حلول مفصلة لسلسلة التمارين حول

تحويل لابلاس.

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التمرين الأول : إيجاد تحويل لابلاس لكل من الدوال التالية مع تحديد مجال الوجود

① لدينا : $L(e^{\omega t})(P) = \dfrac{1}{P-\omega}$ حيث $Re(P) > Re(\omega)$

إذا : $L(2e^{-6t})(P) = 2 L(e^{-6t})(P) = \dfrac{2}{P+6}$; $Re(P) > -6$

② $L(5e^{2t}) = 5 L(e^{2t})(P) = \dfrac{5}{P-2}$; $Re(P) > 2$

③ لدينا : $L(t^n)(P) = \dfrac{n!}{P^{n+1}}$ حيث $Re(P) > 0$ مهما n

$L((t^2+1)^2)(P) = L(t^4+2t^2+1)(P) = L(t^4)(P) + 2L(t^2)(P) + L(1)(P)$

$= \dfrac{4!}{P^5} + 2\cdot\dfrac{2!}{P^3} + \dfrac{1}{P} = \dfrac{24+4P^2+P^4}{P^5}$. $Re(P) > 0$

④ لدينا : $L(\cos at)(P) = \dfrac{P}{P^2+a^2}$; $L(\sin at)(P) = \dfrac{a}{P^2+a^2}$

$L(\cos(3t) + \sin(3t))(P) = \dfrac{P}{P^2+9} + \dfrac{3}{P^2+9} = \dfrac{P+3}{P^2+9}$

⑤ لدينا $L(ch\, at)(P) = \dfrac{P}{P^2-a^2}$, $L(sh\, at)(P) = \dfrac{a}{P^2-a^2}$,

$L(ch\,3t + sh(3t))(P) = \dfrac{P}{P^2-9} + \dfrac{3}{P^2-9} = \dfrac{P+3}{P^2-9} = \dfrac{1}{P-3} = L(e^{3t})(P)$

⑥ لدينا : $L(e^{\omega t}\cos at)(P) = \dfrac{P-\omega}{(P-\omega)^2+a^2}$ $L(e^{\omega t}\sin at)(P) = \dfrac{a}{(P-\omega)^2+a^2}$

ومنه : $L(2e^{-2t}\cos(3t))(P) = 2\cdot\dfrac{P+2}{(P+2)^2+9}$ $Re(P) > 3$

⑦ $L(2e^{-5t}\cos 3t + 2e^{-5t}\sin 3t)(P) = 2\left[L(e^{-5t}\cos 3t)(P) + L(e^{-5t}\sin 3t)(P)\right]$

$= 2\left[\dfrac{P+5}{(P+5)^2+9} + \dfrac{3}{(P+5)^2+9}\right] = 2\cdot\dfrac{P+8}{(P+5)^2+9}$.

⑧ $L(e^{-4t} sh(8t))(P) = \displaystyle\int_0^{+\infty} e^{-4t}\cdot\frac{e^{8t}-e^{-8t}}{2} e^{-Pt}\,dt =$

$= \dfrac{1}{2}\displaystyle\int_0^{+\infty}\left(e^{4t}-e^{-12t}\right)e^{-Pt}\,dt = \dfrac{1}{2}\int_0^{+\infty}\left[e^{-(P-4)t} - e^{-(P+12)t}\right]dt = \dfrac{1}{2}\left[\dfrac{1}{P-4} - \dfrac{1}{P+12}\right]$

$= \dfrac{8}{(P-4)(P+12)}$,

⑨ : $e^{-t}\sin^2\dfrac{t}{2} = e^{-t}\left(\dfrac{1-\cos t}{2}\right) = \dfrac{1}{2}e^{-t} + \dfrac{1}{2}e^{-t}\cos t$

$L(e^{-t}\sin^2\frac{t}{2})(P) = \dfrac{1}{2}\cdot\dfrac{1}{P+1} + \dfrac{1}{2}\cdot\dfrac{P+1}{(P+1)^2+1}$

