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Transfert de chaleur (3L)

Module : Année : 21/20 Spécialité : Groupe : Durée : min
Nom et prénom : : الاسم واللقب Matricule:.....

Heat Conduction Rate Equations (Fourier's Law)

- Heat Flux: $q_x'' = -k \frac{dT}{dx} \frac{W}{m^2}$ k : Thermal Conductivity $\frac{W}{m \cdot K}$
- Heat Rate: $q_x = q_x'' A_c W$ A_c : Cross-Sectional Area

Heat Convection Rate Equations (Newton's Law of Cooling)

- Heat Flux: $q'' = h(T_s - T_\infty) \frac{W}{m^2}$ h : Convection Heat Transfer Coefficient $\frac{W}{m^2 \cdot K}$
- Heat Rate: $q = hA_s(T_s - T_\infty) W$ A_s : Surface Area m^2

Heat Radiation emitted ideally by a blackbody surface has a surface emissive power: $E_b = \sigma T_s^4 \frac{W}{m^2}$

- Heat Flux emitted: $E = \epsilon \sigma T_s^4 \frac{W}{m^2}$ where ϵ is the emissivity with range of $0 \leq \epsilon \leq 1$
and $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$ is the Stefan-Boltzmann constant
- Irradiation: $G_{abs} = \alpha G$ but we assume small body in a large enclosure with $\epsilon = \alpha$ so that $G = \epsilon \sigma T_{sur}^4$
- Net Radiation heat flux from surface: $q_{rad}'' = \frac{q}{A} = \epsilon E_b(T_s) - \alpha G = \epsilon \sigma (T_s^4 - T_{sur}^4)$
- Net radiation heat exchange rate: $q_{rad} = \epsilon \sigma A_s (T_s^4 - T_{sur}^4)$ where for a real surface $0 \leq \epsilon \leq 1$

This can ALSO be expressed as: $q_{rad} = h_r A (T_s - T_{sur})$ depending on the application

- where h_r is the radiation heat transfer coefficient which is: $h_r = \epsilon \sigma (T_s + T_{sur})(T_s^2 + T_{sur}^2) \frac{W}{m^2 \cdot K}$
- TOTAL heat transfer from a surface: $q = q_{conv} + q_{rad} = hA_s(T_s - T_\infty) + \epsilon \sigma A_s (T_s^4 - T_{sur}^4) W$



Transfert thermique et Echangeurs de chaleur & Phénomène de Transfert II

Module : **UEM1.1** Année : 21/20 Spécialité : Groupe : Durée : 60 min

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Heat Equation (used to find the temperature distribution)

$$\text{Heat Equation (Cartesian): } \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

If k is constant then the above simplifies to: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$ where $\alpha = \frac{k}{\rho c_p}$ is the *thermal diffusivity*

$$\text{Heat Equation (Cylindrical): } \frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\text{Heat Eqn. (Spherical): } \frac{1}{r^2} \frac{\partial}{\partial r} \left(kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Thermal Circuits

$$\text{Plane Wall: } R_{t,cond} = \frac{L}{kA} \quad \text{Cylinder: } R_{t,cond} = \frac{\ln(r_2/r_1)}{2\pi k L} \quad \text{Sphere: } R_{t,cond} = \frac{(\frac{1}{r_1} - \frac{1}{r_2})}{4\pi k}$$

$$R_{t,conv} = \frac{1}{hA} \quad R_{t,rad} = \frac{1}{h_r A}$$



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Convection Heat Transfer

$$Re = \frac{\rho VL_c}{\mu} = \frac{VL_c}{\nu} \quad [\text{Reynolds Number}] \quad ; \quad \overline{Nu} = \frac{\bar{h}L_c}{k_f} \quad [\text{Average Nusselt Number}]$$

where ρ is the density, V is the velocity, L_c is the characteristic length, μ is the dynamic viscosity, ν is the kinematic viscosity, \dot{m} is the mass flow rate, \bar{h} is the average convection coefficient, and k_f is the fluid thermal conductivity.

Internal Flow

$$Re = \frac{4 \dot{m}}{\pi D \mu} \quad [\text{For Internal Flow in a Pipe of Diameter } D]$$

For Constant Heat Flux [$q''_s = \text{constant}$]: $q_{conv} = q''_s(P \cdot L)$; where P = Perimeter, L = Length

$$T_m(x) = T_{m,i} + \frac{q''_s \cdot P}{\dot{m} \cdot c_p} x$$

For Constant Surface Temperature [$T_s = \text{constant}$]:

If there is only convection between the surface temperature, T_s , and the mean fluid temperature, T_m , use

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{P \cdot x}{\dot{m} \cdot c_p} \bar{h}\right)$$

