$$\tau_{max} = \frac{M_{xmax}}{I_p} R_{max} \Rightarrow I_p = \iint r^2 \, r\,dr\,d\theta \Rightarrow I_p = \frac{r^4}{4}\Big|_0^R 2\pi$$

$$I_p = \frac{R^4 \cdot \pi}{2} = 9{,}81\ 10^6\ mm^4$$

$$\tau_{max} = \frac{2 \cdot 10^6}{9{,}81 \cdot 10^6} \cdot 50 = 10{,}19\ MPa < 40\ MPa \Rightarrow \text{Resistance}$$

est verifiée.

2- La rigidité

$$\alpha_1 (0 \leq x \leq 2) = \int_0^2 \frac{M_x}{G I_p} \cdot dx = \int_0^2 \frac{(4\ 10^6 x \cdot 10^{-3} - 6\ 10^6)}{G I_p} \cdot dx$$

$$= \int_0^2 \frac{4 \cdot 10^6 \, x \, dx \cdot 10^{-3}}{G I_p} - \int_0^2 \frac{6 \cdot 10^6}{G I_p} \cdot dx \Rightarrow \frac{4\ 10^6}{8 \cdot 10^4 \cdot 9{,}81 \cdot 10^6} \int_0^{2000} x\,dx -$$

$$\int_0^{2000} \frac{6 \cdot 10^6}{8\ 10^4 \times 9{,}81\ 10^6} dx \Rightarrow 5{,}01 \cdot 10^{-6} \frac{x^2}{2}\Big|_0^{2000} - 7{,}6\ 10^{-6} \cdot x\Big|_0^{2000}$$

$$= 10{,}02 + 1{,}5 \cdot 10^{-2} = 0{,}01 + 0{,}015 = 0{,}025 \left(\frac{rad}{mm}\right)$$

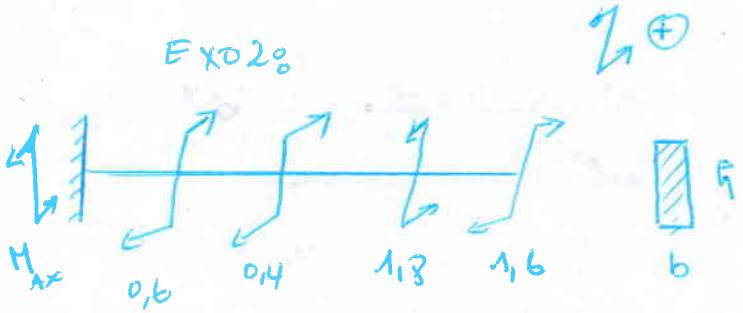
$$[\varphi]_{/L} = 0{,}3^\circ/_m = 0{,}005\ \frac{rad}{m} = 5 \cdot 10^{-6}\ \frac{rd}{mm}$$

$$\alpha_2 (2 \leq x \leq 3) = \int_{2000}^{3000} \frac{M_x}{G I_p} \cdot dx = \frac{2 \cdot 10^6}{8 \cdot 10^4 \cdot 9{,}81\ 10^6} x\Big|_{2000}^{3000} = 0{,}0025\ \frac{rad}{mm}$$

$$\varphi_1 \leq [\varphi] \Rightarrow \varphi_1 = \frac{\alpha_1}{L_1} = \frac{0{,}0025}{2} = 0{,}00125\ \frac{rd}{m} < 0{,}3^\circ$$

$$[\varphi] = 0{,}3^\circ\ rd \quad 0{,}3^\circ\ rd \quad 0{,}0052\ rd \qquad (2)$$

Exo 2 :



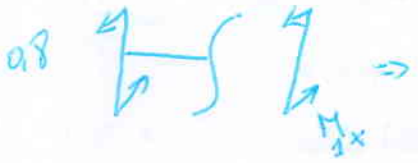
1. Détermination de b et h.