⑩ لدينا $L(t^n e^{\omega t})(P) = \dfrac{n!}{(P-\omega)^{n+1}}$, $Re(P) > Re(\omega)$

إذا $L(e^{-2t}(t^2-1)^2)(P) = L(t^4 e^{-2t})(P) - 2L(t\,e^{-2t})(P) + L(e^{-2t})(P)$

$= \dfrac{4!}{\ldots} - 2\,\dfrac{1!}{\ldots} + \dfrac{1}{\ldots}$.

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التمرين الثاني :

خاصية الانسحاب لتحويل لابلاس : $L(f(t-a))(p) = e^{-ap} \cdot L(f(t))(p)$

1)

$$L(g(t))(p) = L(\mathcal{N}(t-3)\sin(t-3))(p) = e^{-3p} \cdot L(\mathcal{N}(t)\sin(t))(p)$$

$$= e^{-3p} \cdot L(\sin(t))(p) = e^{-3p} \cdot \frac{1}{p^2+1} = \frac{e^{-3p}}{p^2+1}$$

2)

$$L\left(\mathcal{N}\left(t-\tfrac{3}{2}\right)\sin(2t-3)\right)(p) = L\left(\mathcal{N}\left(t-\tfrac{3}{2}\right)\sin 2\left(t-\tfrac{3}{2}\right)\right)(p)$$

$$= e^{-\frac{3}{2}p} \cdot L(\mathcal{N}(t)\cdot\sin 2t)(p) = e^{-\frac{3}{2}p} \cdot \frac{2}{p^2+4}$$

3)

$$L\left(\mathcal{N}\left(t-\tfrac{\pi}{2}\right)\sin(t)\right)(p) = L\left(\mathcal{N}\left(t-\tfrac{\pi}{2}\right)\cos\left(t-\tfrac{\pi}{2}\right)\right)(p)$$

$$= e^{-\frac{\pi}{2}p} \cdot L(\mathcal{N}(t)\cdot\cos t)(p) = e^{-\frac{\pi}{2}p} L(\cos(t))(p) = e^{-\frac{\pi}{2}p} \frac{p}{p^2+1}$$

$$L\left(\mathcal{N}(t)\sin\left(t-\tfrac{\pi}{2}\right)\right)(p) = L(\cancel{\mathcal{N}}(t)(-\cos t))(p) = -L(\cos t)(p)$$

$$= -\frac{p}{p^2+1}$$

التمرين الثالث :

f_1 دورية على $\mathbb{R}_+$ ودورها $T = 2$ اذا

$$L(f_1(t))(p) = \frac{1}{1-e^{-2p}} \cdot \int_0^2 f_1(t)\, e^{-pt}\, dt = \frac{1}{1-e^{-2p}} \cdot \left[\int_0^1 1\cdot e^{-pt} dt + \int_1^2 -1 e^{-pt} dt\right]$$

$$= \frac{1}{1-e^{-2p}} \cdot \left[\left[-\frac{e^{-pt}}{p}\right]_0^1 - \left[-\frac{1}{p}e^{-pt}\right]_1^2\right] = \frac{1}{1-e^{-2p}} \cdot \left[\frac{1-e^{-p}}{p} - \frac{e^{-p}-e^{-2p}}{p}\right]$$

$$= \frac{1}{1-e^{-2p}} \cdot \frac{1-2e^{-p}+e^{-2p}}{p} = \frac{(1-e^{-p})^2}{p(1-e^{-p})(1+e^{-p})}$$

$$= \frac{1-e^{-p}}{p(1+e^{-p})}$$

$$L(f_2(t))(p) = \frac{1}{1-e^{-2p}} \cdot \left[\int_0^2 f_2(t)\, e^{-pt}\, dt\right] = \frac{1}{1-e^{-2p}} \left[\int_0^1 t e^{-pt} dt + \int_1^2 (-t+2) e^{-pt} dt\right]$$

