

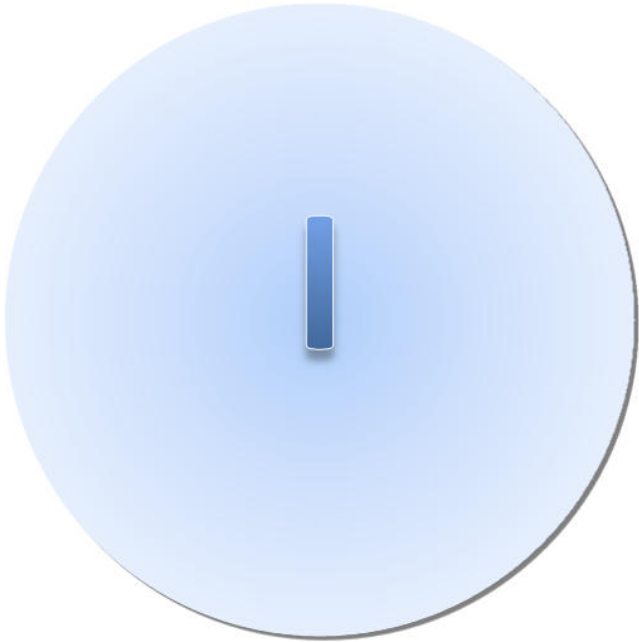
سلسلة الدروس و المحاضرات

# مدخل إلى فيزياء الحالة الصلبة

## الجزء الأول

موجه إلى طلبة السنة الثالثة فيزياء

L.M.D



من إعداد

الدكتور مبروك غوقالي

أستاذ مادة بلادة الشبكات في الفيزياء بالجامعة

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ  
بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ  
بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

بسم الله الرحمن الرحيم, الحمد لله رب العالمين, والصلاة والسلام على أشرف

المرسلين, وعلى آله وصحبه أجمعين

لقد بين لنا الله من خلال النظام الكوني, استمرارية المواد كأشياء, وتكرار الظواهر

كعلاقات سببية, لنراقبها ونذكرها وننتفع بها في حياتنا بعد أن نقف على حقيقة

سلوكها, ونستدل بها على قدرته ووحدانيته, مصداقا لقولته تعالى **سنريهم**

**آياتنا في الأفاق وفي أنفسهم حتى يتبين لهم أنه الحق....(53)** سورة فصلت

والفيزياء تعد دائما في مقدمة العلوم المعنية بدراسة المواد والظواهر الطبيعية

المختلفة, وهي التي تقود التقدم العلمي والتقني للبشر فنظرة سريعة لما يتم حولنا

من إنجازات في مجالات عدة كارتياح الفضاء, وثورة المعلومات, ونظم الاتصالات, وغيرها

كفيلة بإلقاء الضوء على الدور العظيم الذي تضلع به الفيزياء.

وفيزياء الحالة الصلبة-موضوع لهذا الكتاب- هي أحد فروع الفيزياء المعنى بالبحث في

طبيعة المواد الصلبة وخصائصها المختلفة: الميكانيكية والكهربية و المغناطيسية والحرارية

والضوئية وغيرها.

والأجسام الصلبة قد تكون بلورية فتشكل مملكة مترامية الأطراف, رعاياها من المعادن والمواد العازلة و أشباه الموصلات والموصلات الفائقة, وغيرها. وهي تسلك في الظروف المختلفة ضروبا متباينة من السلوك الذي يوحى بمجالات تطبيقية شاسعة. كما أنها قد تكون غيربلورية, ولها هي الأخرى تطبيقاتها الخاصة والكتاب الذي بين أيدينا يحاول أن يصحب القارئ العربي في جولة قصيرة إلى دنيا فيزياء الأجسام الصلبة, حيث اختيرت محتوياته بعناية لكي تلبي احتياجات المقرر الدراسي لمقياس الفيزياء الصلبة I في الجزء الأول منه و مقياس خصائص الأجسام الصلبة في جزئه الثاني وكلاهما خاص بطلاب السنة الثالثة فيزياء **L.M.D**, وقد صيغت فصول هذا الكتاب بشكل مترابط يجعل القارئ لا يجد صعوبة في الفهم و الاسترسال من فصل لآخر

والله نسأل أن يعيننا على عرض محتويات كتابنا هذا بجزأيه الأول والثاني بالطريقة التي تيسر للقارئ فهمها واستيعابها. ونأمل أن يوجهنا القارئ الكريم إذا ما صادفته هنة أو ملحوظة يرى إضافتها هنا أو هناك .. متمنين قول القائل:

إن تجر عيبا نسر الخلالا      جل من لا عيب فيه وعللا

# المحتويات

5		
15		1-1
15		-
15		-
16		2-1
17		3-1
18		4-1
18		1-4-1
20		2-4-1
20	( )	3-4-1
21		4-4-1
22		5-4-1
22		6-4-1
23		7-4-1
24		5-1
25		8-1
29		9-1
30		10-1
30		1-10-1
33		2-10-1
33		3-10-1
34		4-10-1
34		5-10-1
35		6-10-1
36	( )	7-10-1
36	( )	8-10-1
37		11-1

1

المحتويات

## المحتويات

<b>37</b>	(CS)	.1	1 النسب البصرية	
<b>38</b>	(CC)	.2		
<b>39</b>	(CFC)	.3		
<b>41</b>		12-1		
<b>42</b>	(hpc)	13-1		
<b>43</b>	( wigner - seitz )	- 14-1		
<b>45</b>		14-1		
<b>45</b>		1-14-1		
<b>45</b>		2-14-1		
<b>46</b>		3-14-1		
<b>48</b>		4-14-1		
<hr/>				
<b>51</b>		1-2		2 نماذج الأشعة السينية و الشبكة العكسية
<b>51</b>		2-2		
<b>52</b>		3-2		
<b>52</b>		4-2		
<b>53</b>		5-2		
<b>55</b>		6-2		
<b>56</b>		7-2		
<b>58</b>	( )	8-2		
<b>59</b>		1-8-2		
<b>59</b>		2-8-2		
<b>61</b>	-	3-8-2		
<b>63</b>	( )	9-2		
<b>63</b>		1-9-2		
<b>63</b>		2-9-2		

## المحتويات

			<b>2</b>
			<b>إخراج الأشعة السينية و الشبكة المعكوسة</b>
<b>66</b>	( )	<b>3-9-2</b>	
<b>68</b>		<b>4-9-2</b>	
<b>70</b>		<b>5-9-2</b>	
<b>71</b>		<b>6-9-2</b>	
<b>74</b>		<b>7-9-2</b>	
<b>75</b>		<b>8-9-2</b>	
<b>78</b>		<b>9-9-2</b>	
<b>80</b>		<b>10-2</b>	
<b>83</b>		<b>11-2</b>	
			<b>3</b>
			<b>الروابط البورية و الخصائص المرونية</b>
<b>89</b>		<b>1-3</b>	
<b>89</b>		<b>2-3</b>	
<b>92</b>		<b>3-3</b>	
<b>96</b>		<b>4-3</b>	
<b>98</b>		<b>5-3</b>	
<b>99</b>		<b>6-3</b>	
<b>103</b>		<b>6-3</b>	
<b>105</b>		<b>9-3</b>	
<b>105</b>		<b>1-9-3</b>	
<b>109</b>		<b>2-9-3</b>	
<b>111</b>		<b>3-9-3</b>	
<b>114</b>		<b>3-9-3</b>	

## المحتويات

3

الروابط البورصة و الخصائص البورصة

115

5-9-3

118

6-9-3

120

7-9-3

122

7-9-3

4

اهزازات الشبكة البورصة و الخصائص البورصة

137

1-4

137

2-4

3-4

139

(

)

141

1-3-4

145

2-3-4

4-4

147

(

)

154

5-4

159

6-4

162

7-4

162

1-7-4



# المحتويات

4

اهزازات الشبكة السورية و الخصائص المرئية

163

-

165

-

168

-

172

2-7-4

173

3-7-4

175

4-7-4

176

-

178

-

183

الفصل الأول

# البنى البلورية

1-1 مقدمة

:

99%

:

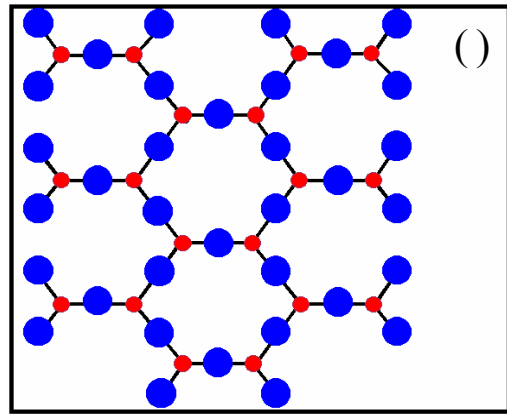
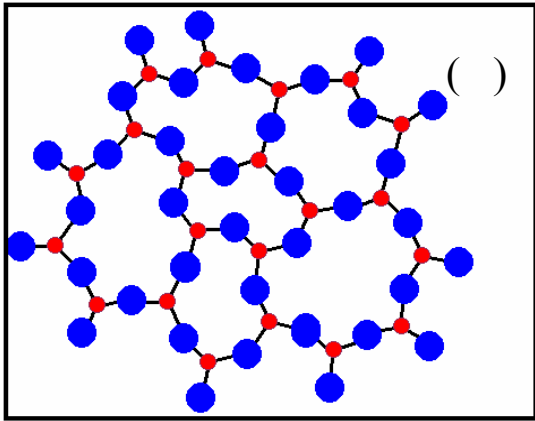
أ - اطواد الصلبة البلورية

" " " "

ب- اطواد الصلبة الابلورية

( )

(1.1).



( ) - ( ) : (1.1)

"

"

:

"

"

2-1 الشبكة البلورية

3-1 البنية البلورية:

)

(

:

-1

-2

-3

-4

( ) ( ) .( )

: ((3.1) )

شبكة بلورية + قاعدة (أساس) = بنية بلورية

( )  $\vec{r}'$

$\vec{r}$

:

(1-1)  $\vec{r}' = \vec{r} + \vec{R}$

( )

( )

$\vec{R}$

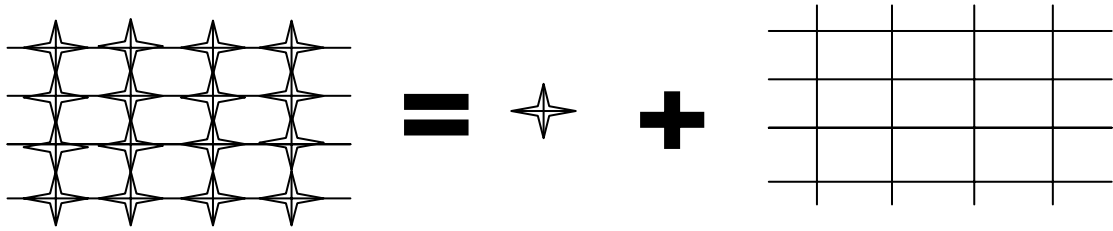
:

(2-1)  $\vec{R} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3$

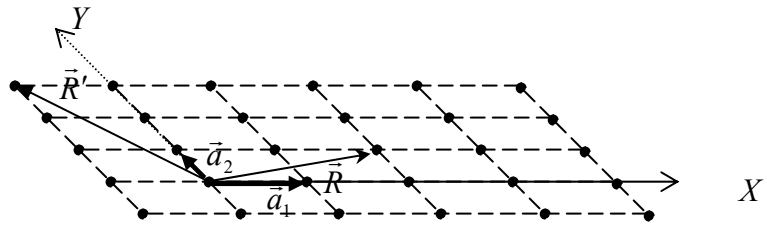
$n_3 \ n_2 \ n_1$

$\vec{a}_3 \ \vec{a}_2 \ \vec{a}_1$  :

. ((4.1) )



:(3.1)



$\vec{R}' = -\vec{a}_1 + 3\vec{a}_2$      $\vec{R} = 2\vec{a}_1 + \vec{a}_2$     :(4.1)

4-1 التناظر البلوري

( )

$\vec{R}$

:

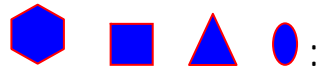
1-4-1 التناظر الدوراني

$\theta = \frac{2\pi}{n}$

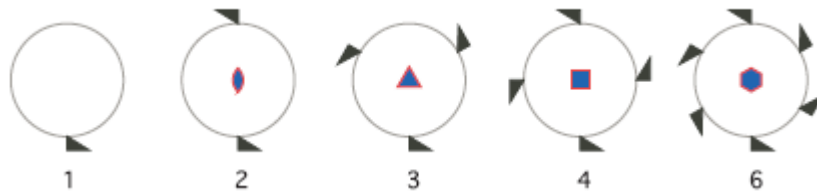
$n$      $A_n$     ((5.1)    )    6·4·3·2·1

$$\frac{\pi}{3} \quad \frac{\pi}{2} \quad \frac{2\pi}{3} \quad \pi \quad 2\pi$$

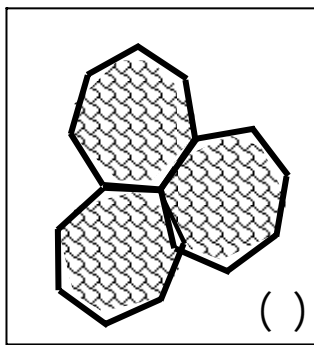
(6.1)



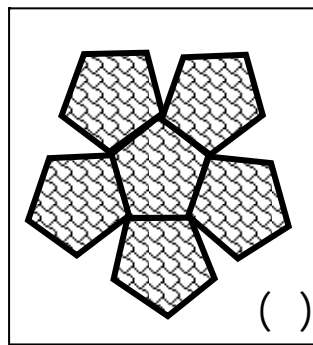
$A_6 \quad A_4 \quad A_3 \quad A_2$



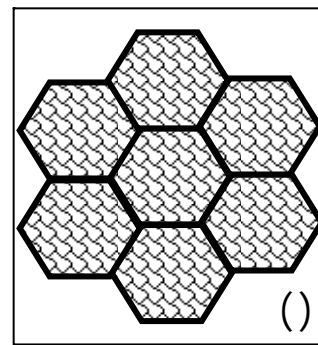
(5.1)



( )



( )



(6.1)

( )

$$a \quad (7.1)$$



(3-1)

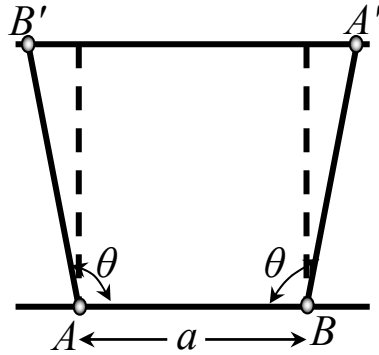
$$B' A' = AB(1 + 2 \cos(\theta)) = a(1 + 2 \cos(\theta))$$

$$a \quad A'B' \quad AB \quad A'B'$$

$$\theta \quad 0 \pm 1 \pm 2 \quad 2 \cos(\theta)$$

$$\theta = \frac{360^\circ}{n}$$

.90° 120° 60° 0° 360° 180°



:(7.1)

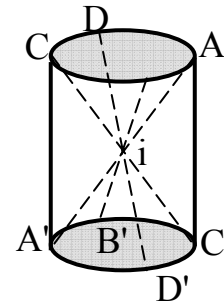
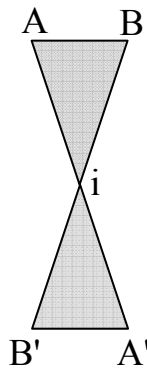
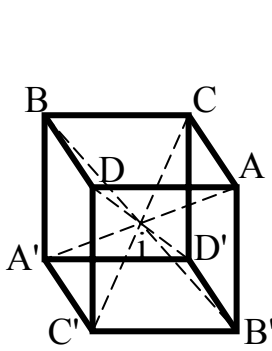
2-4-1 التناظر الانقلابي:

$\vec{r}$

.c (c)

$-\vec{r}$

(8.1)



:(8.1)

3-4-1 التناظر الانعكاسي (المرآتي) وفق مستو:

.m

)AYKD

ADBPYK

ACDKYL

(9.1)

ACFEGYLH

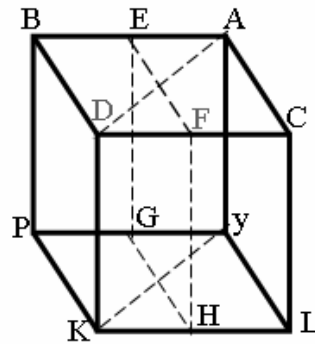
(



.(

)EFHG

EFDBPGHK



:(9.1)

4-4-1 التناظر الدوراني الانقلابي:

$$c \quad A_n \quad \bar{A}_n$$

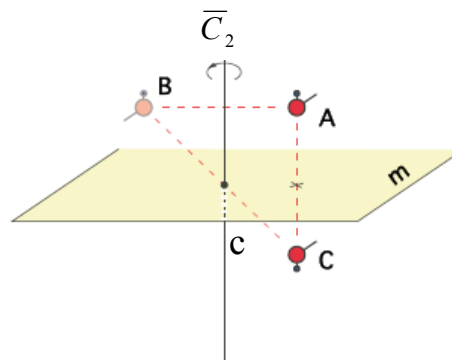
$$n \quad \bar{n}$$

$$c \quad \bar{A}_2 \quad (10.1)$$

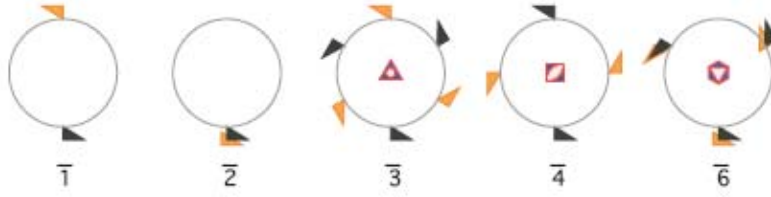
$$.m$$

:  $\bar{A}_6 \quad \bar{A}_4 \quad \bar{A}_3 \quad \bar{A}_2$

.(11.1)



.(10.1)  $\bar{2} \quad m$



:(11.1)

5-4-1 التناظر الدوراني الانعكاسي:

$$\frac{n}{m}$$

$m$

$A_n$

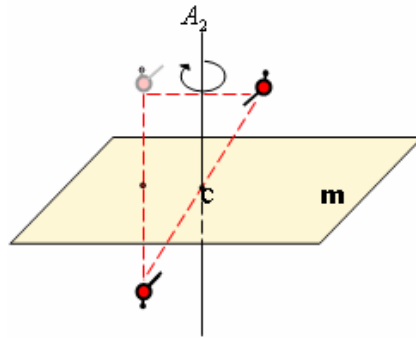
$\cdot nm$

$m$

$A_2$

(12.1)

.c



$$\cdot \frac{2}{m} c$$

:(12.1)

$mmm$

$nm$

:

6-4-1 تمثيل عمليات التناظر بالامتدادات:

$$\sum x'_1, x'_2, x'_3 \quad (X'_1, X'_2, X'_3) \quad (X_1, X_2, X_3) \quad \sum x_1, x_2, x_3 \quad (X_1, X_2, X_3) \quad \cdot (X'_1, X'_2, X'_3)$$

(4-1)

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

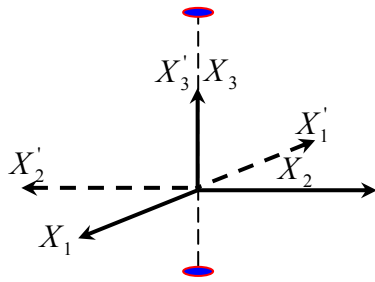
(5-1)

$$C_{ij} = \cos(X'_i, X_j)$$

$j = 1, 2, 3$  و

$i = 1, 2, 3$ :

:(5-1) (4-1)

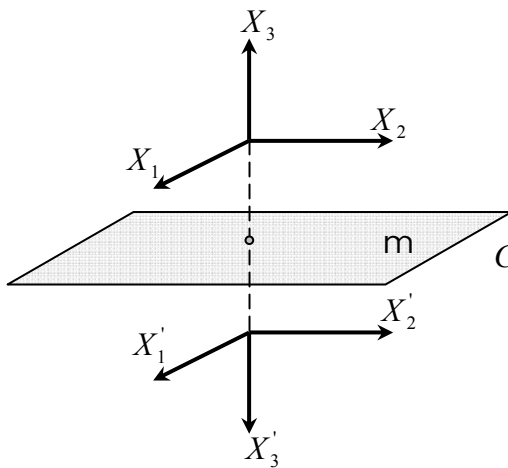


$$C_{ij}(X_3, \pi) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

:  $A_2$

$X_3$

**1**

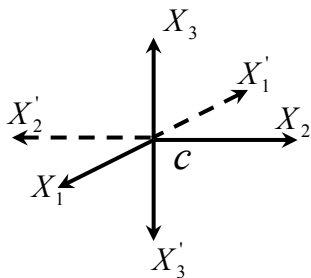


$$C_{ij}(m \perp X_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

:  $X_3$

:  $c$

**2**



$$C_{ij}(i) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

7-4-1 الزمرة النقطية و الزمرة الفضائية:

:

. n -  
 . m -  
 . i -  
 . -  
 :  
 . ... :

5-1 خلية الوحدة:

: ( )

( )

$\vec{a}_3 \quad \vec{a}_2 \quad \vec{a}_1$

(6-1)

$$V_e = \vec{a} \cdot (\vec{b} \times \vec{c})$$

:

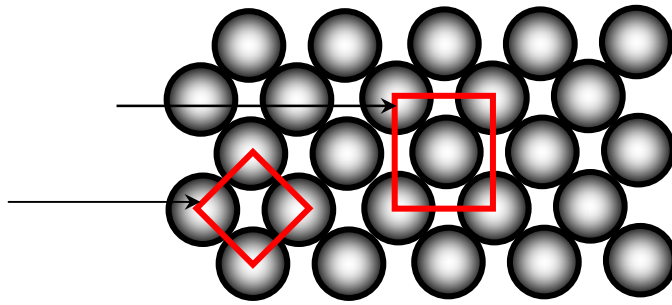
...