$[\sigma] = 60$ MPa ; $\frac{h}{b} = 2$

$\sum \vec{M}_{Ax} = \vec{0} \Rightarrow M_{Ax} - 0,6.10^6 - 0,4.10^6 + 1,8.10^6 - 1,6.10^6 = 0$

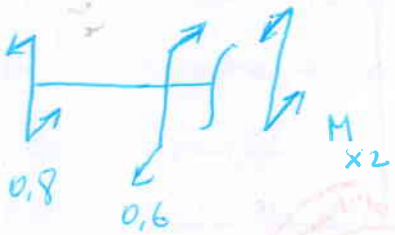
$+M_{Ax} - 0,8\,10^6 = 0 \Rightarrow \boxed{M_{Ax} = 0,8.10^6 \text{ N.mm}}$

- tranche 1 :



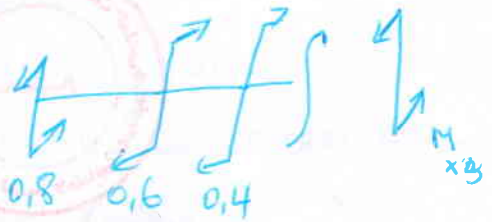
$0,8 + M_{1x} = 0 \Rightarrow M_{x1} = -0,8.10^6 \text{ N.mm}$

- tranche 2 :



$0,8 - 0,6 + M_{x2} = 0 \Rightarrow M_{x2} = -0,2\,10^6 \text{ N.mm}$

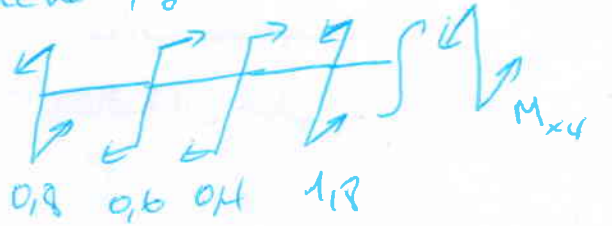
- tranche 3 :



$0,8 - 0,6 - 0,4 + M_{x3} = 0$

$M_{x3} = 0,2\,10^6 \text{ N.mm}$

- tranche 4 :



$0,8 - 0,6 - 0,4 + 1,8 + M_{x4} = 0$

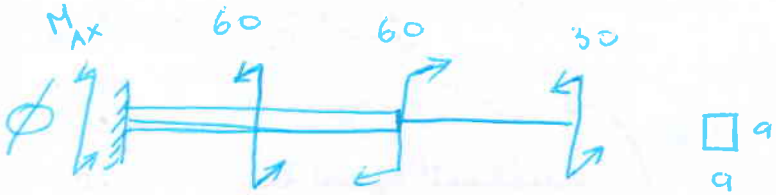
$M_{x4} = -1,6.10^6 \text{ N.mm}$

$$\tau_{max} = \frac{M_{t\,max}}{W_t} \Rightarrow W_t = \varphi\, h\, b^2 \Rightarrow W_t = 2\varphi\, b^3$$

$$\frac{h}{b} = 2 \Rightarrow \varphi = 0{,}246 \Rightarrow W_t = 2 \cdot 0{,}246 \cdot b^3 = 0{,}492 \cdot b^3$$

$$\tau_{max} = \frac{1{,}6 \cdot 10^6}{0{,}492\, b^3} \leqslant 60 \Rightarrow b^3 \geqslant \frac{1{,}6 \cdot 10^6}{0{,}492 \cdot 60} = 54200{,}54$$

$$b \geqslant 37{,}84 \Rightarrow b \simeq 38\ mm \Rightarrow h = 2 \cdot 38 = 76\ mm.$$



$$\Sigma M_x = \vec{0} \Rightarrow M_x + 60 - 60 + 30 = 0 \Rightarrow M_{Ax} = 30\ N.m$$

$$M_{Ax} = 30 \cdot 10^3\ N.mm$$

- Calcule M_x dans chaque tranche :

a- tranche 1 :

$$\Rightarrow -30 + M_{x1} = 0 \Rightarrow \boxed{M_{x1} = 30\,10^3\ N.mm}$$

b- tranche 2 :

$$\Rightarrow -30 + 60 + M_{x2} = 0 \Rightarrow$$

$$\boxed{M_{x2} = -30 \cdot 10^3\ N.mm}$$

c- tranche 3 :

$$-30 + 60 + 60 + M_{x3} = 0 \Rightarrow$$

- Détermination de d°

Section circulaire :