If there are multiple resistances between the outermost temperature, T_∞ , and the mean fluid temperature, T_m , use

$$\frac{T_\infty - T_m(x)}{T_\infty - T_{m,i}} = \exp\left(-\frac{P \cdot x}{\dot{m} \cdot c_p} U\right) = \exp\left(-\frac{1}{\dot{m} \cdot c_p \cdot R_t}\right)$$

Total heat transfer rate over the entire tube length:

$$q_t = \dot{m} \cdot c_p \cdot (T_{m,o} - T_{m,i}) = \bar{h} \cdot A_s \cdot \Delta T_{lm} \text{ or } U \cdot A_s \cdot \Delta T_{lm} \quad ; \quad T_s = \text{constant}$$

Log mean temperature difference: $\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$; $\Delta T_o = T_s - T_{m,o}$; $\Delta T_i = T_s - T_{m,i}$



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Free Convection Heat Transfer

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad [\text{Grashof Number}]$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha} \quad [\text{Rayleigh Number}]$$

Vertical Plates: $\overline{Nu}_L = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr} \right)^{9/16} \right]^{8/27}} \right\}^2 ; \quad [\text{Entire range of } Ra_L; \text{ properties evaluated at } T_f]$

- For better accuracy for Laminar Flow: $\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + \left(\frac{0.492}{Pr} \right)^{9/16} \right]^{4/9}} ; \quad Ra_L \lesssim 10^9 \quad [\text{Properties evaluated at } T_f]$

Inclined Plates: for the top and bottom surfaces of cooled and heated inclined plates, respectively, the equations of the vertical plate can be used by replacing (g) with ($g \cos \theta$) in Ra_L for $0 \leq \theta \leq 60^\circ$.

Horizontal Plates: use the following correlations with $L = \frac{A_s}{P}$ where A_s = Surface Area and P = Perimeter

- Upper surface of Hot Plate or Lower Surface of Cold Plate:

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \quad (10^4 \leq Ra_L \leq 10^7, Pr \geq 0.7) ; \quad \overline{Nu}_L = 0.15 Ra_L^{1/3} \quad (10^7 \leq Ra_L \leq 10^{11}, \text{all } Pr)$$

- Lower Surface of Hot Plate or Upper Surface of Cold Plate:

$$\overline{Nu}_L = 0.52 Ra_L^{1/5} \quad (10^4 \leq Ra_L \leq 10^9, Pr \geq 0.7)$$

Vertical Cylinders: the equations for the Vertical Plate can be applied to vertical cylinders of height L if the following criterion is

met: $\frac{D}{L} \geq \frac{35}{Gr_L^{1/4}}$

Long Horizontal Cylinders: $\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(\frac{0.559}{Pr} \right)^{9/16} \right]^{8/27}} \right\}^2 ; \quad Ra_D \lesssim 10^{12} \quad [\text{Properties evaluated at } T_f]$

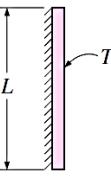
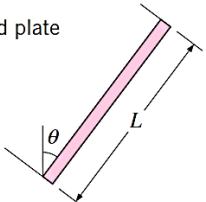
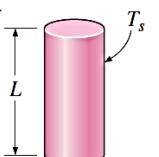
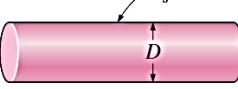
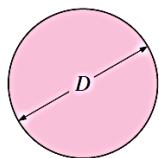
Spheres: $\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + \left(\frac{0.469}{Pr} \right)^{9/16} \right]^{4/9}} ; \quad Ra_D \lesssim 10^{11} ; \quad Pr \geq 0.7 \quad [\text{Properties evaluated at } T_f]$



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Geometry	Characteristic length L_c	Range of Ra	Nu	
Vertical plate		L	$10^4 \text{--} 10^9$ $10^9 \text{--} 10^{13}$ Entire range	$\text{Nu} = 0.59 \text{Ra}_L^{1/4}$ $\text{Nu} = 0.1 \text{Ra}_L^{1/3}$ $\text{Nu} = \left\{ 0.825 + \frac{0.387 \text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}} \right\}^2$ (complex but more accurate)
Inclined plate		L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)		A_s/p	$10^4 \text{--} 10^7$ $10^7 \text{--} 10^{11}$	$\text{Nu} = 0.54 \text{Ra}_L^{1/4}$ $\text{Nu} = 0.15 \text{Ra}_L^{1/3}$
(b) Lower surface of a hot plate (or upper surface of a cold plate)			$10^5 \text{--} 10^{11}$	$\text{Nu} = 0.27 \text{Ra}_L^{1/4}$
Vertical cylinder		L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder		D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{ 0.6 + \frac{0.387 \text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}} \right\}^2$
Sphere		D	$\text{Ra}_D \leq 10^{11}$ ($\text{Pr} \geq 0.7$)	$\text{Nu} = 2 + \frac{0.589 \text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$