(13.1)

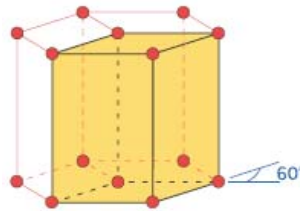
$$\left(1 + \frac{1}{4} \times 4 = 2\right)$$

(14.1)

$$\left(\frac{1}{4} \times 4 = 1\right)$$



:(13.1)



:(14.1)

8-1 تصنيف الشبكات البلورية الفضائية:

" Bravais "

$$(( \quad 32 \quad )) \quad (( \quad 230 \quad ))$$

(1.1) (2.1)

( )

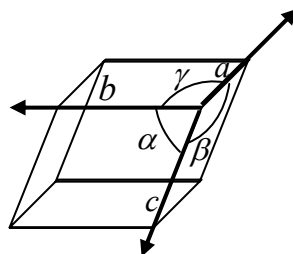
(S) (C) (F)

(BC)

)

$$\bar{c} \quad \bar{b} \quad \bar{a} \quad ) \quad c \quad b \quad a \quad (c) \quad - \quad (15.1)$$

$$\cdot \quad \gamma = (\bar{a}, \bar{b}) \quad \beta = (\bar{c}, \bar{a}) \quad \alpha = (\bar{c}, \bar{b}) \quad \gamma \quad \beta \quad \alpha \quad (\bar{a}_3 \quad \bar{a}_2 \quad \bar{a}_1)$$



:(15.1)



$a = b = c :$

6- الفئة الثلاثية

$\alpha = \gamma = \beta \neq \frac{\pi}{2}$

$- A_2$

$\frac{2}{3} \frac{3A_2}{m} c :$

(c)

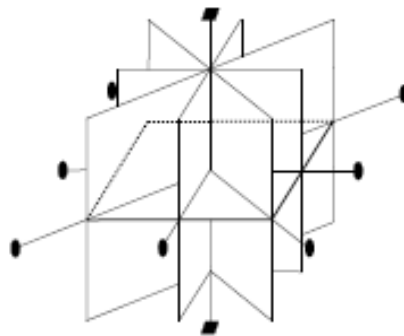
$\alpha = \gamma = \frac{\pi}{2}, \beta = 120^\circ \quad a = b = c :$

7- الفئة السداسية:

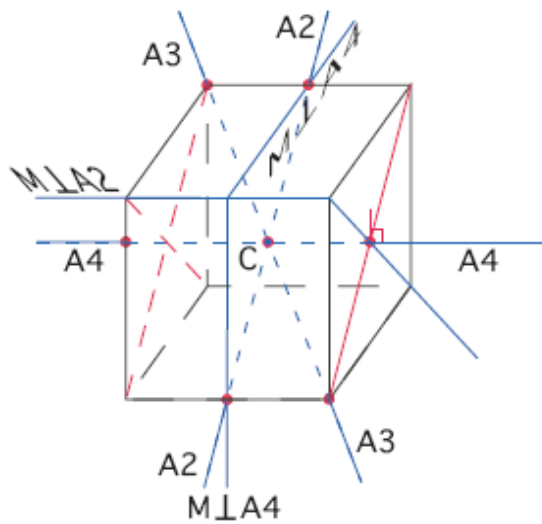
$- A_6$

(c)

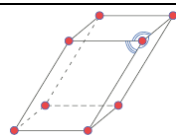

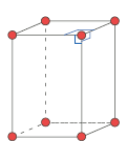
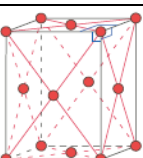
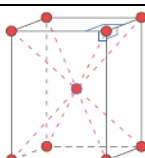
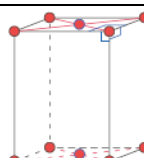
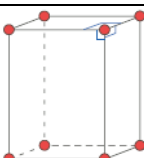
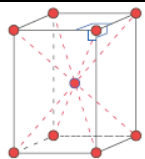
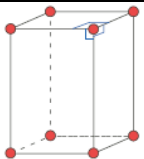
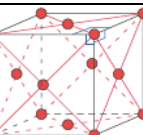
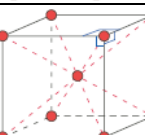
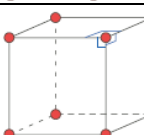

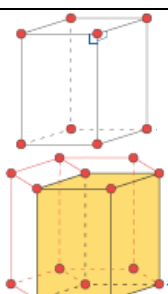
$\frac{6}{m} \frac{2}{m} \frac{2}{m} \quad \frac{A_6}{m} \frac{3A_2}{3m} \frac{3A_2}{3m} c :$



:(16.1)



:(17.1)

	Face centrée	Corps centrée	Base centrée	Simple	
$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq \pi/2$					Triclinique
$a \neq b \neq c$ $\alpha = \gamma = \pi/2 \neq \beta$					Monoclinique
$a \neq b \neq c$ $\alpha = \beta = \gamma = \pi/2$					Orthorhombique
$a = b \neq c$ $\alpha = \beta = \gamma = \pi/2$					Quadratique
$a = b = c$ $\alpha = \beta = \gamma = \pi/2$					Cubique
$a = b = c$ $\alpha = \beta = \gamma \neq \pi/2, < 120^\circ$					Rhomboédrique
$a = b \neq c$ $\alpha = \beta = \pi/2, \gamma = 120^\circ$					Hexagonal

:(1.1)



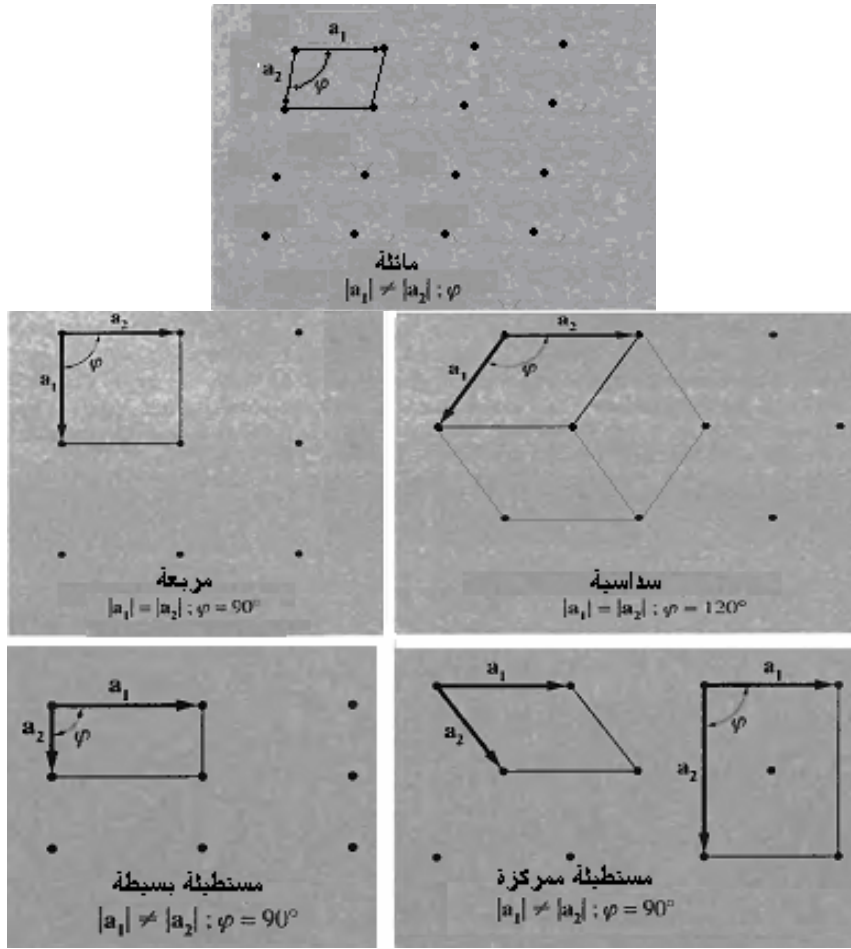
1		$c$	$\bar{1}$	Triclinique
2	$A_2$	$\frac{A_2}{m} c$	$\frac{2}{m}$	Monoclinique
4		$\frac{A_2}{m} \frac{A_2}{m} \frac{A_2}{m} c$	$\frac{2}{m} \frac{2}{m} \frac{2}{m}$	Orthorhombique
2		$\frac{A_4}{m} \frac{2A_2}{2m} \frac{2A_2}{2m} c$	$\frac{4}{m} \frac{2}{m} \frac{2}{m}$	Quadratique
3		$\frac{3A_4}{m} 4A_3 \frac{6A_2}{6m} c$	$\frac{4}{m} \frac{2}{3} \frac{2}{m}$	<b>Cubique</b>
1		$A_3 \frac{3A_2}{3m} c$	$\frac{2}{3} \frac{2}{m}$	Rhomboédrique
1		$\frac{A_6}{m} \frac{3A_2}{3m} \frac{3A_2}{3m} c$	$\frac{6}{m} \frac{2}{m} \frac{2}{m}$	Hexagonal

:(2.1)

## 9-1 تصنيف الشبكات البلورية المستوية :

(c)

 $b \ a$  $\varphi = (\bar{a}, \bar{b}) \quad \varphi \quad b \ a$  $\frac{2\pi}{4}$  $2\pi \quad \pi$  $\frac{2\pi}{6} \quad \frac{2\pi}{3}$  $, 2mm$  $, 2mm$  $, 4mm$ .((16.1) )  $6mm$



(18.1):

10-1 التعريف ببعض خصائص الشبكات البلورية:

1-10-1 تحديد مواضع و متجهات المستويات البلورية:

( )

$c, b, a$

: "Miller "

(X,Y,Z)

: •  
 $\vec{c}, \vec{b}, \vec{a}$

(X,Y,Z)

.c,b,a

( )

.(hkl) :

(-)

(3a : 2b : 1c)

(X,Y,Z)

(19.1)

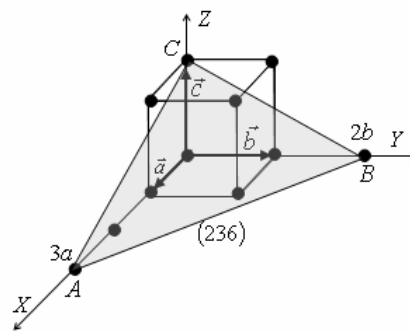
ABC

(2/6, 3/6, 6/6) (6)

(1/3, 1/2, 1)

.(236)

h=2,k=3,l=6



.ABC

:(19.1)

.{hkl}

(20.1)

(100), (010), (001), (100), (010), (001)

{001}

( )

[100] (X)

.[uvw]

:(21.1)

) [001] (Z)

.[010] (Y)

:

<110>

<uvw>

[0 1 1], [0 1 1], [0 1 1], [0 1 1], [1 0 1], [1 0 1], [1 0 1], [1 0 1], [1 1 0], [1 1 0], [1 1 0], [1 1 0]

l = w, k = v, h = u

(hkl)

[uvw]

.(110)

[110]

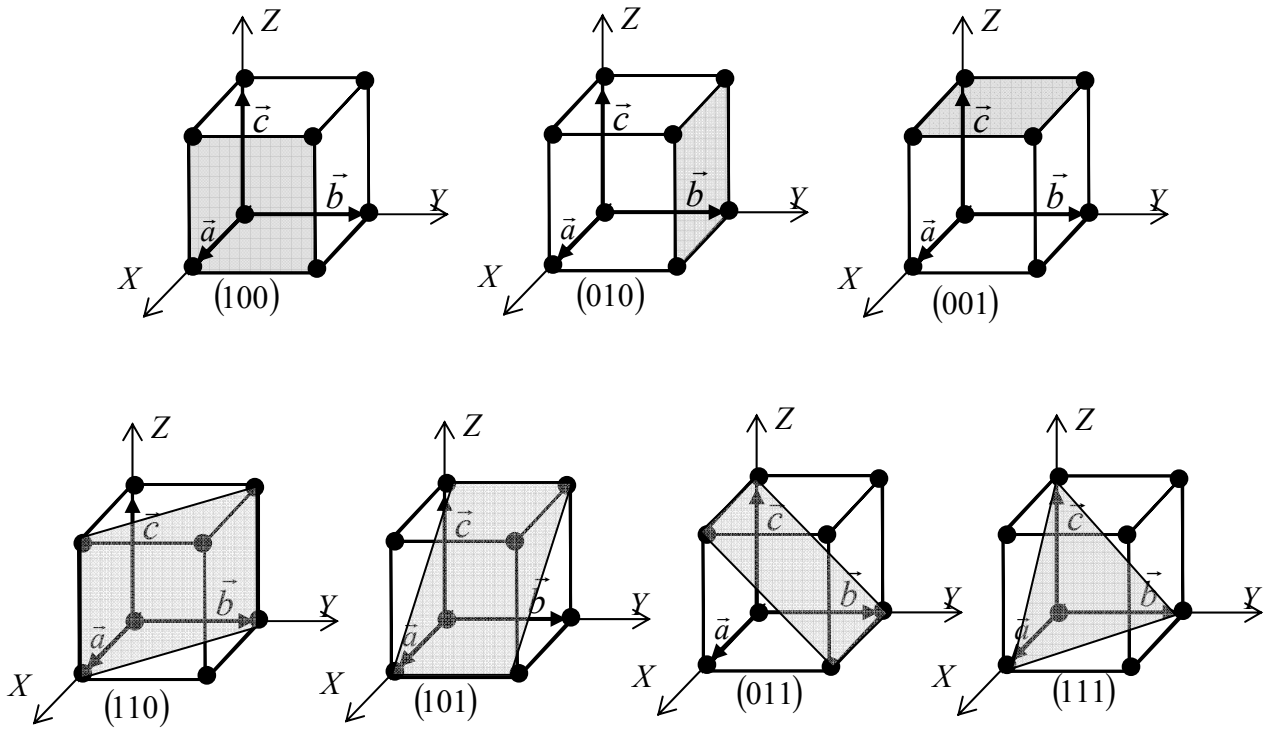
(100)

[100]

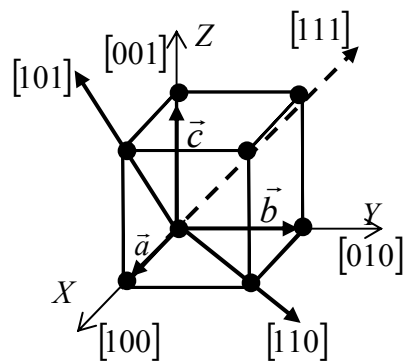
$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ :

(xyz)

.  $(\frac{1}{2}, 0, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, 0), (0, \frac{1}{2}, \frac{1}{2})$  :



:(20.1)



:(21.1)

2-10-1 قرائن ميلر - براخي للفتة السداسية:

$X_1, X_2, X_3$

$(X_1, X_2, X_3, X_4)$

$X_4 \quad 120^\circ$

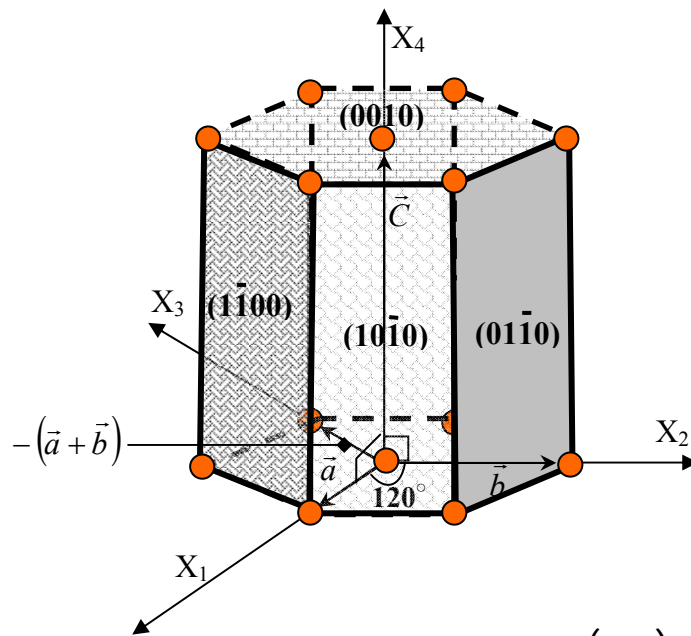
$$- \quad (hki) \quad (22.1) \quad \cdot \quad (h+k+i=0)$$

$(1/2:1/2:-1:1/3) :$

$(2,2,-1,3)$

$(X_1, X_2, X_3, X_4)$

$\cdot (hki) = (3,3,\bar{6},2) \quad 6 :$



:(22.1)

3-10-1 المسافة الفاصلة بين المستويات البلورية المتوازية:

$d_{hkl}$

$a$

$:$

$a$

$l, k, h$

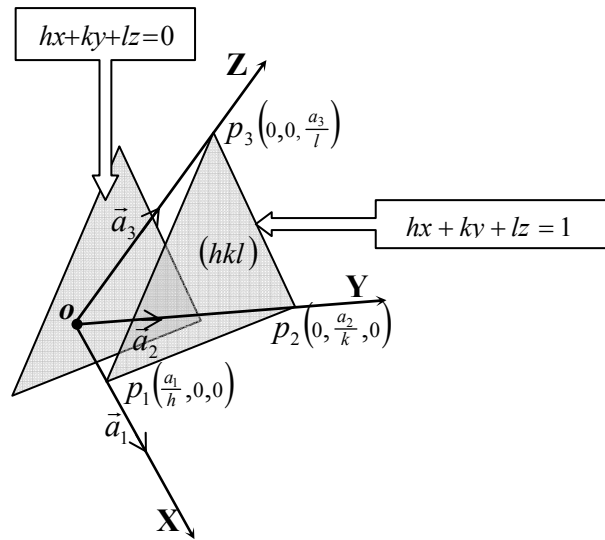
(7-1)

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

(111) (110) (100)



$\cdot (hkl)$



$\cdot hx+ky+lz=0$        $hx + ky + lz = 1$       **:(23.1)**

(14-1)       $hx + ky + lz = m$

$(m = 0, \pm 1, \pm 2, \dots)$        $: m$

$\cdot ((23.1) \quad )$        $(m = \pm 1)$        $(m = 0)$

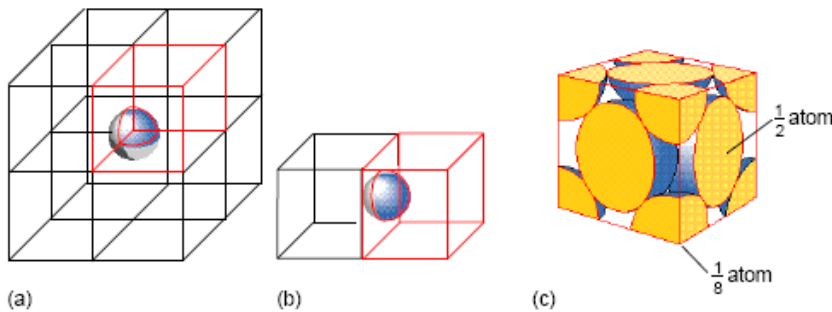
**6-10-1 عدد عقد خلية الوحدة  $n_a$  :**

(24.1)       $( \quad )$

$(1 = 8 \times \frac{1}{8}) :$

$(4 = 3 + 1 = 3 \times \frac{1}{2} + 8 \times \frac{1}{8}) :$

$(3 = 6 \times \frac{1}{2}) :$



**:(24.1)**

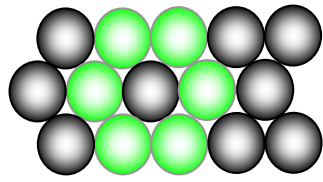
7-10-1 عدد الجوار الأول (عدد التناسق)  $Z$ :

( )

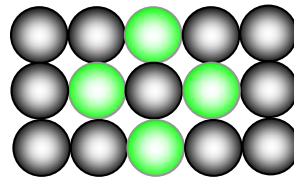
(25.1).