$$= \frac{1}{1-e^{-2p}} \left[\left[-\frac{t}{p}e^{-pt}\right]_0^1 - \int_0^1 -\frac{e^{-pt}}{p} dt + \left[\frac{(-t+2)}{-p}e^{-pt}\right]_1^2 - \int_1^2 \frac{-e^{-pt}}{-p} dt\right]$$

$$= \frac{1}{1-e^{-2p}} \left[\frac{-e^{-p}}{p} + \frac{1}{p}\left[-\frac{1}{p}e^{-pt}\right]_0^1 + \frac{e^{-p}}{p} + \frac{1}{p}\left[\frac{e^{-2p}-e^{-p}}{p}\right]\right]$$

$$= \frac{1}{1-e^{-2p}} \cdot \frac{1}{p}\left[\frac{1-e^{-p}}{p} + \frac{e^{-2p}-e^{-p}}{p}\right] = \frac{1}{p^2} \frac{1-2e^{-p}+e^{-2p}}{1-e^{-2p}} =$$

$$\frac{1}{p^2}\frac{(1-e^{-p})}{(1+e^{-p})}.$$

التمرين الرابع :

$L(f(t))(P) = \frac{5}{P^2+9} = \frac{5}{3} \cdot \frac{3}{P^2+3^2} = \frac{5}{3} L(\sin 3t)(P)$

ومنه $f(t) = \frac{5}{3}\sin 3t$

$L(f(t))(P) = \frac{5P}{P^2+9} = 5 \cdot L(\cos 3t)(P)$ إذا $f(t) = 5\cos 3t$

$L(f(t))(P) = \frac{5P-3}{P^2+9} = 5 \cdot \frac{P}{P^2+9} - \frac{3}{P^2+9} = L(5\cos 3t - \sin 3t)(P)$

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$L(f(t))(P) = \frac{1}{P^3} = \frac{1}{2} \cdot \frac{2!}{P^{2+1}} = \frac{1}{2} L(t^2)(P) \Longrightarrow f(t) = \frac{1}{2}t^2$

$L(f(t))(P) = \frac{1}{P+3} = \frac{1}{P-(-3)} = L(e^{-3t})(P) \Longrightarrow f(t) = e^{-3t}$

$L(f(t))(P) = \frac{5P}{(P^2+9)^2} = \frac{-5}{2}\left(\frac{-2P}{(P^2+3^2)^2}\right) = \frac{-5}{2}\left(\frac{1}{P^2+3^2}\right)' = -\frac{5}{2} \cdot \frac{1}{3} \frac{dL(\sin 3t)(P)}{dP}$

$= \frac{-5}{6} \cdot (-L(t\sin 3t)(P)) = L\left(\frac{5}{6}t\sin 3t\right)(P)$

إذا $f(t) = \frac{5}{6}t\sin 3t$

لأن : $L(tf(t))(P) = -\frac{d(L(f(t))(P))}{dP}$

التمرين الخامس :

$F(P) = L(f(t))(P) = \frac{1}{(P^2+\omega^2)^2} = \frac{1}{P^2+\omega^2} \cdot \frac{1}{P^2+\omega^2}$

$= \frac{1}{\omega^2} \cdot \frac{\omega}{P^2+\omega^2} \cdot \frac{\omega}{P^2+\omega^2} = \frac{1}{\omega^2}\left[L(\sin\omega t)(P)\right]^2 = \frac{1}{\omega^2} L(\sin\omega t * \sin\omega t)(P)$

ومنه :

$f(t) = \frac{1}{\omega^2} \int_0^{\infty} \sin(\omega x) \cdot \sin_\omega(t-x)\, dx =$

$= \frac{1}{\omega^2} \int_0^t \sin(\omega x) \cdot [\sin\omega t \cos\omega x - \sin\omega x \cos\omega t]\, dx$