$I_p = \frac{\pi R^4}{2}$

$\tau_{max} = \frac{M_{xmax}}{I_p} \cdot R \Rightarrow \frac{2 \cdot 30.10^3}{\pi R^4} R \leq 60$

$R^3 \geq \frac{60.10^3}{60.3,14} \Rightarrow R \geq 6,82 \Rightarrow R \simeq 7 \Rightarrow d = 14mm$

* Détermination de a° :

$\tau_{max} = \frac{M_{xmax}}{W_t} \Rightarrow W_t = \varphi a^3 \Rightarrow \varphi = 0,208$

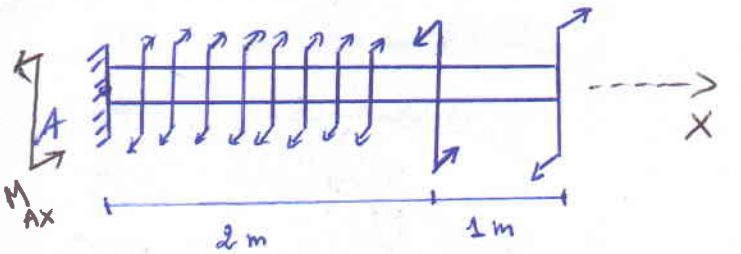
$\frac{30.10^3}{0,208.a^3} \leq 60 \Rightarrow a^3 \geq \frac{30.10^3}{0,208.60} = a \geq 13,39 \Rightarrow$

$a_{mm} = 14 mm$

- torsion -

EXO 1 :

$G = 8.10^4 N/mm^2$; $[\tau] = 0,7 \times [\sigma] = 40 N/mm^2$; $[\varphi]_{/L} = 0,3°/m$

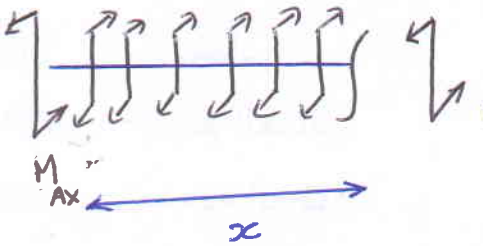


1 - calcule de M_x dans chaque tranche :

1-1 - Calcule de Reaction en A

$$M_{AX} - 4.2 + 4 - 2 = 0 \Rightarrow \boxed{M_{AX} = 6.10^6 N.mm}$$

$0 \leqslant x \leqslant 2\,m$



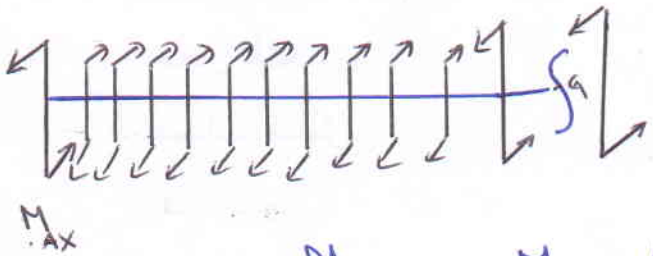
$$\Sigma M_x = \vec{0} \Rightarrow$$

$$M_{AX} - 4.10^6.x + M_{x1} = 0$$

$$\Rightarrow M_{x1} = -M_{AX} + 4.10^6.x \Rightarrow \boxed{M_{x1} = 4.10^6.x - 6.10^6 N.mm}$$

$$M_{x1\,max}\,(x=2) = 4.10^6.2 - 6.10^6 = 2.10^6 N.mm$$

$2 \leqslant x \leqslant 3$



$$\Sigma M_x = \vec{0} \Rightarrow$$

$$M_{AX} - 4.10^6.2 + 4.10^6 + M_{x2} = 0$$

$$M_{x2} = -M_{AX} + 4.10^6.2 - 4.10^6 = -6\,10^6 + 8\,10^6 - 4\,10^6$$

$$\boxed{M_{x2} = -2\,10^6 N.mm}$$