$R_z$ .

\_\_\_\_\_



6:

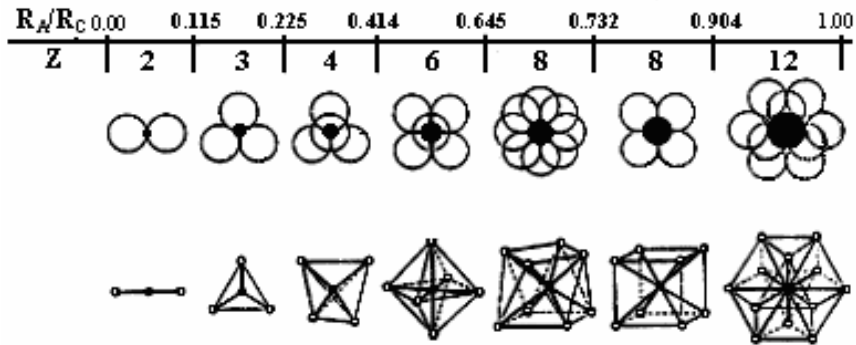


4:

( ) : (25.1)

(26.1)

$R_A/R_C$  ( / ) ( / )



$R_A/R_C$

: (26.1)

8-10-1 عامل التصبئة (الرص)  $F_R$ :

\_\_\_\_\_

(15-1)

$$F_R = \sum_i \frac{n_a^i v_a^i}{V}$$

\_\_\_\_\_



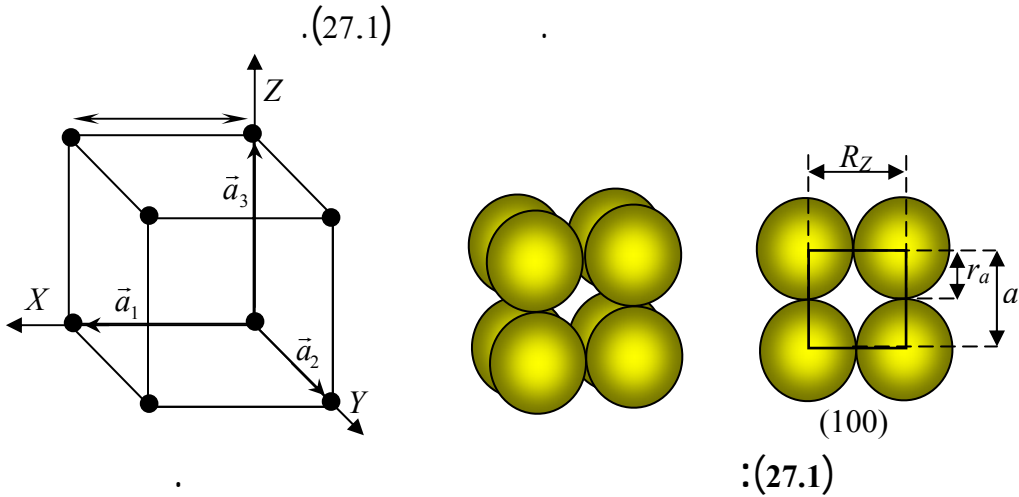
ملاحظة:  $n_a^i$  : عدد الذرات في الخلية  $i$  :  $v_a^i$  : حجم الذرة  $i$  :  $V$  : حجم الخلية :  $m_i$  : كتلة الذرة  $i$  :  $\rho$  : الكثافة : (15-1)

$$(16-1) \quad \rho = \sum_i \frac{n_a^i m_a^i}{V}$$

11-1 دراسة شبكات الفئة المكعبة:

1. الشبكة المكعبة البسيطة (CS):

( )



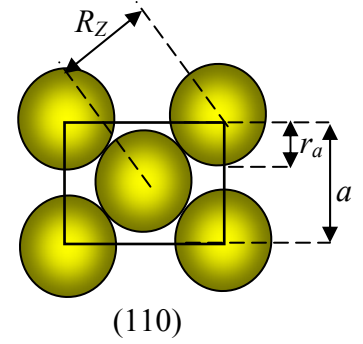
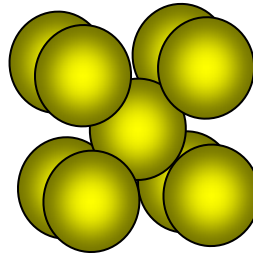
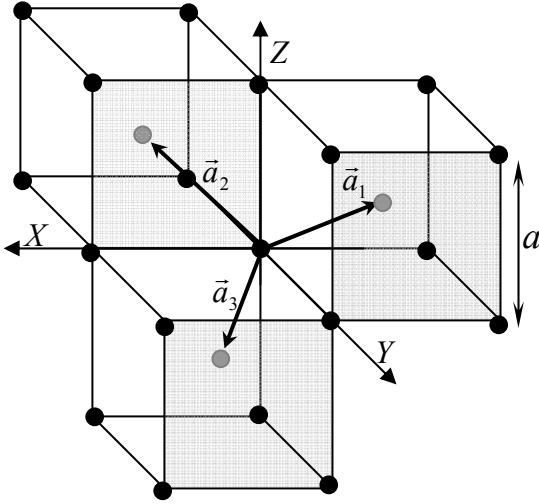
- خصائص الشبكة المكعبة البسيطة:

1.  $\vec{a}_1 = a\vec{i}, \vec{a}_2 = a\vec{j}, \vec{a}_3 = a\vec{k}$  :
2.  $\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3 = n_1a\vec{i} + n_2a\vec{j} + n_3a\vec{k}$  :
3.  $V_e = \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) = a\vec{i} \cdot (a\vec{j} \times a\vec{k}) = a^3$  :
4.  $n_a = \frac{1}{8} \times 8 = 1$  : (0,0,0)
5.  $z = 6$  :
6.  $R_z = 2r_a = a$  :  $r_a$  :
7.  $F_R^{CS} = \frac{n_a v_a}{V} = \frac{1 \times \frac{4}{3} \pi r_a^3}{a^3} = \frac{\frac{4}{3} \pi (\frac{a}{2})^3}{a^3} = \frac{\pi}{6} = 0.52$  :

$$\sigma_{hkl} = \frac{n_{hkl} S_a}{S_{hkl}} = \frac{(4 \times \frac{1}{4}) \pi r_a^2}{a^2} = \frac{\pi (\frac{a}{2})^2}{a^2} = \frac{\pi}{4} = 0.78 : \{100\} \quad .8$$

2. الشبكة المكعبة الممركزة (CC):

(27.1).



:(28.1)

- خصائص الشبكة المكعبة الممركزة:

$$\vec{a}_3 = \frac{a}{2}(\vec{i} + \vec{j} - \vec{k}) \quad \vec{a}_2 = \frac{a}{2}(\vec{i} - \vec{j} + \vec{k}) \quad \vec{a}_1 = \frac{a}{2}(-\vec{i} + \vec{j} + \vec{k}) : \quad .1$$

: : : : : .2

$$a_1 = a_2 = a_3 = \frac{\sqrt{3}}{2} a, \quad \gamma = \beta = \alpha = \arccos \left( \frac{\vec{a}_1 \cdot \vec{a}_2}{\|\vec{a}_1\| \|\vec{a}_2\|} \right) = \arccos \left( -\frac{1}{3} \right) = 109.47^\circ : \quad .3$$

$$\vec{R} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3 = \frac{a}{2} \left( (-n_1 + n_2 + n_3) \vec{i} + (n_1 - n_2 + n_3) \vec{j} + (n_1 + n_2 - n_3) \vec{k} \right) : \quad .4$$

$$V_e = \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) = \frac{a^3}{2} : \quad .5$$

$$\left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) (0,0,0) \quad n_a = \frac{1}{8} \times 8 + 1 = 2 : \quad .6$$

$$z = 8 : \quad .6$$

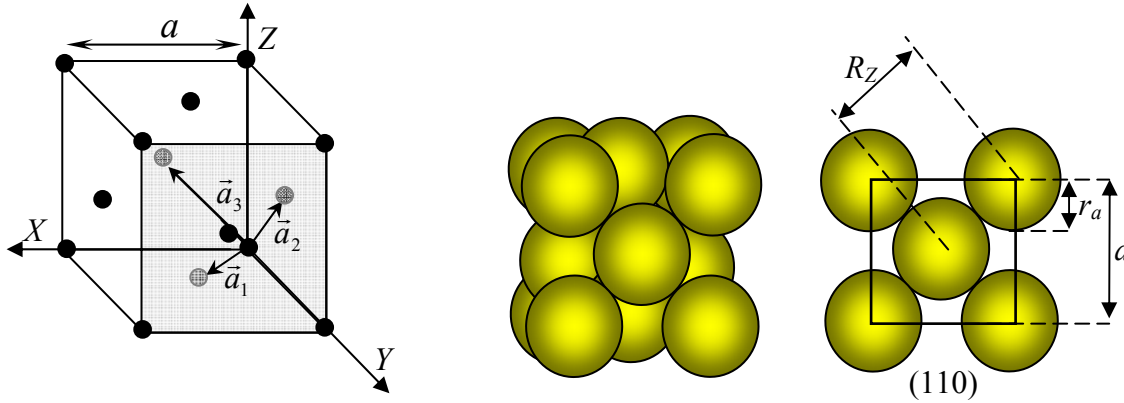
$$:r_a: \quad R_z = 2r_a = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}}{2} a : \quad .7$$

$$F_R^{CC} = \frac{n_a v_a}{V} = \frac{2 \times \frac{4}{3} \pi r_a^3}{a^3} = \frac{\frac{8}{3} \pi \left(\frac{\sqrt{3}}{4} a\right)^3}{a^3} = \frac{\pi \sqrt{3}}{8} = 0.68 : \quad .8$$

$$\sigma_{hkl} = \frac{n_{hkl} S_a}{S_{hkl}} = \frac{(4 \times \frac{1}{4} + 1) \pi r_a^2}{\sqrt{2} a^2} = \frac{2 \pi \left(\frac{\sqrt{3}}{4} a\right)^2}{\sqrt{2} a^2} = \frac{3 \pi}{8 \sqrt{2}} = 0.83 : \{110\} \quad .9$$

3. الشبكة المكعبة الممركزة الأوجه (CFC)

(29.1)



(29.1)

- خصائص الشبكة المكعبة الممركزة الأوجه:

$$\bar{a}_3 = \frac{a}{2}(\bar{i} + \bar{j}) \quad \bar{a}_2 = \frac{a}{2}(\bar{i} + \bar{k}) \quad \bar{a}_1 = \frac{a}{2}(\bar{j} + \bar{k}) : \quad .1$$

: : : : : .2

$$a_1 = a_2 = a_3 = \frac{\sqrt{2}}{2} a, \quad \gamma = \beta = \alpha = \arccos\left(\frac{\bar{a}_1 \cdot \bar{a}_2}{\|\bar{a}_1\| \|\bar{a}_2\|}\right) = \arccos\left(\frac{1}{2}\right) = 60^\circ$$

3. شعاع الانسحاب الأساسي:

$$\vec{R} = n_1 \bar{a}_1 + n_2 \bar{a}_2 + n_3 \bar{a}_3 = \frac{a}{2} \left( (n_2 + n_3) \bar{i} + (n_1 + n_3) \bar{j} + (n_1 + n_2) \bar{k} \right)$$

$$. a^3 \quad V_e = \bar{a}_1 \cdot (\bar{a}_2 \times \bar{a}_3) = a^3 / 4 : \quad .4$$

$$\left(\frac{1}{2}, 0, \frac{1}{2}\right) \quad \left(0, \frac{1}{2}, \frac{1}{2}\right) \quad (0, 0, 0) \quad n_a = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4 : \quad .5$$

$$\cdot \left( \frac{1}{2}, \frac{1}{2}, 0 \right)$$

$$z = 12 : \quad .6$$

$$: r_a : \quad R_z = 2r_a = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{2}}{2} a : \quad .7$$

$$F_R^{CFC} = \frac{n_a v_a}{V} = \frac{4 \times \frac{4}{3} \pi r_a^3}{a^3} = \frac{\frac{16}{3} \pi \left(\frac{\sqrt{3}}{4} a\right)^3}{a^3} = \frac{\pi \sqrt{2}}{6} = 0.74 : \quad .8$$

9. المستويات الأكثر كثافة هي المستويات {111} :

$$\sigma_{hkl} = \frac{n_{hkl} s_a}{s_{hkl}} = \frac{(4 \times \frac{1}{4} + 1) \pi r_a^2}{\frac{\sqrt{3} a^2}{2}} = \frac{4 \pi \left(\frac{\sqrt{2}}{4} a\right)^2}{\sqrt{3} a^2} = \frac{\pi}{2\sqrt{3}} = 0.9$$

(CFC)	(CC)	(CS)	( : a )	*
$a^3$	$a^3$	$a^3$	.	*
4	2	1	.	*
$\frac{4}{a^3}$	$\frac{2}{a^3}$	$\frac{1}{a^3}$	.	*
12	8	6	.	*
$a\sqrt{2}/2$	$a\sqrt{3}/2$	$a$	.	*
6	6	12	.	*
$a$	$a$	$a\sqrt{2}$	.	*
{111}	{110}	{100}	.	*

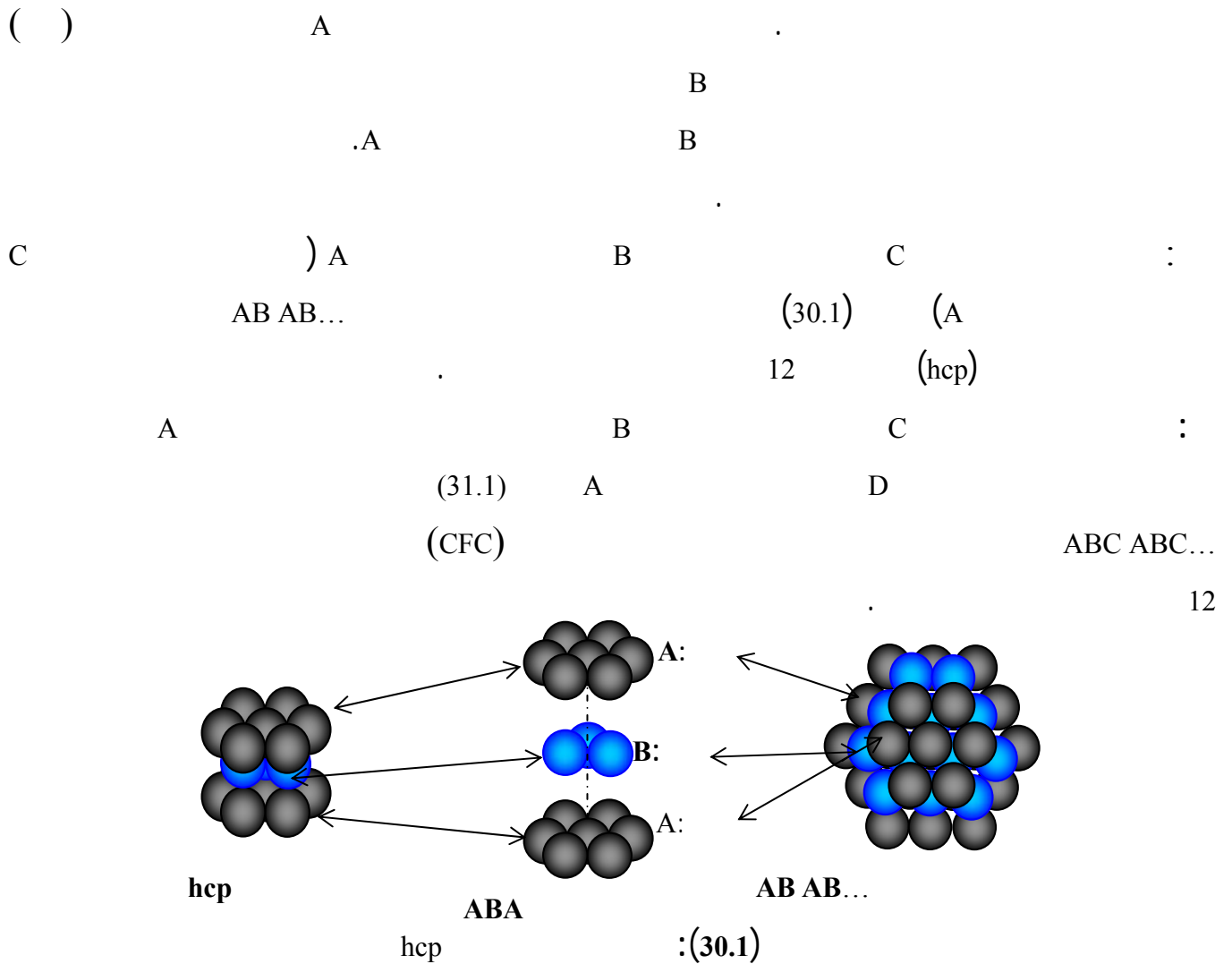
:(3.1)

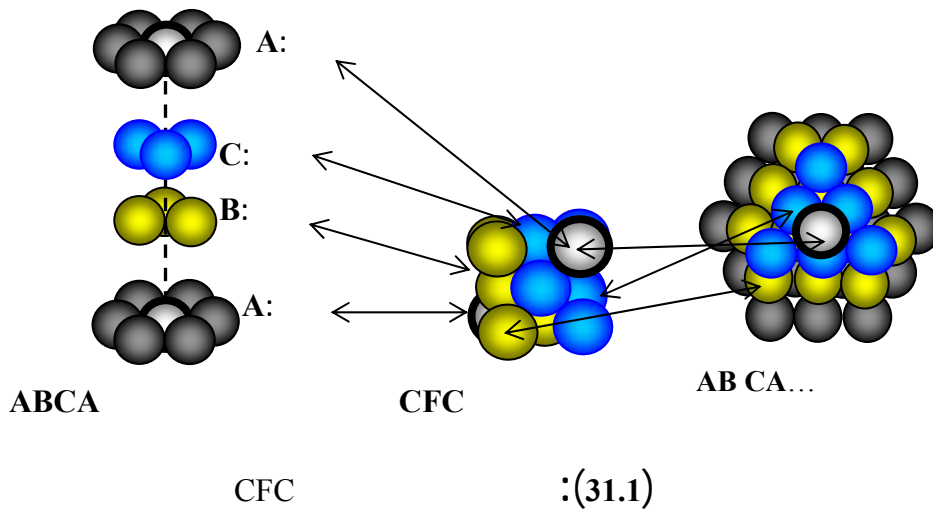
(4.1)

(CFC)		(CC)		(CS)	
$a(\text{Å})$		$a(\text{Å})$		$a(\text{Å})$	
3.15	<i>Mo</i>	5.26	<i>Ar</i>		
2.87	<i>Fe</i>	4.5	<i>Al</i>		
5.2	<i>Ba</i>	5.58	<i>Ca</i>		
3.31	<i>Ta</i>	5.30	<i>Ac</i>		
3.2	<i>V</i>	4.95	<i>Pb</i>		
3.16	<i>W</i>	3.92	<i>Pt</i>		
				(α)	<i>Po</i>

:(4.1)

12-1 التنبئة المتراصة:



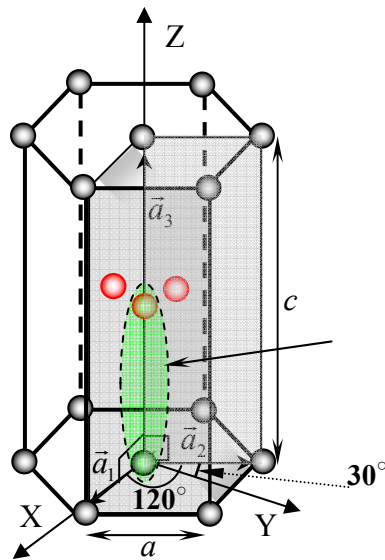


13-1 الشبكة السداسية المترابطة (hpc):

(32.1)  $(\frac{2}{3}, \frac{1}{3}, \frac{1}{2})$  (0,0,0)

$(\frac{2}{3}, \frac{1}{3}, \frac{1}{2})$  :

$(\frac{2}{3}, \frac{1}{3}, \frac{1}{2})$   $(\frac{2}{3}, \frac{1}{3}, \frac{1}{2})$



(32.1):