$= \frac{1}{\omega^2}\left[\sin\omega t \int_0^t \frac{\sin(2\omega x)}{2} dx + \cos\omega t \int_0^t \left\{\frac{\cos 2\omega x}{2} - \frac{1}{2}\right\} dx\right]$

$= \frac{1}{\omega^2}\left[\sin\omega t \left[-\frac{1}{4\omega}\cos(2\omega x)\right]_0^t + \cos\omega t \left[\frac{1}{4\omega}\sin(2\omega x) - \frac{1}{2}x\right]_0^t\right]$

$= \frac{1}{\omega^2}\left[\sin\omega t\left[\frac{1-\cos 2\omega t}{4\omega}\right] + \cos\omega t\left[\frac{1}{4\omega} \cdot \sin 2\omega t - \frac{1}{2}t\right]\right]$

$= \frac{1}{\omega^2}\left[\frac{\sin\omega t}{4\omega} - \frac{\sin\omega t \cos 2\omega t}{4\omega} + \frac{\cos\omega t \sin 2\omega t}{4\omega} - \frac{1}{2}t\cos\omega t\right]$

$= \frac{1}{\omega^2}\left[\frac{\sin\omega t}{4\omega} + \frac{\sin\omega t}{4\omega} - \frac{1}{2}t\cos\omega t\right] = \frac{1}{2\omega^3}\sin\omega t - \frac{1}{2\omega^2}t\cos\omega t$

$L(g(t))(P) = \frac{\omega}{(P^2+\omega^2)^2} \Rightarrow g(t) = \frac{1}{2\omega^2}\sin\omega t - \frac{1}{2\omega}t\cos\omega t$

العمرين السادس : حساب التابع f حيث $L(f(t))(p) = F(p)$

$$\text{a)}\quad L(f(t))(p) = \frac{p^2+3p+1}{(p^2+4)^2} = \frac{p^2+4}{(p^2+4)^2} + \frac{3}{2}\cdot\frac{2p}{(p^2+4)^2} - \frac{3}{(p^2+4)^2}$$

$$= \frac{1}{2}\cdot\frac{2}{p^2+4} + \frac{3}{2}\left(\frac{1}{p^2+4}\right)' - 3\cdot\frac{1}{(p^2+4)^2}$$

لدينا :

$$\frac{2}{p^2+2^2} = L(\sin 2t)(p) \quad , \quad \frac{1}{(p^2+4)^2} = \frac{\sin 2t}{2\times 8} - \frac{t\cos 2t}{2\times 4}$$

(حسب التمرين 5)

$$\left(\frac{1}{p^2+4}\right)' = -L\left(\frac{1}{2}t\sin 2t\right)(p) = L\left(-\frac{1}{2}t\sin 2t\right)(p)$$

ومنه

$$L(f(t))(p) = L\left(\frac{1}{2}\sin 2t\right)(p) - \frac{3}{2}L\left(-\frac{1}{2}t\sin 2t\right) - 3L\left(\frac{\sin 2t}{16} - \frac{t\cos 2t}{8}\right)(p)$$

$$= L\left[\frac{1}{2}\sin 2t + \frac{3}{4}t\sin 2t - \frac{3}{16}\sin 2t + \frac{3t}{8}\cos 2t\right](p)$$

$$= L\left(\left[\frac{3}{4}t + \frac{5}{16}\right]\sin 2t + \frac{3}{8}t\cos 2t\right)(p)$$

