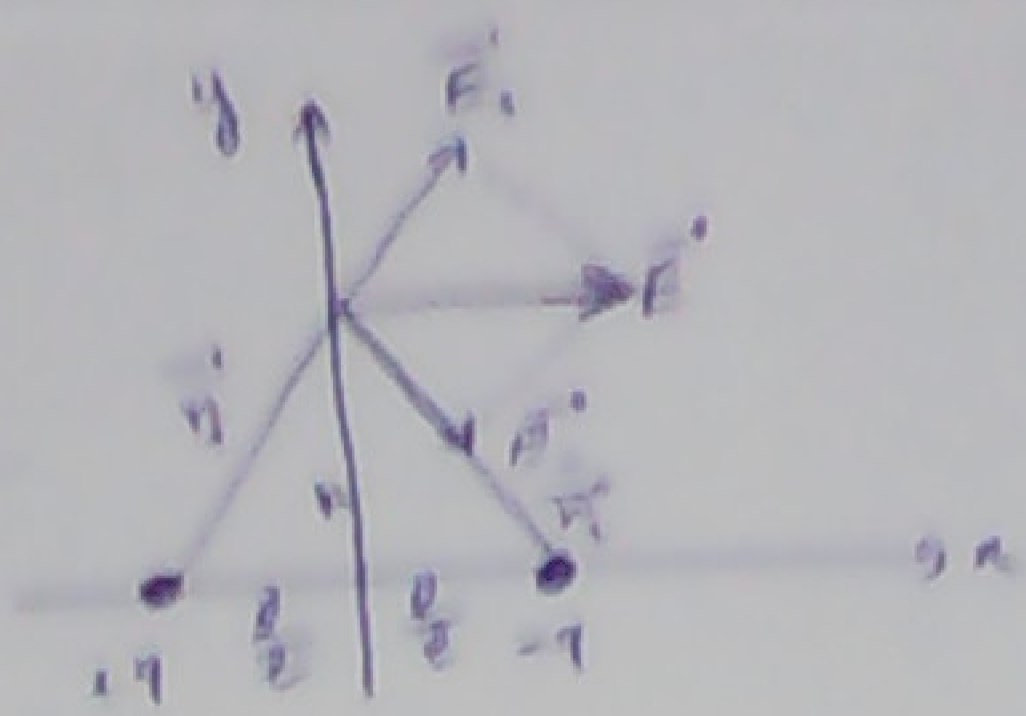


المنطقة $0<r<a$: مكان تواجد الشحنة الموجبة $+q$.

المنطقة $r>a$: " " " السالبة $-q$.

$q^+ = \frac{q_0}{e}$

$q^- = \frac{-q_0}{e}$



التمرين (5):

$$\vec{E} = \vec{E}_+ + \vec{E}_- = k\frac{q}{r_1^2}\vec{r}_1 - k\frac{q}{r_2^2}\vec{r}_2$$

$$r_1 = r_2 \rightarrow \vec{E} = k\frac{q}{r_1^3}(\vec{r}_1 - \vec{r}_2)$$

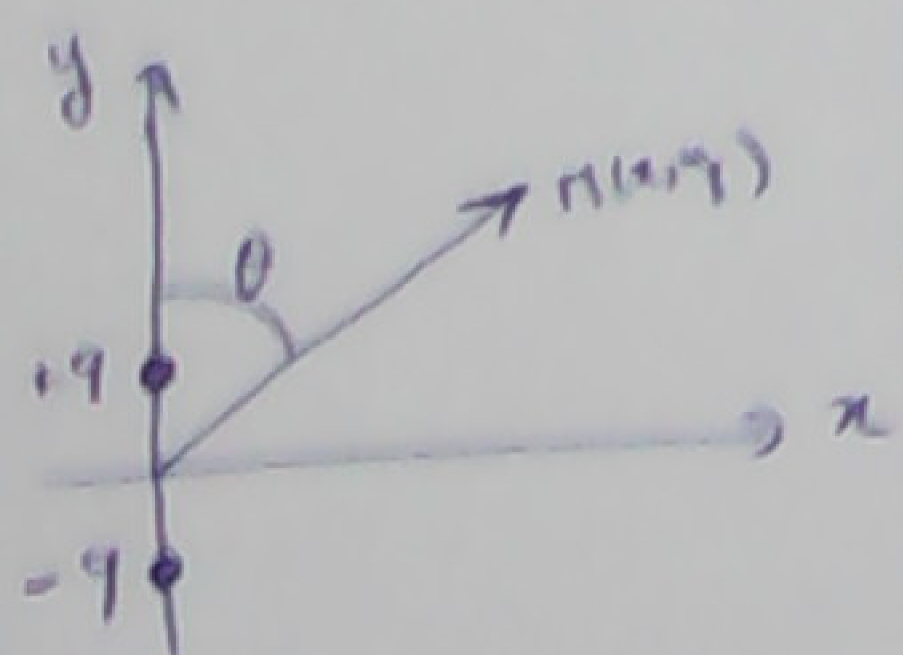
$$= \frac{kq}{r_1^3}(l\vec{i})$$

$$= \frac{kql}{\left[r^2+\left(\frac{l}{2}\right)^2\right]^{3/2}}\vec{i}$$

$$\rightarrow \vec{E} = \frac{kql}{r^3\left[1+\frac{\left(\frac{l}{2}\right)^2}{r^2}\right]^{3/2}}\vec{i}$$

$$r \gg \frac{l}{2} \longrightarrow \boxed{\vec{E} = \frac{kql}{r^3}\vec{i}}$$

التمرين (6):



ننطلق من العبارة المبرهنة في الدرس:

$$\boxed{V = \frac{kp\cos\theta}{r^2}}$$

لدينا: $\cos\theta = \frac{y}{r}$ و $r = \sqrt{x^2+y^2}$ ومنه:

$$V = kp\frac{y}{(x^2+y^2)^{3/2}} \longrightarrow (*)$$

ومن جهة أخرى لدينا $\vec{E} = -\vec{grad}\,V = -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j}$ ومنه:

$$E_x = \frac{-\partial V}{\partial x} = \ldots\ldots$$

$$E_y = \frac{-\partial V}{\partial y} = \ldots\ldots$$