خصائص الشبكة السداسية المترابطة (hcp) :

$$\vec{a}_1 = a\vec{i}, \vec{a}_2 = \frac{\sqrt{3}}{2}a\vec{j} - \frac{1}{2}a\vec{i}, \vec{a}_3 = c\vec{k} : \quad .1$$

$$: \quad : \quad .2$$

$$a_1 = a_2 = a, \quad a_3 = c, \quad \beta = \alpha = 90^\circ, \gamma = 120^\circ$$

$$\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3 = n_1 \frac{a}{2}(2n_1 - n_2)\vec{i} + \frac{\sqrt{3}}{2}n_2a\vec{j} + n_3c\vec{k} : \quad .3$$

$$\frac{c}{a} = \sqrt{\frac{8}{3}} = 1.63 : \quad : c/a \quad .4$$

$$V_e = \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) = \frac{\sqrt{3}}{2}a^2c = \sqrt{2}a^3 : \quad .5$$

$$. \frac{3\sqrt{3}}{2}a^2c = 3\sqrt{2}a^3 :$$

$$2: \quad . n_a = \frac{1}{6} \times 12 + \frac{1}{2} \times 2 + 3 = 6 : \quad .6$$

$$z = 12 : \quad .7$$

$$: r_a : \quad R_z = 2r_a = a : \quad .8$$

$$F_R^{hcp} = \frac{n_a v_a}{V} = \frac{6 \times \frac{4}{3} \pi r_a^3}{3\sqrt{2}a^3} = \frac{\pi \sqrt{2}}{6} = 0.74 : \quad .9$$

(5.1)

hcp :					
$c(\text{Å})$	$a(\text{Å})$		$c(\text{Å})$	$a(\text{Å})$	
6.07	3.75	La	3.58	2.29	Be
5.21	3.21	Mg	5.62	2.98	Cd
5.27	3.31	Sc	5.59	3.56	Er
5.73	3.65	Y	5.78	3.64	Gd
5.69	3.60	Tb	5.83	3.57	He
4.95	2.66	Zn	5.62	3.58	Ho

:(5.1)

14-1 خلية ويغنز - زائتس (wigner - seitz) :

( ) ( )

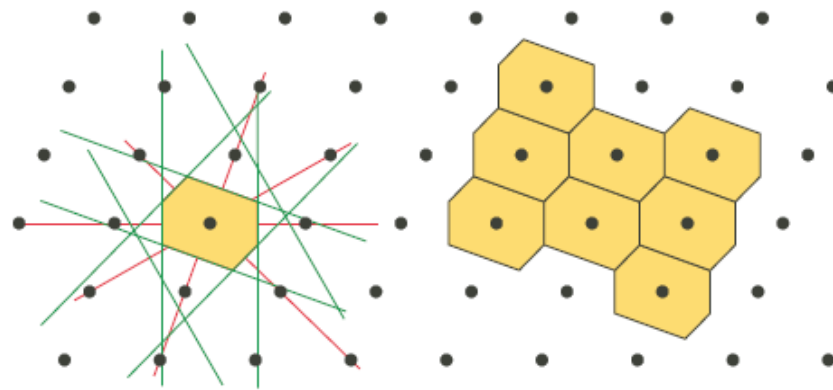
( )

:

.1

( ) ((33.1)) . 2

( ) 3



:(33.1)

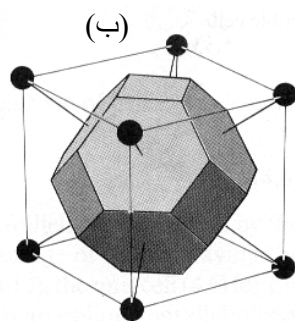
CFC

(34.1)

.CFC

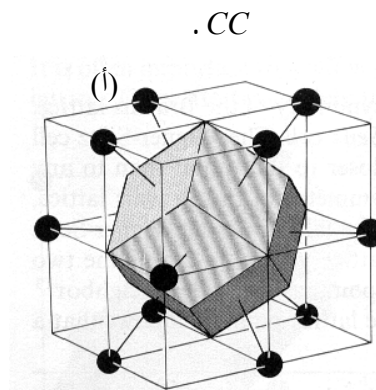
( )

CC



(i) CFC

(ب) CC



:(34.1)



14-1 بعض البنى البلورية المشهورة:

1-14-1 بنية اطاس:

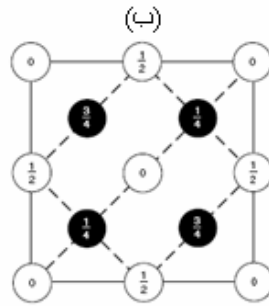
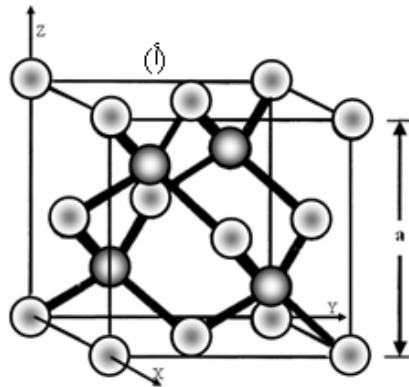
$\cdot \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) (0,0,0) :$

(z = 4)

$\cdot ( \quad ) R_z = 2r_c = \frac{\sqrt{3}}{4} a$

$\left(\frac{1}{4}, \frac{3}{4}, \frac{3}{4}\right) \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) \left(\frac{1}{2}, \frac{1}{2}, 0\right) \left(\frac{1}{2}, 0, \frac{1}{2}\right) \left(0, \frac{1}{2}, \frac{1}{2}\right) (0,0,0)$

$\cdot \left(\frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right) \left(\frac{3}{4}, \frac{1}{4}, \frac{3}{4}\right)$



$\cdot Z \quad ( \quad )$

$( \quad ) : (35.1)$

(F = 0.34) % 34

Z (X,Y) (35.1)

Z

2-14-1 بنية كلوريد السيزيوم CsCl:

CsCl

$\cdot \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

Cl<sup>-</sup>

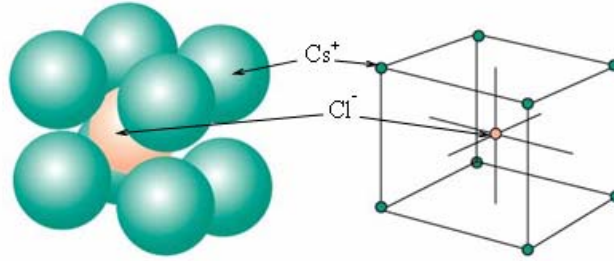
(0,0,0)

Cs<sup>+</sup>

(z = 8)

Cl<sup>-</sup> Cs<sup>+</sup>

$$R_z = r_{Cl^-} + r_{Cs^+} = \frac{\sqrt{3}}{2} a$$



.CsCl

(3.1)

3-14-1 بنية كلوريد الصوديوم NaCl:

( )

·  $(\frac{1}{2}, 0, 0)$

(Cl<sup>-</sup>)

(0,0,0)

(Na<sup>+</sup>)

(Na<sup>+</sup>)

(Cl<sup>-</sup>)

:

NaCl

·  $(0, 0, \frac{1}{2})$

$(0, \frac{1}{2}, 0)$

$(\frac{1}{2}, 0, 0)$

$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$

: Na<sup>+</sup>

$(\frac{1}{2}, \frac{1}{2}, 0)$

$(\frac{1}{2}, 0, \frac{1}{2})$

$(0, \frac{1}{2}, \frac{1}{2})$

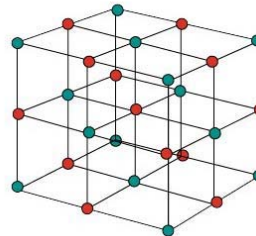
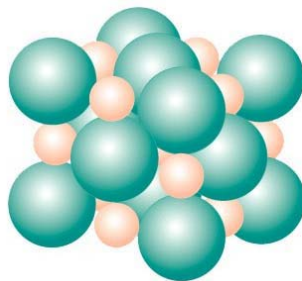
(0,0,0): Cl<sup>-</sup>

:

(z = 6)

Cl<sup>-</sup> Na<sup>+</sup>

$$R_z = r_{Cl^-} + r_{Na^+} = \frac{a}{2}$$



. NaCl

(3.1)

4-14-1 بنية كبريت الزنك ZnS:

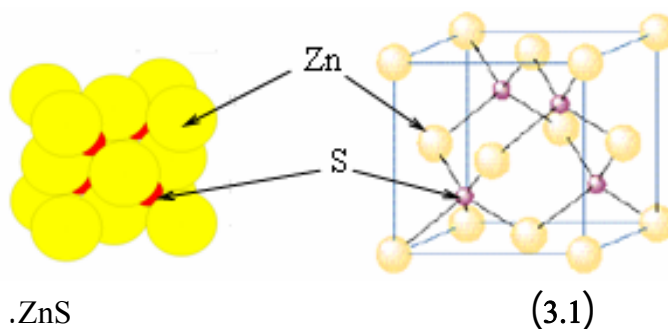
$$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right) (0,0,0) : \quad (S^-) \quad (Zn^+) ,$$

$$\left(\frac{1}{2}, \frac{1}{2}, 0\right) \left(\frac{1}{2}, 0, \frac{1}{2}\right) \left(0, \frac{1}{2}, \frac{1}{2}\right) (0,0,0) :$$

$$\left(\frac{3}{4}, \frac{3}{4}, \frac{1}{4}\right) \left(\frac{3}{4}, \frac{1}{4}, \frac{3}{4}\right) \left(\frac{1}{4}, \frac{3}{4}, \frac{3}{4}\right) \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$$

$$(z = 4) \quad (S^-) \quad (Zn^+)$$

$$R_z = r_{Zn} + r_S = \frac{\sqrt{3}}{4} a$$



(6.1)

ZnS		NaCl		CsCl			
$a(\text{Å})$		$a(\text{Å})$		$a(\text{Å})$		$a(\text{Å})$	
5.41	ZnS	5.64	NaCl	4.12	CsCl	3.57	C
6.09	ZnTe	5.35	KF	4.29	CsBr	5.43	Si
5.82	CdS	5.91	CaSe	4.57	CsI	5.66	Ge
6.08	HgSe	5.55	AgCl	3.83	TlCl	6.49	(α)-Sn
5.62	AlSb	4.21	MgO	3.97	TlBr		

:(6.1)

الفصل الثاني

# انعراج الأشعة السينية والشبكة المعكوسة

## 1-2 مقدمة:

( - )

:

P

h

 $\lambda$ 

(1-2)

$$\lambda = \frac{h}{p}$$

 $\vec{a}, \vec{b}, \vec{c}$ 

## 2-2 انعراج النيوتونات:

(p)

(1-2)

:

(2-2)

$$E_n = \frac{p^2}{2m} = \frac{h^2}{2m_n \lambda_n^2} \Rightarrow \lambda_n = \frac{h}{\sqrt{2m_n E_n}}$$

$$(m_n = 1.675 \times 10^{-27} \text{ Kg}) \quad (2-2)$$

:

(3-2)

$$\lambda_n \approx \frac{0.28}{\sqrt{E_n}} \text{ \AA}$$

$$(E_n = 0.08 \text{ eV})$$

$$0.025 \text{ eV}$$

$$KT$$

$$. 4000 \text{ m/s}$$

3-2 انعراج الإلكترونات:

)

(

$$: (m_e = 9.1 \times 10^{-31} \text{ Kg}) \quad (2-2)$$

$$(4-2) \quad \lambda_e = \frac{12.25 \text{ \AA}}{\sqrt{E_e}}$$

$$. 150 \text{ eV}$$

4-2 الأشعة السينية المستعملة في تحليل البنية البلورية:

$$. (1 \rightarrow 10 \text{ \AA}) \quad - \quad -$$

( )

:

:

$$(5-2) \quad E = \hbar\omega = h\nu = h\frac{c}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

$$, hc = 1240 \text{ ev.nm} \quad (5-2)$$

$$(1\text{ev} = 1.602 \times 10^{-19}) \quad (\text{Kev}) \quad \left(1\text{\AA} = 10^{-10} \text{m}\right)$$

:

$$(6-2) \quad \lambda = \frac{1240 \text{ [ev.nm]}}{E \text{ [Kev]}} = \frac{12.4}{E} \text{\AA}$$

(10-50Kev)

### 5-2 إنتاج الأشعة السينية:

$$((1.2) \quad )$$

$$((2.2) \quad )$$

$$. ((3.2) \quad )$$

( )

$\gamma \quad \beta \quad \alpha$

$\gamma \quad \beta \quad ,1$

$\alpha \quad ,$

M

3 2

.K<sub>α</sub>

K

L

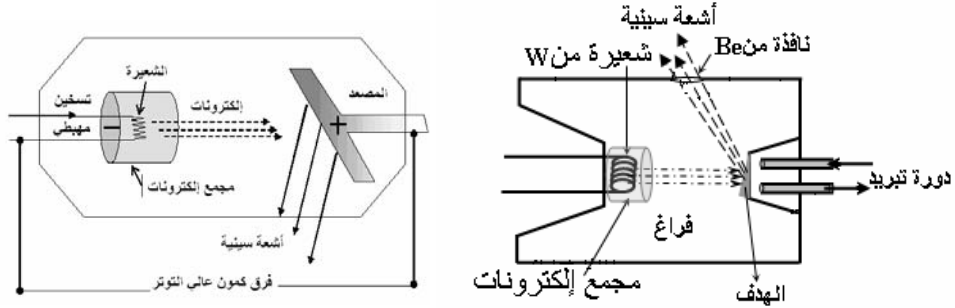
.K<sub>β</sub>

K

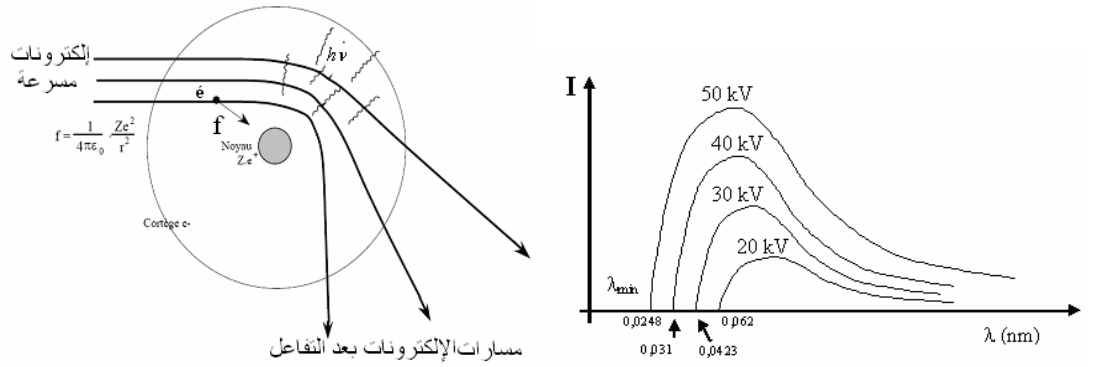
$$(1.2)$$

W

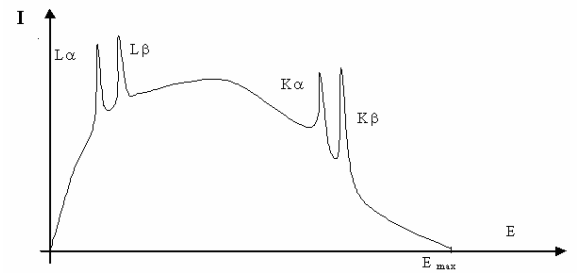
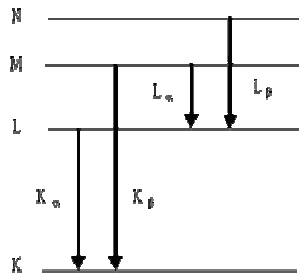
Be



:(1.2)



:(2.2)



:(3.2)



## 6-2 إمتصاص الأشعة السينية:

 $\mu$  $(I_o)$ 

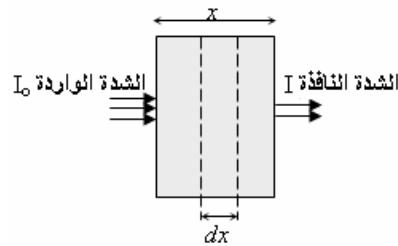
. (2)

: ((4.2) ) (I)

$$I - I_o = dI = -\mu I dx \Rightarrow \int_{I_o}^I \frac{dI}{I} = \int_0^x \mu dx \Rightarrow I = I_o e^{-\mu x}$$

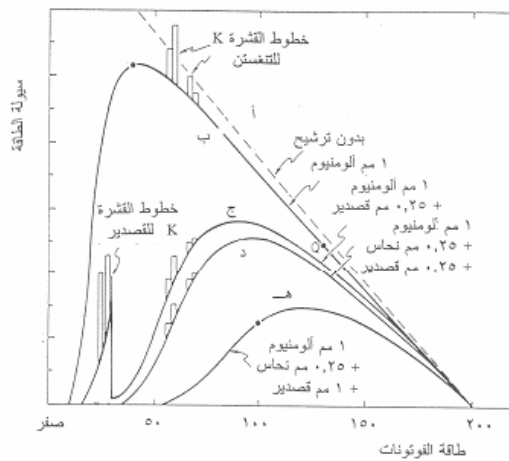
(7-2)

: x :



: (4.2)

. ((5.2) )



: (5.2)

7-2 علاقة براغ في انعراج الأشعة السينية :

1913

:

)

-

(

( )

-

((5.2) )

(

)

-

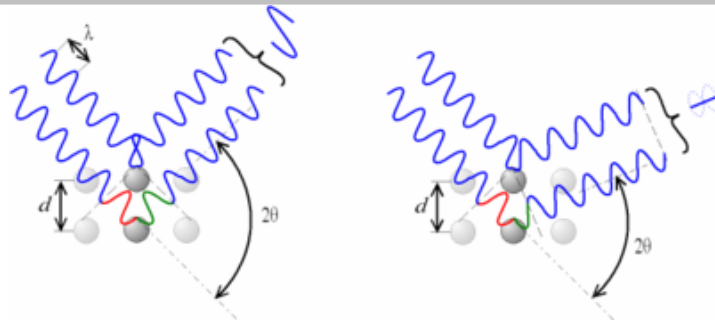
( )

-

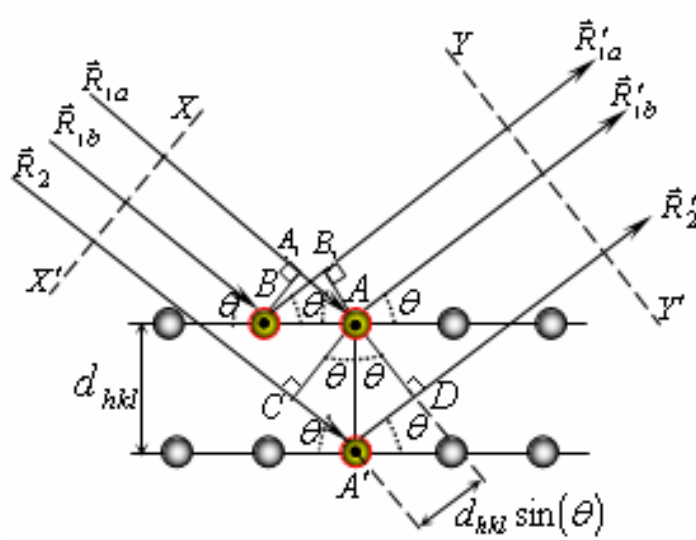
...