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Résumé des corrélations de transfert de chaleur par convection pour le flux externe

Correlation	Geometry	Conditions ^c
$\delta = 5x Re_x^{-1/2}$	Flat plate	Laminar, T_f
$C_{f,x} = 0.664 Re_x^{-1/2}$	Flat plate	Laminar, local, T_f
$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$	Flat plate	Laminar, local, $T_f, Pr \geq 0.6$
$\delta_t = \delta Pr^{-1/3}$	Flat plate	Laminar, T_f
$\bar{C}_{f,x} = 1.328 Re_x^{-1/2}$	Flat plate	Laminar, average, T_f
$\bar{Nu}_x = 0.664 Re_x^{1/2} Pr^{1/3}$	Flat plate	Laminar, average, $T_f, Pr \geq 0.6$
$Nu_x = 0.564 Pe_x^{1/2}$	Flat plate	Laminar, local, $T_f, Pr \leq 0.05, Pe_x \geq 100$
$C_{f,x} = 0.0592 Re_x^{-1/5}$	Flat plate	Turbulent, local, $T_f, Re_x \leq 10^8$
$\delta = 0.37x Re_x^{-1/5}$	Flat plate	Turbulent, $T_f, Re_x \leq 10^8$
$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$	Flat plate	Turbulent, local, $T_f, Re_x \leq 10^8, 0.6 \leq Pr \leq 60$
$\bar{C}_{f,L} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$	Flat plate	Mixed, average, $T_f, Re_{x,c} = 5 \times 10^5, Re_L \leq 10^8$
$\bar{Nu}_L = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$	Flat plate	Mixed, average, $T_f, Re_{x,c} = 5 \times 10^5, Re_L \leq 10^8, 0.6 \leq Pr \leq 60$
$\bar{Nu}_D = C Re_D^m Pr^{1/3}$ (Table 7.2)	Cylinder	Average, $T_f, 0.4 \leq Re_D \leq 4 \times 10^5, Pr \geq 0.7$
$\bar{Nu}_D = C Re_D^m Pr^n (Pr/Pr_s)^{1/4}$ (Table 7.4)	Cylinder	Average, $T_\infty, 1 \leq Re_D \leq 10^6, 0.7 \leq Pr \leq 500$
$\bar{Nu}_D = 0.3 + [0.62 Re_D^{1/2} Pr^{1/3} \times [1 + (0.4/Pr)^{2/3}]^{-1/4} \times [1 + (Re_D/282,000)^{5/8}]^{4/5}]$	Cylinder	Average, $T_f, Re_D Pr \geq 0.2$
$\bar{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4} \times (\mu/\mu_s)^{1/4}$	Sphere	Average, $T_\infty, 3.5 \leq Re_D \leq 7.6 \times 10^4, 0.71 \leq Pr \leq 380, 1.0 \leq (\mu/\mu_s) \leq 3.2$



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Résumé des corrélations de convection pour l'écoulement dans un tube circulaire

Correlation	Conditions
$f = 64/Re_D$	Laminar, fully developed
$Nu_D = 4.36$	Laminar, fully developed, uniform q''_s
$Nu_D = 3.66$	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \frac{0.0668 Gz_D}{1 + 0.04 Gz_D^{2/3}}$	Laminar, thermal entry (or combined entry with $Pr \geq 5$), uniform $T_s, Gz_D = (D/x) Re_D Pr$
$\overline{Nu}_D = \frac{\frac{3.66}{\tanh[2.264 Gz_D^{-1/3} + 1.7 Gz_D^{-2/3}]} + 0.0499 Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432 Pr^{1/6} Gz_D^{-1/6})}$	Laminar, combined entry, $Pr \geq 0.1$, uniform $T_s, Gz_D = (D/x) Re_D Pr$
$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right]$	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$	Turbulent, fully developed, smooth walls, $3000 \leq Re_D \leq 5 \times 10^6$
$Nu_D = 0.023 Re_D^{4/5} Pr^n$	Turbulent, fully developed, $0.6 \leq Pr \leq 160$, $Re_D \geq 10,000$, $(L/D) \geq 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14}$	Turbulent, fully developed, $0.7 \leq Pr \leq 16,700$, $Re_D \geq 10,000$, $L/D \geq 10$
$Nu_D = \frac{(f/8)(Re_D - 1000) Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	Turbulent, fully developed, $0.5 \leq Pr \leq 2000$, $3000 \leq Re_D \leq 5 \times 10^6$, $(L/D) \geq 10$
$Nu_D = 4.82 + 0.0185(Re_D Pr)^{0.827}$	Liquid metals, turbulent, fully developed, uniform q''_s , $3.6 \times 10^3 \leq Re_D \leq 9.05 \times 10^5$, $3 \times 10^{-3} \leq Pr \leq 5 \times 10^{-2}$, $10^2 \leq Re_D Pr \leq 10^4$
$Nu_D = 5.0 + 0.025(Re_D Pr)^{0.8}$	Liquid metals, turbulent, fully developed, uniform T_s , $Re_D Pr \geq 100$