إذا :

$$f(t) = L^{-1}\left(\frac{p^2+3p+1}{(p^2+4)^2}\right) = \left(\frac{3}{4}t + \frac{5}{16}\right)\sin 2t + \frac{3}{8}t\cos 2t$$

$$\text{b)}\quad \frac{p+3}{(p^2+6p+18)^2} = \frac{p+3}{[(p+3)^2+3^2]^2} = -\frac{1}{2}\cdot\frac{-2(p+3)}{[(p+3)^2+3^2]^2}$$

$$= -\frac{1}{2}\left(\frac{1}{(p+3)^2+3^2}\right)' = -\frac{1}{2}\left[L\left(\frac{1}{\omega}\sin\omega t\cdot e^{-\omega t}\right)(p)\right]'$$

$$= -\frac{1}{2}\left[-L\left(\frac{t}{\omega}\sin\omega t\cdot e^{-\omega t}\right)(p)\right] \quad \text{حيث } (\omega = 3)$$

$$= L\left(\frac{t}{6}\sin 3t\cdot e^{-3t}\right)(p)$$

ومنه

$$L^{-1}\left(\frac{p+3}{(p^2+6p+18)^2}\right) = \frac{t}{6}\sin 3t\, e^{-3t}$$

على كل لك استخدام الخاصية

$$L(e^{\omega t}f(t))(p) = L(f(t))(p-\omega)$$

إذا :

$$L(f(t))(p) = \frac{p+3}{((p+3)^2+3^2)^2} = L(e^{-3t}\cdot g(t))(p)$$

حيث

$$L(g(t))(p) = \frac{p}{(p^2+3^2)^2} = -\frac{1}{2}\cdot\frac{-2p}{(p^2+3^2)^2} = -\frac{1}{2}\left(\frac{1}{p^2+3}\right)'$$

$$= -\frac{1}{2}\left(\frac{1}{3}L(\sin 3t)\right)' = +\frac{1}{6}L(t\sin 3t)(p)$$

$$= L\left(\frac{t}{6}\sin 3t\right) \longrightarrow g(t) = \frac{t}{6}\sin 3t$$

إذا

$$f(t) = \frac{t}{6}e^{-3t}\sin 3t$$

c) $\dfrac{3}{(p^2+6p+18)^2} = \dfrac{3}{((p+3)^2+3^2)^2} = L\left(e^{-3t}\cdot g(t)\right)$, $L(g(t))(p) = \dfrac{3}{(p^2+3^2)^2}$

حسب ما سبق في التمرين ③ إذاً $g(t) = \frac{1}{18}\sin 3t - \frac{1}{6}t\cos 3t$

$$L^{-1}\left(\frac{3}{(p^2+6p+18)^2}\right) = \left(\frac{1}{18}\sin 3t - \frac{1}{6}t.\cos 3t\right).e^{-3t}.$$

d) $\dfrac{9p+7}{(p^2+2p+2)^2} = 9\,\dfrac{p+1}{((p+1)^2+1)^2} - 2\,\dfrac{1}{((p+1)^2+1)^2} = 9.L(e^{-t}\cdot g(t))(p) - 2L(e^{-t}h(t))(p)$

حيث $L(g(t))(p) = \dfrac{p}{(p^2+1)^2}$, $L(h(t))(p) = \dfrac{1}{(p^2+1)^2}$

لدينا : $g(t) = \frac{1}{2}t\sin t$ $\quad h(t) = \frac{1}{2}\sin t - \frac{1}{2}t\cos t$

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$$L^{-1}\left(\frac{9p+7}{(p^2+2p+2)^2}\right) = \frac{9te^{-t}}{2}\sin t - 2e^{-t}\left(\frac{\sin t - t\cos t}{2}\right) =$$

$$= \frac{9}{2}t\,e^{-t}\sin t + 2t\,e^{-t}\cos t - e^{-t}\sin t$$

$$= t.e^{-t}\left(\frac{9}{2}\sin t + 2\cos t\right) - e^{-t}\sin t.$$

التمرين السابع

1) $\begin{cases} y''(t) + 2y'(t) + 5y(t) = e^{-t}\sin t \\ y'(0) = 0 \quad y(0) = 1 \end{cases}$