((6.2) )

-



:(6.2)



:(7.2)

$$d_{hkl} \sin(\theta) = n\lambda \quad (7.2)$$

$$AA_1 - BB_1 = AB \cos(\theta) - AB \cos(\theta) = 0$$

(8-2)

$\vec{R}'_{1a}$

$$\vec{R}'_2 \quad \vec{R}'_{1a} \quad A \quad \vec{R}'_2 \quad \vec{R}'_{1a} \quad (7.2) \quad A'$$

$$CA' + A'D = 2CA' = n\lambda$$

$$\sin(\theta) = \frac{CA'}{d_{hkl}} \Rightarrow CA' = d_{hkl} \sin(\theta)$$

$$2CA' = 2d_{hkl} \sin(\theta) = n\lambda$$

$$(9-2) \quad 2d_{hkl} \sin(\theta) = n\lambda$$

:  $\lambda$  ,:  $\theta$  ,:  $n$  $d_{hkl}$ 

$$(9-2)$$

n

8-2 الطرق التجريبية لانعراج الأشعة (الأمواج) السينية على البلورات:

$$(2d \sin \theta = n\lambda)$$

(λ)

(λ) (θ)

(θ)

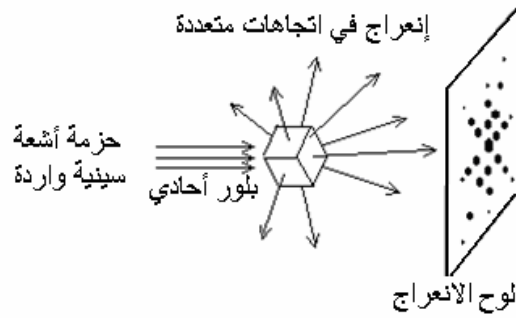
{hkl}

## 1-8-2 طريقة فون لاوي (von Laue):

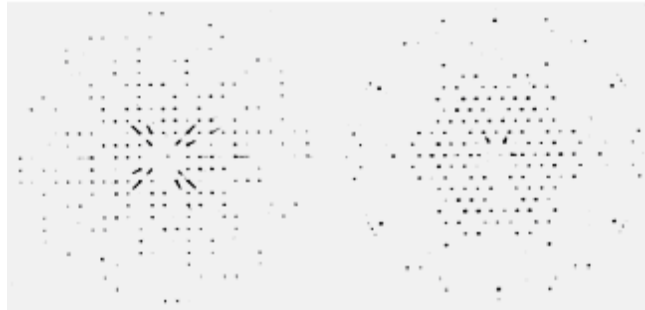
$$(0.2 - 3 \text{ \AA})$$

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (9.2)$$

$$\lambda \sin^2 \theta = d_{hkl} \quad (9.2)$$



:(8.2)



:(9.2)

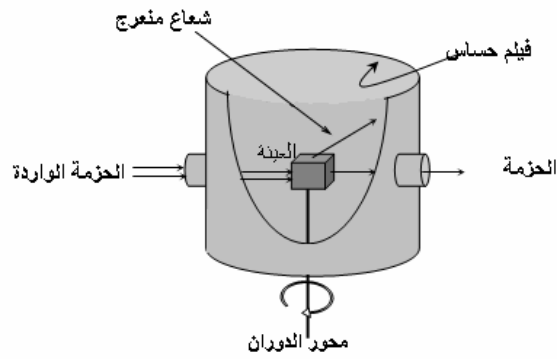
## 2-8-2 طريقة البلورة الدوارة:

$$(\theta)$$

$$(d_{hkl})$$

.( )

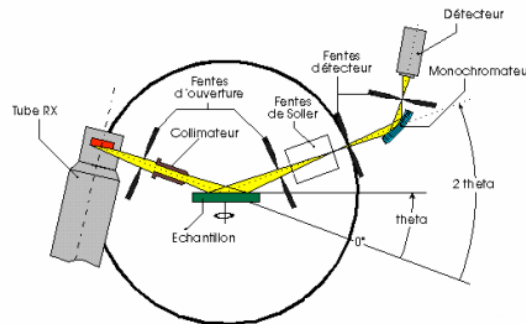
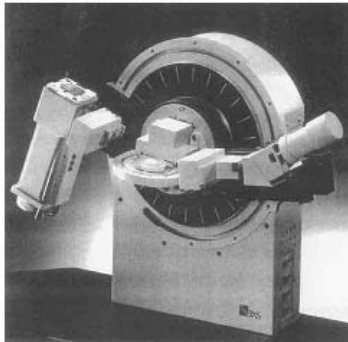
.((10.2)



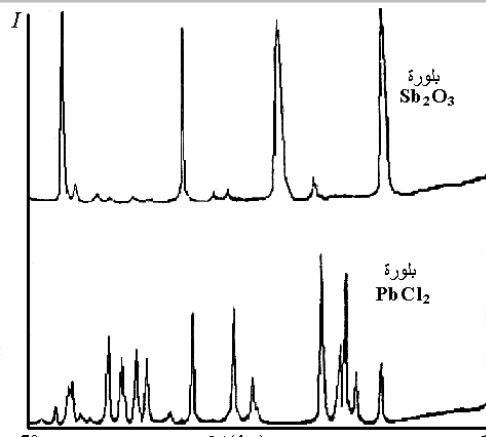
:(10.2)

(12.2)

.((11.2)

.PbCl<sub>2</sub> Sb<sub>2</sub>O<sub>3</sub>

:(11.2)



.  $PbCl_2$   $Sb_2O_3$  : (12.2)

3-8-2 طريقة المسحوق أو طريقة ديبياي-شرر Debye-scherrer :

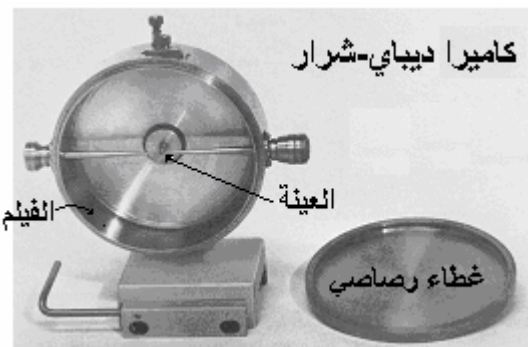
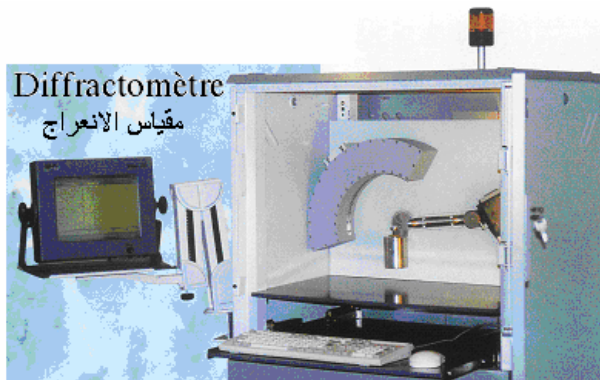
( )

( $\theta$ )

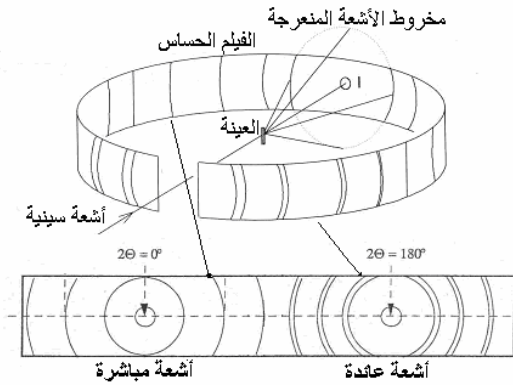
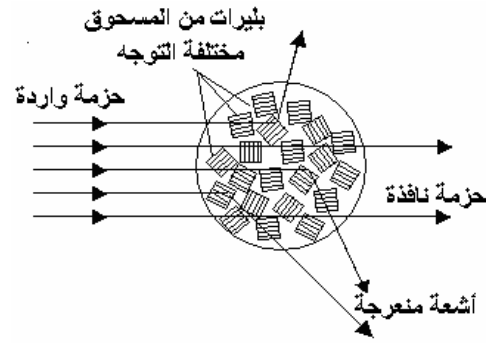
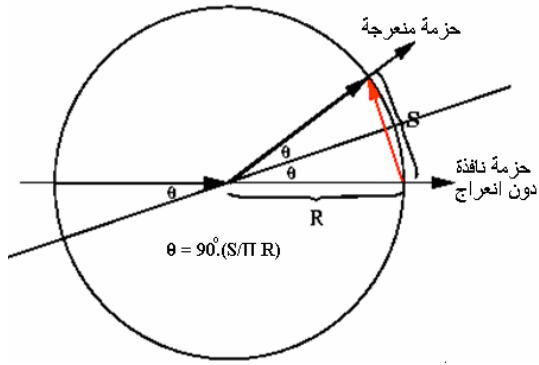
.((13.2) )

( )

.(12.2)

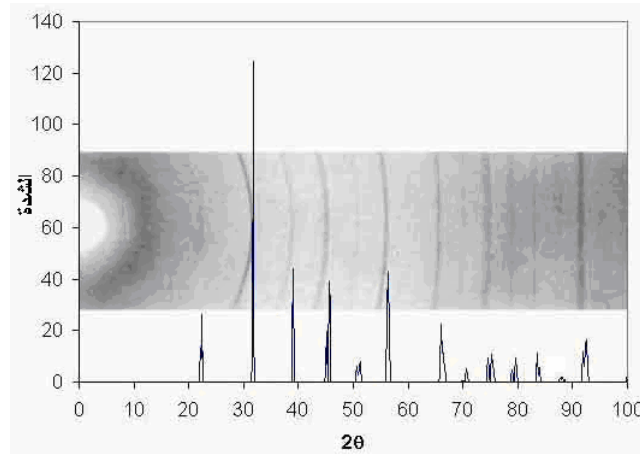


انعراج الأشعة السينية والشبكة المعكوسة



:(13.2)

(14.2)



:(14.2)



## 9-2 الشبكة المعكوسة (المقلوبة):

( ... )

$$\vec{K} = \frac{2\pi}{\lambda}$$

. ( )

## 1-9-2 مفهوم الشبكة المعكوسة :

$$(\sin \theta_{hkl} = n\lambda/2d_{hkl}):$$

$$(\sin(\theta_{hkl}))$$

$$(d_{hkl}) \quad ( )$$

$$(\sin(\theta_{hkl}))$$

## 2-9-2 خصائص الشبكة المعكوسة:

$$(d_{hkl})$$

$$( )$$

$$\frac{2\pi}{d_{hkl}}$$

:

( $\vec{K}$ )

( )  $\vec{G}_{g_1 g_2 g_3}$  -

:

$$(8-2) \quad \vec{G} = \vec{A}_1 g_1 + \vec{A}_2 g_2 + \vec{A}_3 g_3$$

$g_1, g_2, g_3$   $\vec{A}_1, \vec{A}_2, \vec{A}_3$

$\vec{A}_1, \vec{A}_2, \vec{A}_3$  -

:  $\vec{a}_1, \vec{a}_2, \vec{a}_3$  ( )

$$(9-2) \quad \begin{array}{lll} \vec{A}_1 \cdot \vec{a}_1 = 2\pi & \vec{A}_1 \cdot \vec{a}_2 = 0 & \vec{A}_1 \cdot \vec{a}_3 = 0 \\ \vec{A}_2 \cdot \vec{a}_2 = 2\pi & \vec{A}_2 \cdot \vec{a}_1 = 0 & \vec{A}_2 \cdot \vec{a}_3 = 0 \\ \vec{A}_3 \cdot \vec{a}_3 = 2\pi & \vec{A}_3 \cdot \vec{a}_1 = 0 & \vec{A}_3 \cdot \vec{a}_2 = 0 \end{array}$$

$\frac{\vec{a}_2 \times \vec{a}_3}{\|\vec{a}_2 \times \vec{a}_3\|}$

$\vec{A}_1$

$\vec{A}_3$

$\frac{\vec{a}_3 \times \vec{a}_1}{\|\vec{a}_3 \times \vec{a}_1\|}$

$\vec{A}_2$

$\frac{\vec{a}_1 \times \vec{a}_2}{\|\vec{a}_1 \times \vec{a}_2\|}$

$(\vec{a}_3 \times \vec{a}_1 / \vec{a}_3 \times \vec{a}_1) :$   $(\vec{a}_2 \times \vec{a}_3 / \vec{a}_2 \times \vec{a}_3) :$  (9-2)

:

$(\vec{a}_1 \times \vec{a}_2 / \vec{a}_1 \times \vec{a}_2)$

$$(10-2) \quad \begin{array}{l} \vec{A}_1 \cdot \vec{a}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_2 \times \vec{a}_3} \Rightarrow \vec{A}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} \\ \vec{A}_3 \cdot \vec{a}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \times \vec{a}_2} \Rightarrow \vec{A}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_3 \cdot (\vec{a}_1 \times \vec{a}_2)} \\ \vec{A}_2 \cdot \vec{a}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_3 \times \vec{a}_1} \Rightarrow \vec{A}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_2 \cdot (\vec{a}_3 \times \vec{a}_1)} \end{array}$$

$\vec{a}_1(\vec{a}_2 \times \vec{a}_3) \quad \vec{a}_2(\vec{a}_3 \times \vec{a}_1) \quad \vec{a}_3(\vec{a}_1 \times \vec{a}_2) \quad (\vec{A}_1, \vec{A}_2, \vec{A}_3) \quad (10-2)$

:

$$(11-2) \quad V_e = \vec{a}_1(\vec{a}_2 \times \vec{a}_3) = \vec{a}_2(\vec{a}_3 \times \vec{a}_1) = \vec{a}_3(\vec{a}_1 \times \vec{a}_2)$$

$(\vec{A}_1, \vec{A}_2, \vec{A}_3)$  -

$$(\vec{K}) \quad (\vec{G})$$

$$V_e^*$$

$$: \quad (10-2) \quad V_e$$

$$(12-2) \quad V_e^* V_e = (\vec{A}_1 \cdot (\vec{A}_2 \times \vec{A}_3)) (\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)) = \begin{vmatrix} \vec{A}_1 \cdot \vec{a}_1 & \vec{A}_1 \cdot \vec{a}_2 & \vec{A}_1 \cdot \vec{a}_3 \\ \vec{A}_2 \cdot \vec{a}_1 & \vec{A}_2 \cdot \vec{a}_2 & \vec{A}_2 \cdot \vec{a}_3 \\ \vec{A}_3 \cdot \vec{a}_1 & \vec{A}_3 \cdot \vec{a}_2 & \vec{A}_3 \cdot \vec{a}_3 \end{vmatrix} = (2\pi)^3$$

$$: \quad \vec{R}$$

$$\vec{G}$$

$$(13-2) \quad \begin{aligned} \vec{G} \cdot \vec{R} &= (\vec{A}_1 g_1 + \vec{A}_2 g_2 + \vec{A}_3 g_3) \cdot (n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3) \\ &= 2\pi (g_1 n_1 + g_2 n_2 + g_3 n_3) \\ &= 2\pi m \end{aligned}$$

$$m :$$

$$h, k, l$$

$$\vec{G}_{hkl}$$

$$:$$

$$(\vec{G}_{hkl}) \perp \vec{G}_{hkl} : \{hkl\}$$

$$(hkl) \quad \vec{G}_{hkl} = h\vec{A}_1 + k\vec{A}_2 + l\vec{A}_3 : \quad h, k, l$$

$$\vec{G}_{hkl}$$

$$: ((15.2) \quad )$$

$$( \quad )$$

$$\vec{p}_1 \vec{p}_2 = \left( \frac{\vec{a}_2}{k} \right) - \left( \frac{\vec{a}_1}{h} \right) \quad , \quad \vec{p}_1 \vec{p}_3 = \left( \frac{\vec{a}_3}{l} \right) - \left( \frac{\vec{a}_1}{h} \right)$$

$$\vec{G}_{hkl} \cdot \vec{p}_1 \vec{p}_2 = (h\vec{A}_1 + k\vec{A}_2 + l\vec{A}_3) \cdot \left( \left( \frac{\vec{a}_2}{k} \right) - \left( \frac{\vec{a}_1}{h} \right) \right) = -2\pi + 2\pi = 0 \Rightarrow \vec{G}_{hkl} \perp \vec{p}_1 \vec{p}_2$$

$$\vec{G}_{hkl} \cdot \vec{p}_1 \vec{p}_3 = (h\vec{A}_1 + k\vec{A}_2 + l\vec{A}_3) \cdot \left( \left( \frac{\vec{a}_3}{l} \right) - \left( \frac{\vec{a}_1}{h} \right) \right) = -2\pi + 2\pi = 0 \Rightarrow \vec{G}_{hkl} \perp \vec{p}_1 \vec{p}_3$$

$$(14-2) \quad \boxed{(hkl) \perp \vec{G}_{hkl}} : \quad \vec{G}_{hkl} \perp \vec{p}_1 \vec{p}_2 \quad \vec{G}_{hkl} \perp \vec{p}_1 \vec{p}_3 :$$

$$d_{hkl}$$

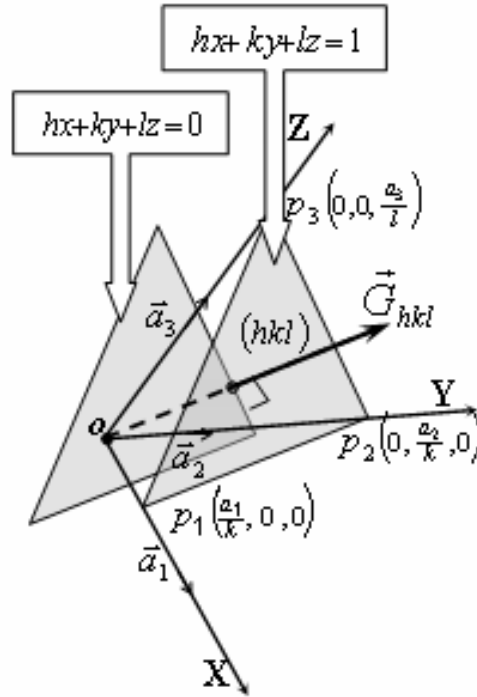
$$\vec{G}_{hkl}$$

$$\|\vec{OP}\| = d_{hkl} : \quad (15.2) \quad :$$

$$\vec{G}_{hkl} \cdot \vec{Op}_1 = (h\vec{A}_1 + k\vec{A}_2 + l\vec{A}_3) \cdot \left( \frac{\vec{a}_1}{h} \right) = 2\pi :$$

$$\vec{G}_{hkl} \cdot \vec{Op}_1 = \|\vec{G}_{hkl}\| \|\vec{Op}_1\| \cos(\vec{G}_{hkl}, \vec{Op}_1) = \|\vec{G}_{hkl}\| \|\vec{OP}\| = \|\vec{G}_{hkl}\| d_{hkl} :$$

$$(15-2) \quad \|\vec{G}_{hkl}\| d_{hkl} = 2\pi \Rightarrow \|\vec{G}_{hkl}\| = \frac{2\pi}{d_{hkl}} :$$



$$\cdot (hkl) \quad h, k, l \quad : (15.2)$$

### 3-9-2 حساب القيم المعكوسة (المقلوبة):

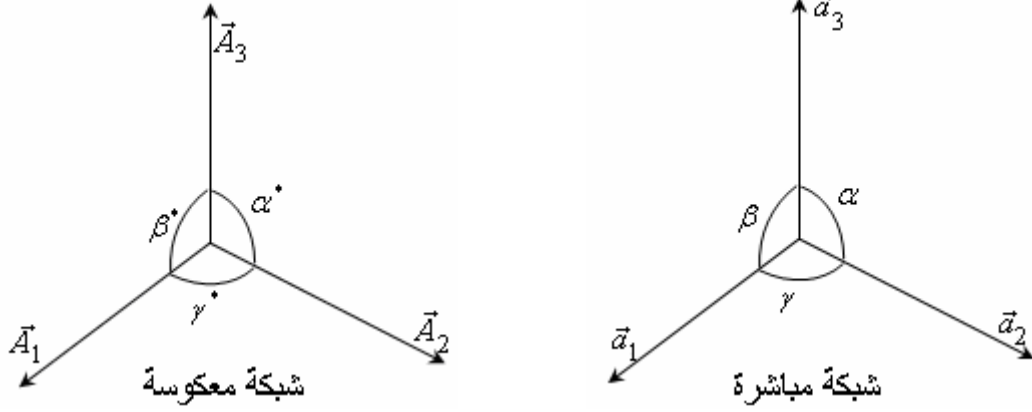
$$\begin{aligned} \alpha &= (\vec{a}_2, \vec{a}_3) : & \vec{a}_1, \vec{a}_2, \vec{a}_3 : \\ \alpha^* &= (\vec{A}_2, \vec{A}_3) : & \vec{A}_1, \vec{A}_2, \vec{A}_3 \end{aligned} \quad \begin{aligned} \cdot \gamma &= (\vec{a}_1, \vec{a}_2) & \beta &= (\vec{a}_3, \vec{a}_1) \\ \cdot \gamma^* &= (\vec{A}_1, \vec{A}_2) & \beta^* &= (\vec{A}_3, \vec{A}_1) \end{aligned}$$

$$: ((15.2))$$

### • حساب الزوايا المعكوسة:

$$: (10-2) \quad V_e = \vec{a}_1(\vec{a}_2 \times \vec{a}_3) = \vec{a}_2(\vec{a}_3 \times \vec{a}_1) = \vec{a}_3(\vec{a}_1 \times \vec{a}_2) :$$

$$(16-2) \quad \vec{A}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{V_e}, \quad \vec{A}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{V_e}, \quad \vec{A}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{V_e}$$



:(15.2)

:  $(\vec{A}_1 \cdot \vec{A}_2)$ 

$$(17-2) \quad \vec{A}_1 \cdot \vec{A}_2 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{V_e} \cdot 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{V_e} = \frac{4\pi^2}{V_e^2} (\vec{a}_2 \times \vec{a}_3) \cdot (\vec{a}_3 \times \vec{a}_1)$$

:

$$(18-2) \quad (\vec{a}_2 \times \vec{a}_3) \cdot (\vec{a}_3 \times \vec{a}_1) = (\vec{a}_2 \cdot \vec{a}_3) (\vec{a}_3 \cdot \vec{a}_1) - (\vec{a}_2 \times \vec{a}_1) \cdot \vec{a}_3^2 = a_2 a_3^2 a_1 (\cos(\alpha) \cos(\beta) - \cos(\gamma))$$

: (17-2)

$$(19-2) \quad \vec{A}_1 \cdot \vec{A}_2 = \frac{4\pi^2}{V_e^2} a_1 a_2 a_3^2 (\cos(\alpha) \cos(\beta) - \cos(\gamma))$$

:

$$\vec{A}_1 \cdot \vec{A}_2 = \|\vec{A}_1\| \|\vec{A}_2\| \cos(\gamma^*) = \frac{4\pi^2}{V_e^2} \|\vec{a}_2 \times \vec{a}_3\| \|\vec{a}_3 \times \vec{a}_1\| \cos(\gamma^*)$$

$$(20-2) \quad \vec{A}_1 \cdot \vec{A}_2 = \frac{4\pi^2}{V_e^2} a_1 a_2 a_3^2 \sin(\alpha) \sin(\beta) \cos(\gamma^*)$$