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Useful conversion factors

Physical quantity	Symbol	SI to English conversion	English to SI conversion
Length	<i>L</i>	1 m = 3.2808 ft	1 ft = 0.3048 m
Area	<i>A</i>	1 m ² = 10.7639 ft ²	1 ft ² = 0.092903 m ²
Volume	<i>V</i>	1 m ³ = 35.3134 ft ³	1 ft ³ = 0.028317 m ³
Velocity	<i>v</i>	1 m/s = 3.2808 ft/s	1 ft/s = 0.3048 m/s
Density	<i>ρ</i>	1 kg/m ³ = 0.06243 lb _m /ft ³	1 lb _m /ft ³ = 16.018 kg/m ³
Force	<i>F</i>	1 N = 0.2248 lb _f	1 lb _f = 4.4482 N
Mass	<i>m</i>	1 kg = 2.20462 lb _m	1 lb _m = 0.45359237 kg
Pressure	<i>p</i>	1 N/m ² = 1.45038 × 10 ⁻⁴ lb _f /in ²	1 lb _f /in ² = 6894.76 N/m ²
Energy, heat	<i>q</i>	1 kJ = 0.94783 Btu	1 Btu = 1.05504 kJ
Heat flow	<i>q</i>	1 W = 3.4121 Btu/h	1 Btu/h = 0.29307 W
Heat flux per unit area	<i>q/A</i>	1 W/m ² = 0.317 Btu/h · ft ²	1 Btu/h · ft ² = 3.154 W/m ²
Heat flux per unit length	<i>q/L</i>	1 W/m = 1.0403 Btu/h · ft	1 Btu/h · ft = 0.9613 W/m
Heat generation per unit volume	<i>q̇</i>	1 W/m ³ = 0.096623 Btu/h · ft ³	1 Btu/h · ft ³ = 10.35 W/m ³
Energy per unit mass	<i>q/m</i>	1 kJ/kg = 0.4299 Btu/lb _m	1 Btu/lb _m = 2.326 kJ/kg
Specific heat	<i>c</i>	1 kJ/kg · °C = 0.23884 Btu/lb _m · °F	1 Btu/lb _m · °F = 4.1869 kJ/kg · °C
Thermal conductivity	<i>k</i>	1 W/m · °C = 0.5778 Btu/h · ft · °F	1 Btu/h · ft · °F = 1.7307 W/m · °C
Convection heat-transfer coefficient	<i>h</i>	1 W/m ² · °C = 0.1761 Btu/h · ft ² · °F	1 Btu/h · ft ² · °F = 5.6782 W/m ² · °C
Dynamic		1 kg/m · s = 0.672 lb _m /ft · s	
Viscosity	<i>μ</i>	= 2419.2 lb _m /ft · h	1 lb _m /ft · s = 1.4881 kg/m · s
Kinematic viscosity and thermal diffusivity	<i>ν, α</i>	1 m ² /s = 10.7639 ft ² /s	1 ft ² /s = 0.092903 m ² /s

Important physical constants

Avogadro's number	$N_0 = 6.022045 \times 10^{26}$ molecules/kg mol
Universal gas constant	$\mathcal{R} = 1545.35 \text{ ft} \cdot \text{lbf/lbm} \cdot \text{mol} \cdot ^\circ\text{R}$ = 8314.41 J/kg mol · K = 1.986 Btu/lbm · mol · °R = 1.986 kcal/kg mol · K
Planck's constant	$h = 6.626176 \times 10^{-34} \text{ J} \cdot \text{sec}$
Boltzmann's constant	$k = 1.380662 \times 10^{-23} \text{ J/molecule} \cdot \text{K}$ = 8.6173×10^{-5} eV/molecule · K
Speed of light in vacuum	$c = 2.997925 \times 10^8 \text{ m/s}$
Standard gravitational acceleration	$g = 32.174 \text{ ft/s}^2$ = 9.80665 m/s ²
Electron mass	$m_e = 9.1095 \times 10^{-31} \text{ kg}$
Charge on the electron	$e = 1.602189 \times 10^{-19} \text{ C}$
Stefan-Boltzmann constant	$\sigma = 0.1714 \times 10^{-8} \text{ Btu/hr} \cdot \text{ft}^2 \cdot \text{R}^4$ = $5.669 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$
1 atm	= $14.69595 \text{ lbf/in}^2 = 760 \text{ mmHg at } 32^\circ\text{F}$ = $29.92 \text{ inHg at } 32^\circ\text{F} = 2116.21 \text{ lbf/ft}^2$ = $1.01325 \times 10^5 \text{ N/m}^2$