نطبق تحويل لابلاس على طرفي المعادلة ونستخدم خواصه (الخطية) والخواص الأخرى فنجد

4)

$$L(y'' + 2y' + 5y)(p) = L(e^{-t}\sin t)(p)$$

إذا :

$$L(y'') + 2L(y') + 5L(y) = L(e^{-t}\sin t)$$

$$\left[p^2L(y) - p.y(0) - y'(0)\right] + 2\left(pL(y) - y(0)\right) + 5L(y) = \frac{1}{(p+1)^2+1}$$

ومنه

$$(p^2+2p+5)L(y) - p - 2 = \frac{1}{p^2+2p+2}$$

إذاً

$$L(y) = \left[\frac{1}{p^2+2p+2} + p + 2\right]\frac{1}{(p^2+2p+5)}$$

$$= \frac{p^3+4p^2+6p+5}{(p^2+2p+2)(p^2+2p+5)} = A(p)$$

$$= \frac{a_1p+a_2}{p^2+2p+2} + \frac{a_3p+a_4}{p^2+2p+5}$$

$-1-i$, $-1+i$ هي p^2+2p+2 جذور (او اقطاب)

$-1-2i$ $\quad -1+2i$ هي p^2+2p+5 اقطاب

$$\lim_{p\to(-1+i)} (p^2+2p+2)A(p) = \lim_{p\to(-1+i)} (a_1p+a_2)$$

$$\frac{1}{3} = a_1(-1+i) + a_2 = (a_2-a_1) + a_1 i$$

$a_1 = 0$, $a_2 = \frac{1}{3}$: إذن

$$\lim_{p \to -1+2i} (p^2+2p+5) A(p) = \lim_{p \to -1+2i} (a_3 p + a_4)$$

$$\lim_{p \to -1+2i} \frac{(p^2+2p+2)(p+2)+1}{p^2+2p+2} = a_3(-1+2i)+a_4 = (a_4 - a_3) + 2a_3 i$$

$$\frac{(-3)(-1+2i+2)+1}{(-3)} = (a_4 - a_3) + 2a_3 i$$

$$+\frac{2}{3} + 2i = \frac{3-6i-6+1}{-3} = (a_4 - a_3) + 2a_3 i$$

إذن $a_3 = 1$ $a_4 = 1 + \frac{2}{3} = \frac{5}{3}$

$$A(p) = \frac{1}{3} \cdot \frac{1}{(p+1)^2+1} + \frac{p+1}{(p+1)^2+2^2} + \frac{2}{3} \frac{1}{(p+1)^2+2^2}$$

$$= L\left(\frac{1}{3} e^{-t} \sin t + e^{-t} \cos 2t + \frac{1}{3} e^{-t} \sin 2t\right)(p)$$

ومنه : $y(t) = e^{-t} \left[\frac{1}{3} \sin t + \frac{1}{3} \sin 2t + \cos 2t \right)$.

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2) $y'(t) = 2y(t) + t e^{2t}$, $y(0) = 1$

نطبق تحويل لابلاس على الطرفين فنجد:

$$p L(y) - y(0) = 2L(y) + L(t e^{2t})(p)$$

$$(p-2) L(y) = L(t e^{2t})(p) + 1 = \frac{1!}{(p-2)^2} + 1 = \frac{1}{(p-2)^2} + 1$$

$$L(y) = \frac{1}{(p-2)^3} - \frac{1}{p-2} = \frac{1}{2} \cdot \frac{2!}{(p-2)^{2+1}} + \frac{1}{p-2}$$

$$= \frac{1}{2} L(t^2 e^{2t})(p) + L(e^{2t})(p) = L\left(\frac{t^2}{2} e^{2t} + e^{2t}\right)(p)$$

ومنه $y(t) = \left(\frac{t^2}{2} + 1\right) e^{2t}$

التمرين 08 :