: (20-2) (19-2)

$$(21-2) \quad \cos(\gamma^*) = \frac{\cos(\alpha) \cos(\beta) - \cos(\gamma)}{\sin(\alpha) \sin(\beta)}$$

:  $\cos(\beta^*) \cos(\alpha^*)$ 

$$(22-2) \quad \cos(\alpha^*) = \frac{\cos(\beta) \cos(\gamma) - \cos(\alpha)}{\sin(\beta) \sin(\gamma)}$$

$$(23-2) \quad \cos(\beta^*) = \frac{\cos(\gamma) \cos(\alpha) - \cos(\beta)}{\sin(\lambda) \sin(\alpha)}$$

## • حساب الثوابت المعكوسة :

:

$$V_e^2 = [\vec{a}_1 (\vec{a}_2 \times \vec{a}_3)]^2 = \begin{vmatrix} \vec{a}_1 \cdot \vec{a}_1 & \vec{a}_1 \cdot \vec{a}_2 & \vec{a}_1 \cdot \vec{a}_3 \\ \vec{a}_2 \cdot \vec{a}_1 & \vec{a}_2 \cdot \vec{a}_2 & \vec{a}_2 \cdot \vec{a}_3 \\ \vec{a}_3 \cdot \vec{a}_1 & \vec{a}_3 \cdot \vec{a}_2 & \vec{a}_3 \cdot \vec{a}_3 \end{vmatrix} \Rightarrow$$

$$(24-2) \quad V_e^2 = (a_1 a_2 a_3)^2 (1 + 2 \cos(\alpha) \cos(\beta) \cos(\gamma) - \cos^2(\alpha) - \cos^2(\beta) - \cos^2(\gamma)) : \|\vec{A}_1\|^2$$

$$\|\vec{A}_1\|^2 = \frac{4\pi^2}{V_e^2} \|\vec{a}_2 \times \vec{a}_3\|^2$$

$$\|\vec{A}_1\|^2 = \frac{4\pi^2}{(a_1 a_2 a_3)^2 (1 + 2 \cos \alpha \cos \beta \cos \gamma - (\cos \alpha)^2 - (\cos \beta)^2 - (\cos \gamma)^2)} (a_2 a_3)^2 (\sin(\alpha))^2$$

$$(25-2) \quad \|\vec{A}_1\|^2 = \frac{(\sin(\alpha))^2}{(1 + 2 \cos \alpha \cos \beta \cos \gamma - \cos^2(\alpha) - \cos^2(\beta) - \cos^2(\gamma))} \left( \frac{2\pi}{a_1} \right)^2 : \|\vec{A}_3\|^2 \|\vec{A}_2\|^2$$

$$(26-2) \quad \|\vec{A}_2\|^2 = \frac{(\sin(\beta))^2}{(1 + 2 \cos \alpha \cos \beta \cos \gamma - \cos^2(\alpha) - \cos^2(\beta) - \cos^2(\gamma))} \left( \frac{2\pi}{a_2} \right)^2$$

$$(27-2) \quad \|\vec{A}_3\|^2 = \frac{(\sin(\gamma))^2}{(1 + 2 \cos \alpha \cos \beta \cos \gamma - \cos^2(\alpha) - \cos^2(\beta) - \cos^2(\gamma))} \left( \frac{2\pi}{a_3} \right)^2$$

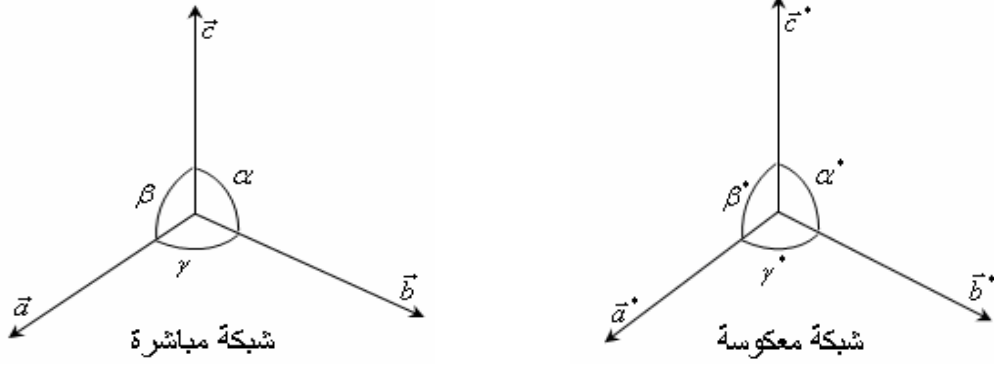
4-9-2 العلاقة العامة للمسافة الفاصلة بين المستويات البلورية المتوازية ( $d_{hkl}$ ): $d_{hkl}$ 

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) (X,Y,Z)

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,  $\gamma = (\vec{a}, \vec{b})$   $\beta = (\vec{c}, \vec{a})$   $\alpha = (\vec{b}, \vec{c})$  :  $\vec{c}, \vec{b}, \vec{a}$  $\beta^* = (\vec{c}^*, \vec{a}^*)$   $\alpha^* = (\vec{b}^*, \vec{c}^*)$  :  $\vec{c}^*, \vec{b}^*, \vec{a}^*$  :.((16.2) )  $\gamma^* = (\vec{a}^*, \vec{b}^*)$



:(16.2)

$$(d_{hkl})$$

.  $\alpha \neq \beta \neq \gamma$  و  $a \neq b \neq c$  :

$$(28-2) \quad \|\vec{G}_{hkl}\| = \frac{2\pi}{d_{hkl}} \Rightarrow \frac{1}{(d_{hkl})^2} = \frac{\|\vec{G}_{hkl}\|^2}{(2\pi)^2} \Rightarrow \frac{1}{(d_{hkl})^2} = \frac{\vec{G}_{hkl} \cdot \vec{G}_{hkl}}{(2\pi)^2}$$

$$\vec{G}_{hkl} \cdot \vec{G}_{hkl} = (h\vec{a}_1^* + k\vec{a}_2^* + l\vec{a}_3^*) \cdot (h\vec{a}_1^* + k\vec{a}_2^* + l\vec{a}_3^*)$$

$$\vec{G}_{hkl} \cdot \vec{G}_{hkl} = h^2 \|\vec{a}_1^*\|^2 + k^2 \|\vec{a}_2^*\|^2 + l^2 \|\vec{a}_3^*\|^2 + 2\|\vec{a}_1^*\| \|\vec{a}_2^*\| \cos \gamma^* + 2\|\vec{a}_2^*\| \|\vec{a}_3^*\| \cos \alpha^* + 2\|\vec{a}_3^*\| \|\vec{a}_1^*\| \cos \beta^*$$

(27-2) (26-2) (25-2) (23-2) (22-2) (21-2)

(28)

$$(29-2) \quad \frac{1}{(d_{hkl})^2} = \frac{a^2 b^2 c^2}{v^2} \left( \frac{h^2 \sin^2(\alpha)}{a^2} + \frac{k^2 \sin^2(\beta)}{b^2} + \frac{l^2 \sin^2(\gamma)}{c^2} + \frac{2hk}{ab} (\cos(\alpha)\cos(\beta) - \cos(\gamma)) + \frac{2kl}{bc} (\cos(\beta)\cos(\gamma) - \cos(\alpha)) + \frac{2hl}{ac} (\cos(\gamma)\cos(\alpha) - \cos(\beta)) \right)$$

$$(30-2) \quad v^2 = (abc)^2 (1 + 2\cos(\alpha)\cos(\beta)\cos(\gamma) - \cos^2(\alpha) - \cos^2(\beta) - \cos^2(\gamma))$$

$$\alpha = \gamma = \frac{\pi}{2} \neq \beta \text{ و } a \neq b \neq c \therefore .1$$

$$(31-2) \quad \frac{1}{(d_{hkl})^2} = \frac{1}{\sin^2(\beta)} \left( \frac{h^2}{a^2} + \frac{k^2 \sin^2(\beta)}{b^2} + \frac{l^2}{c^2} - \frac{2hl}{ac} (\cos(\beta)) \right)$$

$$\alpha = \gamma = \beta = \frac{\pi}{2} \text{ و } a \neq b \neq c : \quad .2$$

$$(32-2) \quad \frac{1}{(d_{hkl})^2} = \left( \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right)$$

$$\alpha = \gamma = \beta = \frac{\pi}{2} \text{ و } a = b \neq c \therefore \quad .3$$

$$(33-2) \quad \frac{1}{(d_{hkl})^2} = \left( \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2} \right)$$

$$\alpha = \gamma = \beta = \frac{\pi}{2} \text{ و } a = b = c : \quad .4$$

$$(34-2) \quad \frac{1}{(d_{hkl})^2} = \left( \frac{h^2 + k^2 + l^2}{a^2} \right)$$

$$\alpha = \gamma = \beta \neq \frac{\pi}{2} < 120^\circ \text{ و } a = b = c : \quad .5$$

$$(35-2) \quad \frac{1}{(d_{hkl})^2} = \frac{(h^2 + k^2 + l^2) \sin^2(\alpha) + 2(hk + kl + hl)(\cos^2(\alpha) - \cos(\alpha))}{a^2(1 + 2 \cos^3(\alpha) - 3 \cos^2(\alpha))}$$

$$\alpha = \beta = \frac{\pi}{2}, \gamma = 120^\circ \text{ و } a = b \neq c : \quad .6$$

$$(36-2) \quad \frac{1}{(d_{hkl})^2} = \frac{4}{3} \left( \frac{h^2 + hk + k^2}{a^2} \right) + \frac{l^2}{c^2}$$

### 5-9-2 إنشاء شبكة مستوية معكوسة لشبكة مستوية مباشرة:

$$\gamma \quad \bar{a}_1, \bar{a}_2 \quad .1$$

$$(17.2) \quad d_{010} \quad d_{100} \quad (010) \quad (100)$$

$$\bar{A}_1, \bar{A}_2 \quad \bar{a}_2 \quad \bar{a}_1 \quad .2$$

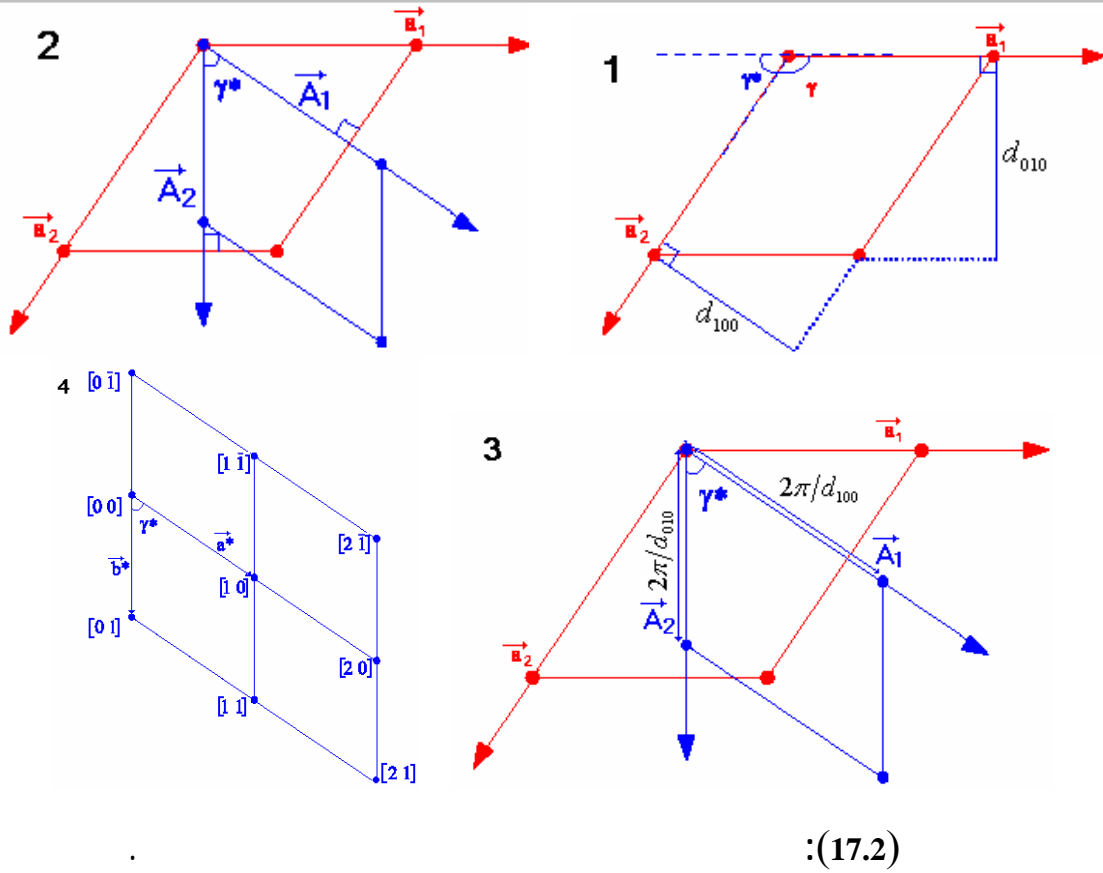
.  $\gamma^*$

$$, \quad \|\bar{A}_2\| = 2\pi/d_{010} \quad d_{hkl} \quad \bar{A}_1, \bar{A}_2 \quad .3$$

$$\|\bar{A}_1\| = 2\pi/d_{100}$$

$$\bar{G}_{hk} = h\bar{A}_1 + k\bar{A}_2 \quad .4$$



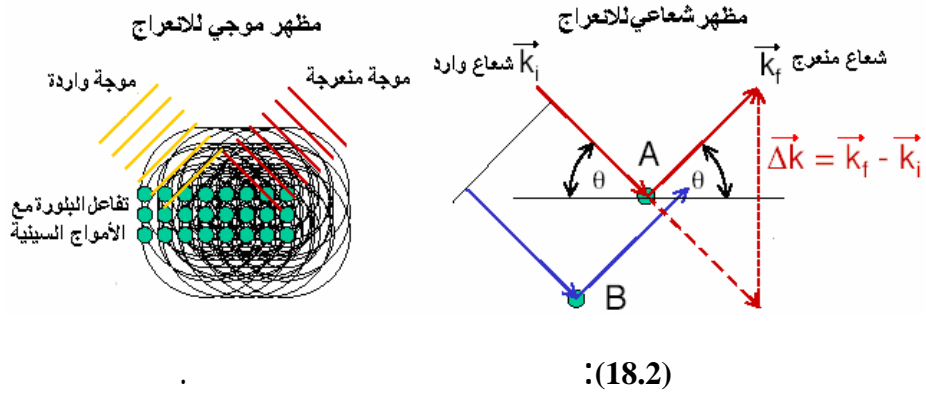


6-9-2 شروط فون لاوي للانعراج:

$$(\lambda_f) \quad (\lambda_i) \quad \left( \|\vec{K}_f\| = 2\pi/\lambda_f \right) \quad \left( \|\vec{K}_i\| = 2\pi/\lambda_i \right)$$

$$: \quad (18.2) \quad \vec{K} \quad \vec{K}$$

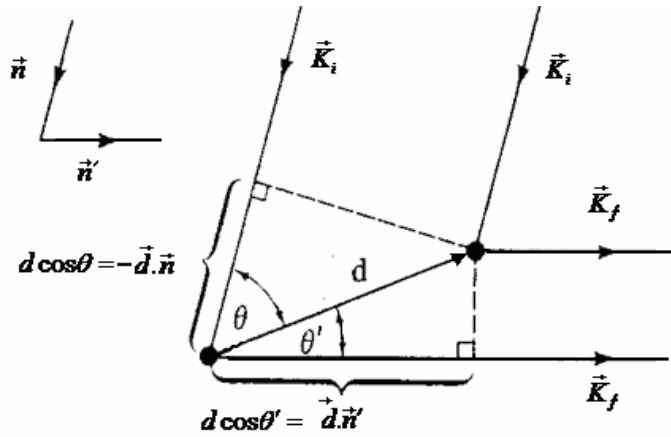
$$(37-2) \quad \Delta\vec{K} = \vec{K}_f - \vec{K}_i$$



$$d \vec{n}' \cdot \vec{K}_f - d \vec{n} \cdot \vec{K}_i, \vec{d} = a_i (i=1, 2, 3): \quad (19.2)$$

$\vec{n}' \cdot \vec{n} :$

$$(38-2) \quad d \cos \theta + d \cos \theta' = \vec{d} \cdot (\vec{n}' - \vec{n})$$



$$(39-2) \quad \vec{d} \cdot (\vec{n}' - \vec{n}) = m \lambda$$

$$\left( \frac{2\pi}{\lambda} \right) \quad (39-2)$$

$$\vec{d} \cdot \left[ \left( \frac{2\pi}{\lambda} \right) \cdot \vec{n}' - \left( \frac{2\pi}{\lambda} \right) \cdot \vec{n} \right] = 2\pi m$$

$$\vec{d} \cdot (\vec{K}_f - \vec{K}_i) = 2\pi m$$

$$(40-2) \quad \vec{d} \cdot (\Delta \vec{k}) = 2\pi m$$

$$: \quad \vec{d} = \vec{a}_i (i=1, 2, 3) :$$

$$(41-2) \quad \vec{a}_1 \cdot (\Delta \vec{k}) = 2\pi m_1$$

$$(42-2) \quad \vec{a}_2 \cdot (\Delta \vec{k}) = 2\pi m_2$$

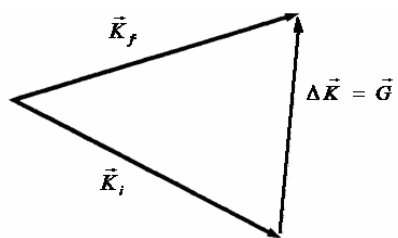
$$(43-2) \quad \vec{a}_3 \cdot (\Delta \vec{k}) = 2\pi m_3$$

$$, \quad (\overline{\Delta K}) \quad , \quad (43-2) \quad (42-2) \quad (41-2)$$

$$: \quad ((20.2) \quad ) \quad (\overline{\Delta K}) \quad (13-2) \quad \vec{G}$$

$$(44-2) \quad \Delta \vec{k} = \vec{G}$$

$$(45-2) \quad \vec{K}_f = \vec{K}_i + \vec{G}$$



:(20.2)

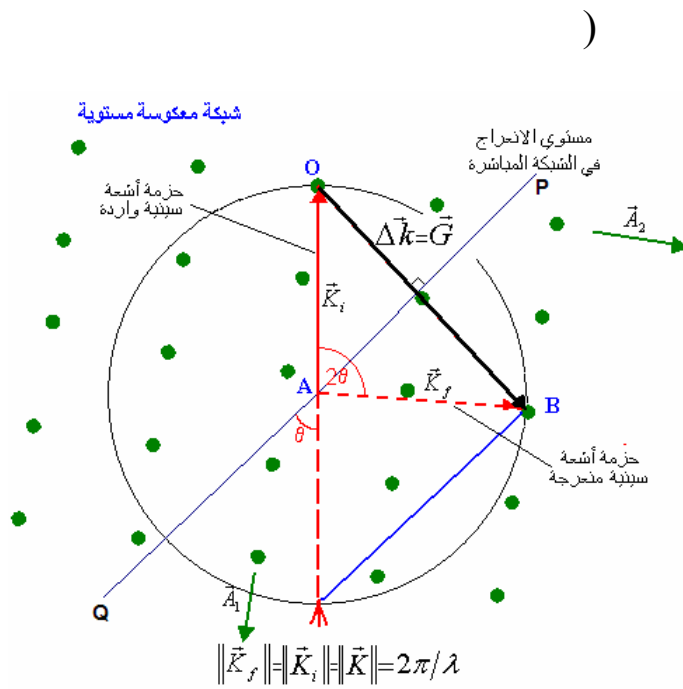
$$: \quad (45-2)$$

$$(46-2) \quad K_f^2 = K_i^2 + G^2 + 2\vec{K}_i \cdot \vec{G}$$

$$: \quad (46-2) \quad \|\vec{K}_f\| = \|\vec{K}_i\| = \|\vec{K}\| = k$$

$$(47-2) \quad G^2 + 2\vec{K} \cdot \vec{G} = 0 \quad (47-2)$$

7-9-2 إنشاء إيوالد (Ewald):



(21.2): إنشاء إيوالد.

$\vec{AO} = \vec{K}_i$  , A  
 O A  $\|\vec{AO}\| = \|\vec{K}_i\| = k = 2\pi/\lambda$   
 $\|\vec{AO}\|$  A  $\lambda$   
 A  $\vec{AB}$  , ((21.2) )  
 )  $(\vec{AB} = \vec{K}_f)$  B  
 ) o  $\vec{OB}$  .  $(\vec{K}_f)$   $\vec{K}_i$   
 ( ( )

(PQ) (A)  $\|\vec{G}\|$   $\vec{OB} = \vec{G} = \Delta\vec{k}$

(A)

: ( )  $\theta$

(48-2)  $\|\vec{G}\| = \|\Delta\vec{k}\| = \frac{2\pi}{d} \Rightarrow d = \frac{2\pi}{\|\vec{G}\|}$

: (21.2)

$$\sin \theta = \frac{\frac{1}{2}\|\vec{G}\|}{\|\vec{K}\|} \Rightarrow \|\vec{G}\| = 2\|\vec{K}\| \sin \theta \Rightarrow \frac{2\pi}{d} = 2 \cdot \frac{2\pi}{\lambda} \sin \theta \Rightarrow$$

(49-2)

$$\lambda = 2d \sin \theta$$

: (n)

$$(n=1)$$

(49-2)

(50-2)

$$n\lambda = 2d \sin \theta$$

: (21.2)

(51-2)

$$\Delta\vec{k} = \vec{G} = \vec{K}_f - \vec{K}_i \Rightarrow \vec{K}_f = \vec{K}_i + \vec{G}$$

: (51-2)

(52-2)

$$K_f^2 = K_i^2 + G^2 + 2\vec{K}_i \cdot \vec{G}$$

: (52-2)  $\|\vec{K}_f\| = \|\vec{K}_i\| = \|\vec{K}\| = k$  : (21.2)

(53-2)

$$G^2 + 2\vec{K} \cdot \vec{G} = 0$$

(53-2)

### 8-9-2 مناطق بريلوان (Brillouin):

(Wigner-Seitz) -

$\vec{G}$

(47-2)

:  $-\vec{G} \quad \vec{G} \quad (47-2)$

$-\vec{G}$



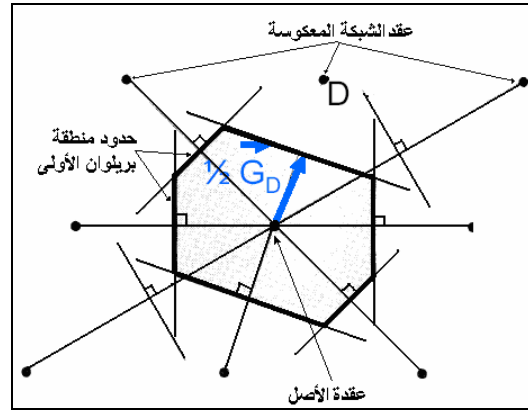
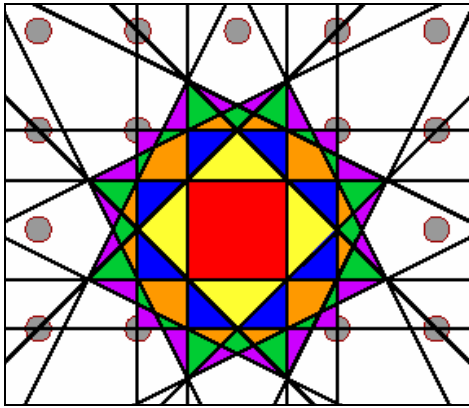
:

( )

 $\bar{G}$ 

)

.(23.2)



:(23.2)

• بعض خصائص مناطق بريلوان:

.1

.2

.3

## 9-9-2 معكوس شبكات الفئة المكعبة :

:

## 1. معكوس الشبكة المكعبة البسيطة (CS):

$$\vec{a}_1 = a\vec{i} \quad , \quad \vec{a}_2 = a\vec{j} \quad , \quad \vec{a}_3 = a\vec{k} \quad :$$

:

$$\vec{A}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{a^2}{a^3} (\vec{j} \times \vec{k}) = \frac{2\pi}{a} \vec{i}$$

$$\vec{A}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{a^2}{a^3} (\vec{k} \times \vec{i}) = \frac{2\pi}{a} \vec{j}$$

$$\vec{A}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{a^2}{a^3} (\vec{i} \times \vec{j}) = \frac{2\pi}{a} \vec{k}$$

$$\vec{A}_1, \vec{A}_2, \vec{A}_3$$

$$. 2\pi/a$$

$$2\pi/a$$

$$V_{SB}^{CS} = \vec{A}_1 \cdot (\vec{A}_2 \times \vec{A}_3) = \left(\frac{2\pi}{a}\right)^3 : \quad \pm \vec{A}_1 = \pm \frac{2\pi}{a} \vec{i}, \pm \vec{A}_2 = \pm \frac{2\pi}{a} \vec{j}, \pm \vec{A}_3 = \pm \frac{2\pi}{a} \vec{k} :$$

## 2. معكوس الشبكة المكعبة الممركزة (CC):

:

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$$\vec{a}_3 = \frac{a}{2}(\vec{i} + \vec{j} - \vec{k}) \quad \vec{a}_2 = \frac{a}{2}(\vec{i} - \vec{j} + \vec{k}) \quad \vec{a}_1 = \frac{a}{2}(-\vec{i} + \vec{j} + \vec{k})$$

:

$$(56-2) \quad \vec{A}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{\left(\frac{a^2}{4}\right)}{\left(\frac{a^3}{2}\right)} \left( (\vec{i} - \vec{j} + \vec{k}) \times (\vec{i} + \vec{j} - \vec{k}) \right) = \frac{\pi}{a} (\vec{k} + \vec{j} + \vec{k} + \vec{i} + \vec{j} - \vec{i})$$

$$= \frac{2\pi}{a} (\vec{k} + \vec{j})$$

:



$$(57-2) \quad \vec{A}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi}{a} (\vec{i} + \vec{k})$$

$$(58-2) \quad \vec{A}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi}{a} (\vec{j} + \vec{i})$$

CFC

$\vec{A}_1, \vec{A}_2, \vec{A}_3$

)  $4\pi/a$

.(

((24.2) )

:  $\frac{2\pi}{a} (\pm \vec{j} \pm \vec{i})$  ,  $\frac{2\pi}{a} (\pm \vec{i} \pm \vec{k})$  ,  $\frac{2\pi}{a} (\pm \vec{k} \pm \vec{j})$  :

$$. V_{SB}^{CC} = \vec{A}_1 \cdot (\vec{A}_2 \times \vec{A}_3) = 2 \left( \frac{2\pi}{a} \right)^3$$

3. معكوس الشبكة المكعبة الممركزة الأوجه (CFC):

:

$$\vec{a}_3 = \frac{a}{2} (\vec{i} + \vec{j}) \quad \vec{a}_2 = \frac{a}{2} (\vec{i} + \vec{k}) \quad \vec{a}_1 = \frac{a}{2} (\vec{j} + \vec{k})$$

:

$$(59-2) \quad \vec{A}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{\left( \frac{a^2}{4} \right)}{\left( \frac{a^3}{4} \right)} ((\vec{i} + \vec{k}) \times (\vec{i} + \vec{j})) = \frac{2\pi}{a} (\vec{k} + \vec{j} - \vec{i}) = \frac{2\pi}{a} (-\vec{i} + \vec{j} + \vec{k})$$

:

$$(60-2) \quad \vec{A}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi}{a} (\vec{i} - \vec{j} + \vec{k})$$

$$(61-2) \quad \vec{A}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = \frac{2\pi}{a} (\vec{i} + \vec{j} - \vec{k})$$

CC

$$\vec{A}_1, \vec{A}_2, \vec{A}_3$$

$$.4\pi/a$$

CC

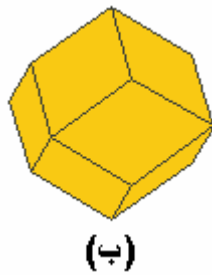
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$$((24.2) \quad ) ( \quad )$$

$$\frac{2\pi}{a} (\pm \vec{i} \pm \vec{k} \pm \vec{j}) :$$

$$.V_{SB}^{CFC} = \vec{A}_1 \cdot (\vec{A}_2 \times \vec{A}_3) = 4 \left( \frac{2\pi}{a} \right)^3 :$$

$$\pm \frac{4\pi}{a} \vec{k}, \pm \frac{4\pi}{a} \vec{j}, \pm \frac{4\pi}{a} \vec{i} :$$



. (ب) CC

(ج) CFC

:(24.2)

10-2 عامل البنية :

$$( \quad )$$

$$(j) \quad \vec{a}, \vec{b}, \vec{c}$$

$$. (\vec{r}_j = x_j \vec{a} + y_j \vec{b} + z_j \vec{c}) :$$

:

$$(\vec{R}_{m,n,p})$$

:

$$. \vec{R}_{0,0,0}$$

$$( \quad )$$

$$(\vec{R}_{m,n,p} = m \vec{a} + n \vec{b} + p \vec{c})$$

$$\vec{r}_j$$

$$. ( \quad )$$

$$((25.2)) \quad (\vec{r}_j + \vec{R}_{m,n,p}) : (\vec{R}_{m,n,p})$$

$$(62-2) \quad C_j (\vec{R} - (\vec{r}_j + \vec{R}_{m,n,p}))$$

$\vec{R} :$

$$(63-2) \quad N(\vec{r}') = \sum_{j=1}^S C_j (\vec{R} - (\vec{r}_j + \vec{R}_{m,n,p}))$$

$$\vec{r}' = \vec{R} - (\vec{r}_j + \vec{R}_{m,n,p}) :$$

$$(64-2) \quad \Omega = \sum_{mnp}^{M^3} \int_{\text{خلية}} N(\vec{r}') e^{i\vec{R} \cdot \vec{\Delta k}} dv$$

$$.M^3 \quad ( \quad ) \quad (m n p) :$$

$$\Omega = \sum_{mnp}^{M^3} \sum_j^S \int_{\text{خلية}} C_j(\vec{r}') dv e^{i(\vec{r}' + \vec{R}_{m,n,p} + \vec{r}_j) \cdot \vec{\Delta k}}$$

$$(65-2) \quad \Omega = \sum_{mnp}^{M^3} \sum_j^S f_j e^{i(\vec{R}_{m,n,p} + \vec{r}_j) \cdot \vec{\Delta k}}$$

$$(66-2) \quad f_j = \int_{\text{خلية}} C_j(\vec{r}') dv e^{i\vec{r}' \cdot \vec{\Delta k}}$$

(j)  $f_j$

: (65-2)  $\vec{\Delta k} = \vec{G}$ :

$$\Omega = M^3 \sum_j^S f_j e^{i\vec{r}_j \cdot \vec{G}} e^{i\vec{R}_{m,n,p} \cdot \vec{G}} = M^3 \sum_j^S f_j e^{i\vec{r}_j \cdot \vec{G}}$$

( $e^{i\vec{R}_{m,n,p} \cdot \vec{G}} = 1$ ):

:  $\Omega = M^3 F$

$$(67-2) \quad F = \sum_j^S f_j e^{i\vec{r}_j \cdot \vec{G}}$$

:

$$\begin{aligned} \vec{r}_j \cdot \vec{G} &= (x_j \vec{a} + y_j \vec{b} + z_j \vec{c}) (\vec{h} \vec{a}^* + \vec{k} \vec{b}^* + \vec{l} \vec{c}^*) \\ &= 2\pi (x_j h + y_j k + z_j l) \end{aligned}$$

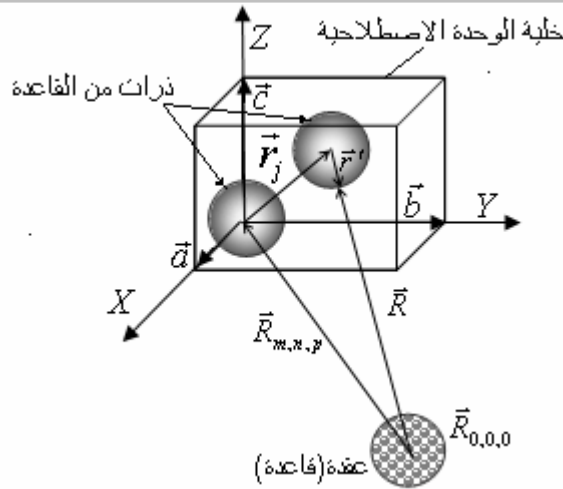
:

$$(68-2) \quad F_{hkl} = \sum_j^S f_j e^{i2\pi(x_j h + y_j k + z_j l)}$$

$F_{hkl}$

(hkl) ( )

(hkl)



(25.2):

11-2 حساب عامل البنية لبعض البنى البلورية:

:(68-2)

• بنية المكعب البسيط (CS):

:(0,0,0)

$$F_{hkl} = f e^{i2\pi(0h+0k+0l)} = f$$

(hkl)                      h, k, l                      F<sub>hkl</sub>                      (F<sub>hkl</sub> ≠ 0)

• بنية المكعب المراكز (CC):

:(0,0,0)                      ((                      (CS)                      (CC)

:( $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ )

$$F_{hkl} = f + f e^{i2\pi(\frac{h}{2} + \frac{k}{2} + \frac{l}{2})} = f(1 + e^{i\pi(h+k+l)})$$

h, k, l                      F<sub>hkl</sub>

(F<sub>hkl</sub> = 2f ≠ 0)                      h + k + l = 2n :                      h + k + l

(F<sub>hkl</sub> = 0)                      h + k + l = 2n + 1 :                      h + k + l

• بنية المكعب الممرز الأوجه (CFC):

))

(0,0,0): (( (CS) (CFC)

:

$$\left(0, \frac{1}{2}, \frac{1}{2}\right) \left(\frac{1}{2}, 0, \frac{1}{2}\right) \left(\frac{1}{2}, \frac{1}{2}, 0\right) :$$

$$F_{hkl} = f \left(1 + e^{i\pi(h+k)} + e^{i\pi(h+l)} + e^{i\pi(k+l)}\right)$$

: h, k, l  $F_{hkl}$

h, k, l  
( $F_{hkl} = 4f \neq 0$ )

( $F_{hkl} = 0$ ) h, k, l

• بنية كلوريد السيزيوم (CsCl):

))

(Cs<sup>+</sup>) (( (Cs<sup>+</sup>) (Cl<sup>-</sup>) (Cs<sup>+</sup>) (CsCl)

:

(Cl<sup>-</sup>) (0,0,0)

$$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) :$$

$$F_{hkl} = f_{Cs^+} + f_{Cl^-} e^{i\pi(h+k+l)}$$

: h, k, l  $F_{hkl}$

( $F_{hkl} = f_{Cs^+} + f_{Cl^-} \neq 0$ ) h+k+l

( $F_{hkl} = 0$ ) h+k+l

• بنية كلوريد الصوديوم (NaCl):

))

(Na<sup>+</sup>) (Cl<sup>-</sup>) (CS) (NaCl)

(Cl<sup>-</sup>) (( (NaCl)

(CFC)

(Na<sup>+</sup>)

$$\cdot \left(0, 0, \frac{1}{2}\right) \left(0, \frac{1}{2}, 0\right) \left(\frac{1}{2}, 0, 0\right) \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) : Cl^- \left(\frac{1}{2}, \frac{1}{2}, 0\right) \left(\frac{1}{2}, 0, \frac{1}{2}\right) \left(0, \frac{1}{2}, \frac{1}{2}\right) (0,0,0) : Na^+$$

:

$$F_{hkl} = f_{Na^+} \left( 1 + e^{i\pi(h+k)} + e^{i\pi(h+l)} + e^{i\pi(k+l)} \right) + f_{Cl^-} \left( e^{i\pi(h+k+l)} + e^{i\pi h} + e^{i\pi k} + e^{i\pi l} \right)$$

:  $h, k, l$

$F_{hkl}$

$$(F_{hkl} = 0)$$

$h, k, l$

$$(F_{hkl} = 4f_{Na^+} + 4f_{Cl^-})$$

$h, k, l$

$$(F_{hkl} = 4f_{Na^+} - 4f_{Cl^-})$$

$h, k, l$

(+)

(26.2)

(-)

$NaCl$	$CsCl$	$CFC$	$CC$	$Cs$	$N^2 = h^2 + k^2 + l^2$	$(hkl)$
-	+	-	-	+	1	(100)
-	+	-	+	+	2	(110)
+	+	+	-	+	3	(111)
+	+	+	+	+	4	(200)
-	+	-	-	+	5	(210)
-	+	-	+	+	6	(211)
+	+	+	+	+	8	(220)
-	+	-	-	+	9	(300)-(221)

:(2.2)

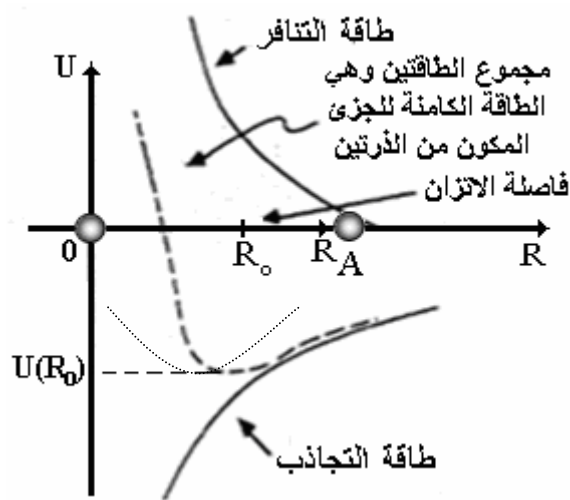
الفصل الثالث

# الروابط البلورية والخصائص المرونية





( ) r A  
F



الشكل (1.3):

$$(1.3) \quad \vec{F} = -\frac{dU}{dR} \vec{R}$$

$$\vec{F} \quad ( ) \quad \frac{dU}{dR} > 0 \quad (1.3)$$

$$\vec{R} \quad \vec{F} \quad \frac{dU}{dR} < 0 \quad \vec{R}$$

$$\vec{R} \quad (1.3) \quad .( )$$

$$( ) \quad \frac{dU}{dR} > 0 \quad R > R_0$$

$$(F)_{R_0} = 0 \quad \frac{dU}{dR} = 0 \quad R = R_0 \quad \frac{dU}{dR} < 0 \quad R < R_0$$

:

$$(2.3) \quad U(R) = \frac{a}{R^m} - \frac{b}{R^n}$$

،  $-b/R^n$   $a/R^m$   $n, m, b, a :$

$$\left( \frac{d^2U}{dR^2} \right)_{R_0} = \beta > 0 :$$

:  $\lambda \exp(-R/\rho) :$

$m > n$

$m$

$a/R^m$

$\rho, \lambda$

$R_0$

(2.3)

:

$$(3.3) \quad F(R_0) = \left( -\frac{du}{dr} \right)_{R=R_0} = 0$$

:  $R = R_0$  (3.3)

(2.3)

$$(4.3) \quad - \left( \frac{bnR_0^{n-1}}{R_0^{2n}} - \frac{amR_0^{m-1}}{R_0^{2m}} \right) = 0$$

: (4.3)

$$(5.3) \quad R_0 = \left( \frac{am}{bn} \right)^{\frac{1}{m-n}}$$

:

(  $R = R_0$  ) (2.3) (5.3)

$$(6.3) \quad U(R_0) = \frac{bnR_0^{m-n}}{mR_0^m} - \frac{b}{R_0^n} = \frac{bn}{m} R_0^{-n} - \frac{b}{R_0^n} = -bR_0^{-n} \left( 1 - \frac{n}{m} \right)$$

$m > n$  أن

$U(R_0)$

(6.3)



$$(8.3) \quad U_{ij} = \frac{a}{R^m} \left( \frac{1}{p_{ij}} \right) - K \frac{q^2}{R} \left( \frac{\mp 1}{p_{ij}} \right)$$

$$: \quad (8.3) \quad j \quad i$$

$$(9.3) \quad U_i = \sum_{j(j \neq i)} U_{ij} = \frac{a}{R^m} A_n - |\alpha| K \frac{q^2}{R}$$

$$(10.3) \quad A_n = \sum_{j(j \neq i)} \left( \frac{1}{p_{ij}} \right)^n$$

$$(11.3) \quad \alpha = \sum_{j(j \neq i)} \left( \frac{\mp 1}{p_{ij}} \right)$$

(Madelung)  $\alpha$   $m$   $A_n$

$\cdot$   
:  
 $2N$

$$(12.3) \quad U_{tot}(R) = \left( \frac{1}{2} \right) 2N U_i = N \left( \frac{a}{R^m} A_n - |\alpha| \frac{Kq^2}{R} \right)$$

$\cdot$   
 $1/2$

$R_0$  ( )

:

$$(13.3) \quad \left( \frac{dU_{tot}(R)}{dR} \right)_{R_0} = 0$$

$$N \left( \frac{-ma}{R_0^{m+1}} A_n + |\alpha| \frac{Kq^2}{R_0^2} \right) = 0$$

$$(14.3) \quad R_0 = \left( \frac{maA_n}{|\alpha|Kq^2} \right)^{\frac{1}{m-1}}$$

: (  $R = R_0$  ) (12.3) (14.3)