إيجاد تحويل لابلاس العكسي:

a) $$A(p) = \frac{p+13}{p^2+2p-8} = \frac{a_1}{p-2} + \frac{a_2}{p+4} = \frac{5}{2} \cdot \frac{1}{p-2} - \frac{3}{2} \cdot \frac{1}{p+4}$$

$$a_1 = \lim_{p \to 2} A(p)(p-2) = \lim_{p \to 2} \frac{p+13}{p+4} = \frac{15}{6} = \frac{5}{2}$$

$$a_2 = \lim_{p \to -4} A(p)(p+4) = \lim_{p \to -4} \frac{p+13}{p-2} = \frac{9}{-6} = -\frac{3}{2}$$

إذن :

$$L^{-1}(A(p)) = \frac{5}{2} e^{2t} - \frac{3}{2} e^{-4t}$$

b) $$A(p) = \frac{2p-1}{p^2+2p-8} = \frac{1}{2} \frac{1}{p-2} + \frac{3}{2} \times \frac{1}{p+4} \Rightarrow L^{-1}(A(p)) = \frac{1}{2} e^{2t} + \frac{3}{2} e^{-4t}$$

تابع التمرين 08

2) نضع: $L(x(t))(P) = X(P)$, $L(y(t))(P) = Y(P)$

بتطبيق تحويل لابلاس على طرفي كل من المعادلتين نجد

$$\begin{cases} L(x'(t))(P) = L(x(t)+5y(t))(P) \\ L(y'(t))(P) = L(x(t)-3y(t))(P) \end{cases} \Leftrightarrow \begin{cases} P\cdot X(P) - x(0) = X(P) + 5Y(P) \\ PY(P) - y(0) = X(P) - 3Y(P) \end{cases}$$

$$\Leftrightarrow \begin{cases} (P-1)X(P) = 5Y(P) + 1 \quad \dots(1) \\ (P+3)Y(P) = X(P) + 2 \quad \dots(2) \end{cases}$$

من (2) نجد $X(P) = (P+3)Y(P) - 2$ نعوض في (1)

$$(P-1)\big((P+3)Y(P) - 2\big) = 5Y(P) + 1$$

$$\big[(P-1)(P+3) - 5\big]Y(P) = \underset{2(P-1)}{+1} \Leftrightarrow Y(P) = \frac{2P-1}{P^2+2P-8}$$

$$X(P) = \frac{(P+3)(2P-1)}{P^2+2P-8} - 2 = \frac{2P^2+5P-3-2P^2-4P+16}{P^2+2P-8}$$

$$= \frac{P+13}{P^2+2P-8}$$

حسابيا نجد: $x(t) = \frac{5}{2}e^{2t} - \frac{3}{2}e^{-4t}$, $y(t) = \frac{1}{2}e^{2t} + \frac{3}{2}e^{-4t}$.

التمرين 09

$$\varphi(t) - \lambda \int_0^t e^{t-y}\cdot \varphi(y)\,dy = f(t)$$

$$L(f(t))(P) = L(\varphi(t))(P) - \lambda\, L(e^t)(P)\cdot L(\varphi(t))(P)$$

$$= L(\varphi(t))(P)\left[1 - \frac{\lambda}{P-1}\right] = \left(\frac{P-(1+\lambda)}{P-1}\right) L(\varphi(t))(P)$$

$$\Leftrightarrow L(\varphi(t))(P) = \frac{P-1}{P-(\lambda+1)} L(f(t))(P) = \left(1 + \frac{\lambda}{P-(\lambda+1)}\right) L(f(t))(P)$$

$$= L(f(t))(P) + \lambda\, L(e^{(\lambda+1)t})(P)\cdot L(f(t))(P)$$

$$= L(f(t))(P) + \lambda\cdot L\left(e^{(\lambda+1)t} * f\right).$$

$$\varphi(t) = f(t) + \lambda \int_0^t e^{(\lambda+1)(t-u)} f(u)\,du.$$

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