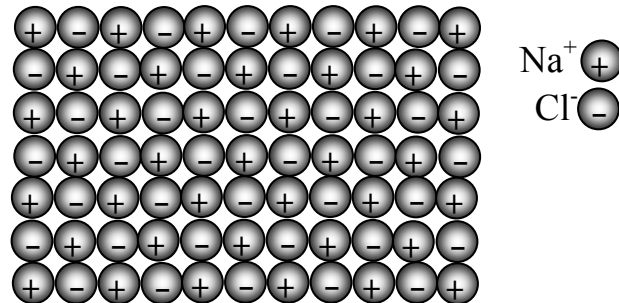
$$(15.3) \quad U_{tot}(R_0) = -|\alpha| \frac{NKq^2}{R_0} \left( 1 - \frac{1}{m} \right)$$

$$) \quad \frac{U_{tot}(R_0)}{N} = -|\alpha| \frac{Kq^2}{R_0} \left( 1 - \frac{1}{m} \right) \quad \left( -|\alpha| \frac{NKq^2}{R_0} \right)$$

$N_a$  . (  $(J/mole)$  )

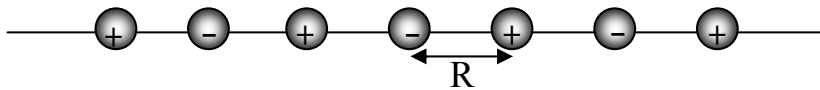
طاقة الالتحام (mole/ KJ)	البلورة	طاقة الالتحام (mole/ KJ)	البلورة
635	بروميد الروبيديوم <i>RbBr</i>	752	كلوريد الصوديوم <i>NaCl</i>
595	أيوديد السيزيوم <i>CsI</i>	650	أيوديد البوتاسيوم <i>KI</i>

الجدول (1.3):

:  $\langle 100 \rangle$ 

الشكل (2.3):

مثال:



الشكل (3.3):

$$(16.3) \quad |\alpha| = \left| \sum_{j(j \neq i)} \left( \frac{\mp 1}{p_{ij}} \right) \right| = \left| \sum_{j(j \neq i)} \left( \frac{\mp 1}{\left( \frac{r_{ij}}{R} \right)} \right) \right| = \left| \sum_{j(j \neq i)} \left( \frac{\mp R}{r_{ij}} \right) \right|$$

$$|\alpha| = \left| \sum_{j(j \neq i)} \left( \frac{\mp 1}{p_{ij}} \right) \right| = 2 \left( 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \right)$$

$$: 0 \quad \ln(1+x)$$

$$\ln(1+x) = \left( x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots \right)$$

$$\ln(1+1) = \ln(2) = \left( 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \right)$$

:

$$(17.3) \quad |\alpha| = |2(\ln(2))| = 1.3863$$

:

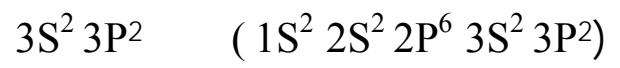
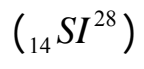
<i>ZnS</i>	<i>CsCl</i>	<i>NaCl</i>	<i>CFC</i>	<i>CC</i>	البنية البلورية
1.638	1.762	1.747	1.792	1.792	ثابت مادلونغ

الجدول (2.3):

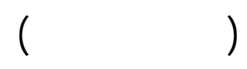
### 4-3 الرابطة التساهمية:

,

)  
)  
(  
(



$$3S \quad (4.3)$$





( )  $SP^3$

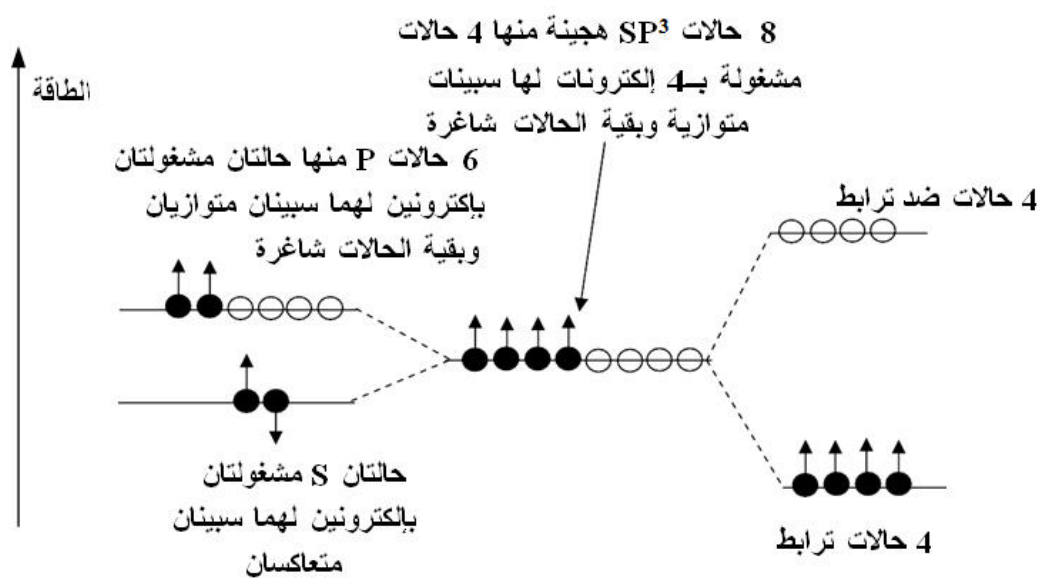
$SP^3$  )

(

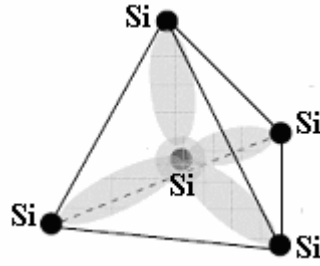
(5.3)

1

2



الشكل (4.3):



الشكل (5.3):

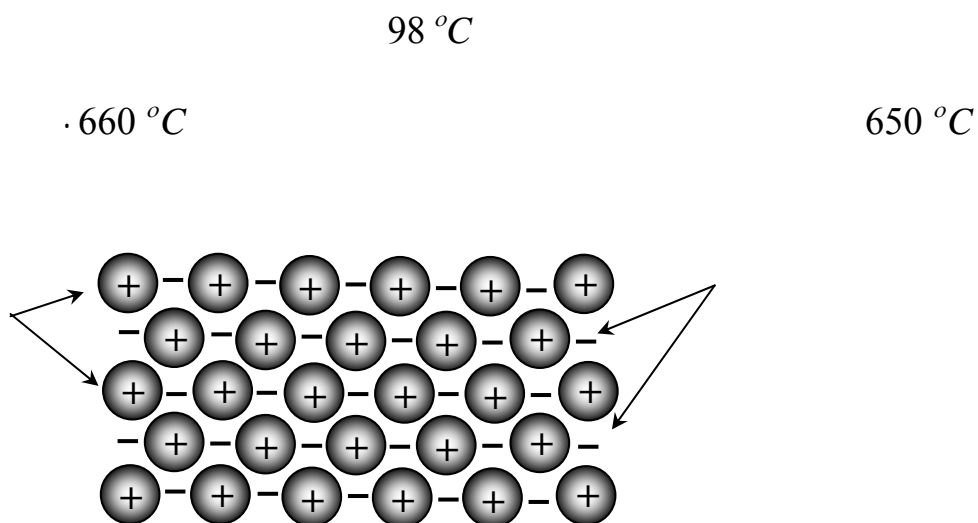
(3.3)

البلورة	طاقة الانتحام ( $KJ / mole$ )	درجة حرارة الانصهار ( $^{\circ}C$ )
الماس C	713	1410
سيلكون Si	450	>3550
جرمانيوم Ge	3.5	-

الجدول (3.3):

5-3 الرابطة المعدنية:

((6.3) )



الشكل (6.3): مخطط مبسط للرابطة المعدنية

(4.3)

درجة حرارة الانصهار (°C)	طاقة الالتحام (KJ / mole)	بلورة
660	324	الألمنيوم Al
1538	406	الحديد Fe
3410	849	التنغستن W

الجدول (4.3):

### 6-3 رابطة فان در فالس (Van Der Waals) أو الرابطة الجزيئية:

0.2ev

(London)

1930

(Heisenberg)

)

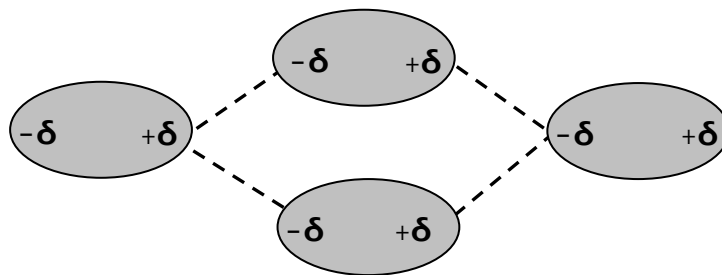
.(

,((7.3) )

)

*CFC*

.(Xe(-112°C) Kr(-156°C) Ar(-189°C) Ne(-249°C)



الشكل (7.3):

درجة حرارة الانصهار (° C)	طاقة الالتحام (KJ / mole)	البلورة
-189	7.7	الأرغون Ar
-101	31	CL <sub>2</sub>
-78	35	NH <sub>3</sub>

الجدول (5.3):

$r_{ij}$   $j$   $i$

: (Lennard-Jones)

$$(18.3) \quad U_{ij}(r_{ij}) = 4\epsilon \left[ \left( \frac{\sigma}{r_{ij}} \right)^{12} - \left( \frac{\sigma}{r_{ij}} \right)^6 \right]$$

$\epsilon$   $\sigma$ :

$$. b = 4\epsilon\sigma^6 \quad a = 4\epsilon\sigma^{12} \quad (2.3)$$

$$(19.3) \quad U_{tot}(R) = \frac{N}{2} \sum_{j(j \neq i)} U_{ij}(r_{ij}) = 2N\epsilon \left[ \left( \frac{\sigma}{R} \right)^{12} A_{12} - \left( \frac{\sigma}{R} \right)^6 A_6 \right]$$

$$: R \quad p_{ij} = \frac{r_{ij}}{R} \quad A_n = \sum_{i \neq j} \left( \frac{1}{p_{ij}} \right)^n :$$

$R_0$

$$(20.3) \quad \left( \frac{dU_{tot}(R)}{dR} \right)_{R_0} = 0 \Rightarrow R_0 = \sigma \left( \frac{2A_{12}}{A_6} \right)^{\frac{1}{6}}$$

: (  $R = R_0$  ) (19.3) (20.3)

$$(21.3) \quad U_{tot}(R_0) = -\frac{2N\varepsilon\sigma^6 A_6}{2} R_0^{-6} = \frac{N\varepsilon A_6^2}{2A_{12}}$$

$$\frac{U_{tot}(R_0)}{N} = \frac{\varepsilon A_6^2}{2A_{12}}$$

$$A_6 \quad A_{12} \quad (6.3)$$

$$A_{12} < A_6$$

<i>CFC</i>	<i>CC</i>	<i>CS</i>	$A_n$
14.45	12.25	8.40	$A_6$
12.13	9.11	6.20	$A_{12}$

. الجدول (6.3): لـ  $A_6$  ,  $A_{12}$

تطبيق:

$a$   $N$   $CFC$   $R$

: ( )

$$(22.3) \quad B = V_0 \left( \frac{d^2 U_{tot}}{dV^2} \right)_{T, V_0} = \left( V \frac{d^2 U_{tot}}{dR^2} \left( \frac{dR}{dV} \right)^2 \right)_{T, R_0}$$

$V_0$  :

$$R = \frac{a}{\sqrt{2}} \quad V = \frac{a^3}{4} N : \quad CFC$$

:  $R_0$

$$(23.3) \quad R_0 = \sigma \left( \frac{2A_{12}}{A_6} \right)^{\frac{1}{6}} = 1.09\sigma$$

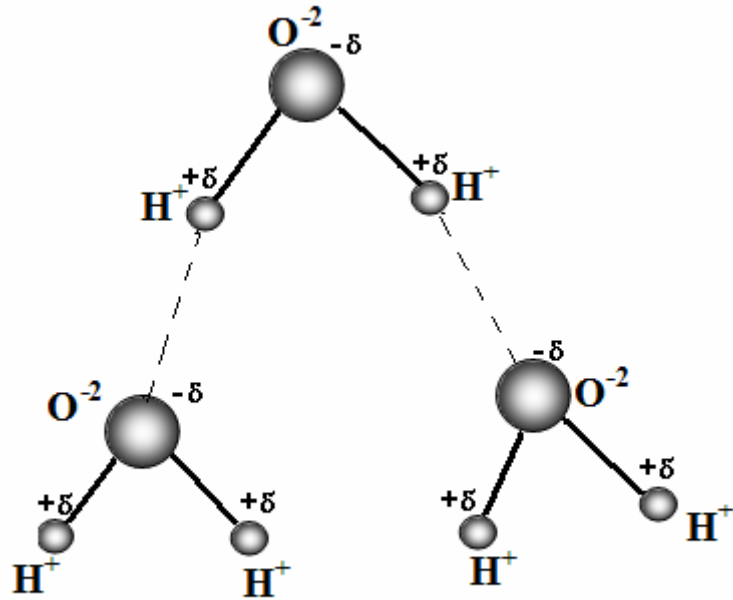
$$(24.3) \quad V = \frac{a^3}{4} N = \frac{N}{\sqrt{2}} R^3$$

: (24.3) و (23.3) (19.3) (22.3)

$$(25.3) \quad B \approx 75 \epsilon / \sigma^3$$

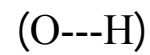
6-3 الرابطة الهيدروجينية:

(8.3)



تنظيم جزيئات الماء بسبب الرابطة الهيدروجينية

الشكل (8.3):



(+δ)

(-δ)

$$(R_{O...H} = 2.76 \text{ \AA})$$

$$.( R_{O-H} = 0.96 \text{ \AA})$$





$$(26.3) \quad U(R) = U(R_0) + \left( \frac{dU}{dR} \right)_{R_0} (R - R_0) + \frac{1}{2} \left( \frac{d^2U}{dR^2} \right)_{R_0} (R - R_0)^2 + \frac{1}{6} \left( \frac{d^3U}{dR^3} \right)_{R_0} (R - R_0)^3 + \dots$$

$$\left( \frac{d^2U}{dR^2} \right)_{R_0} = \beta > 0 \quad (1.3)$$

$$\gamma \left( \frac{d^3U}{dR^3} \right)_{R_0} = -2\gamma < 0 \quad : \quad R = R_0$$

$$(26.3) \quad U(X) = U(R) - U(R_0), \quad (R - R_0) = X$$

$$(27.3) \quad U(X) = \frac{1}{2} \beta X^2 + \frac{1}{3} \gamma X^3$$

$$(27.3) \quad X^2 \gg X^3$$

$$(28.3) \quad U(X) = \frac{1}{2} \beta X^2$$

: A

$$(29.3) \quad f = -\frac{dU(X)}{dX} = -\beta X$$

) X

(

(1.3)

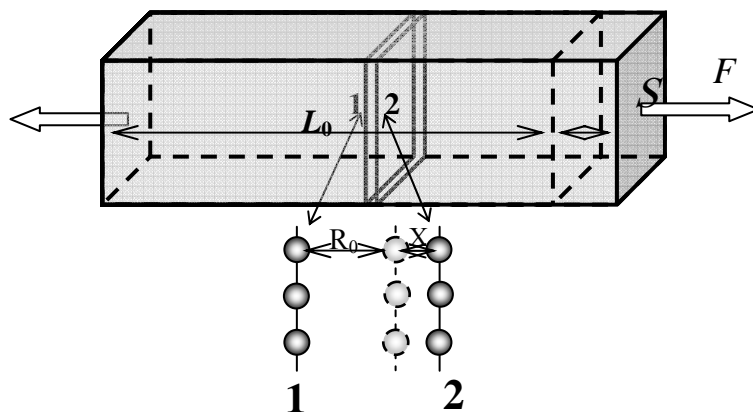
(28.3)

. U(R<sub>0</sub>)

$$L_0 \quad S \quad F$$

$$X : \quad \Delta L = \sum X \quad L$$

$$(9.3) \quad 2 \quad 1$$



الشكل (9.3):

$$F_{\text{int}}$$

$$(30.3) \quad F_{\text{int}} = fN = N\beta X$$

$$S \quad N$$

$$(31.3) \quad \sigma = \frac{F_{\text{int}}}{S} = \frac{N\beta X}{S} = CX$$

$$C = \frac{N\beta}{S}$$

$$R_0 \quad (31.3)$$

$$(32.3) \quad \sigma = R_0 C \frac{X}{R_0} = \frac{R_0 N \beta}{S} \left( \frac{X}{R_0} \right)$$

$$E = \frac{R_0 N \beta}{S}, \varepsilon' = \frac{X}{R_0} \quad (32.3)$$

$$(33.3) \quad \sigma = E \varepsilon'$$

$$(34.3) \quad \varepsilon' = \frac{N'X}{N'R_0} = \frac{\Delta L}{L_0} = \varepsilon \quad (34.3)$$

$$(35.3) \quad \sigma = E \varepsilon$$

$$\text{(Hooke)} \quad (35.3)$$

$$\sigma = E \varepsilon \quad (35.3)$$

$$(7.3)$$

$E (10^9 N m^2)$		المادة
النهاية الصغرى	النهاية العظمى	
64	77	Al
68	194	Cu
135	290	Fe
437	514	Mg
400	400	W

الجدول (7.3): معامل يونغ لبعض المعادن.

### 2-9-3 منحنى الإجهاد والانفعال:

(10.3).

المجال OA: ( )

$$\sigma_e \quad . (\sigma \propto \varepsilon)$$

:

$$\sigma = 0 \Rightarrow \varepsilon = 0$$

(36.3)

$$\sigma \neq 0 \Rightarrow \sigma = \tan(\alpha)\varepsilon \Rightarrow E = \tan(\alpha)$$

$$\sigma > \sigma_e$$

المجال AB:

:

(37.3)

$$\sigma = \Gamma \varepsilon^m$$

( )

: m

Γ :

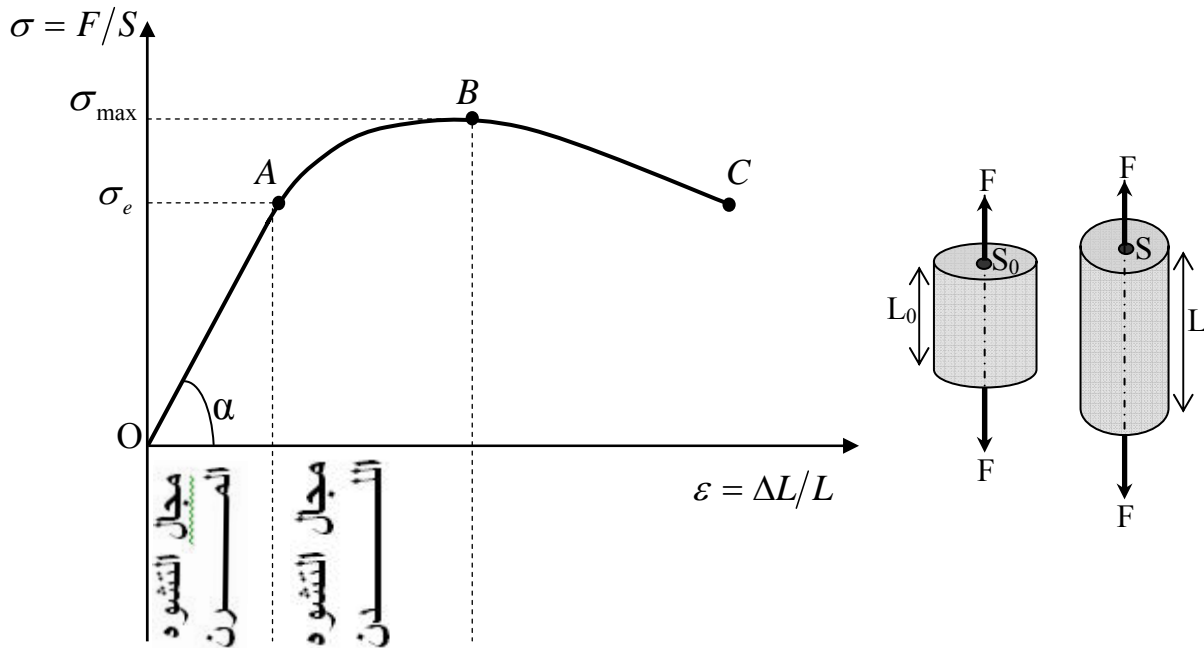
)

$\sigma_{max}$

المجال BC:

(

.C



الشكل (10.3): المنحنى الاسمي إجهاد - انفعال.

3-9-3 معامل بواسون والانفعال الحجمي:

$$(11.3) \quad (XYZ) \quad \sigma_y \quad \varepsilon_y \quad Y \quad \varepsilon_z \quad \varepsilon_x \quad Z \quad X \quad : \quad (34.3)$$

$$(38.3) \quad \varepsilon_y = \frac{\Delta L_y}{L_{y0}} = \frac{L_y - L_{y0}}{L_{y0}} > 0$$

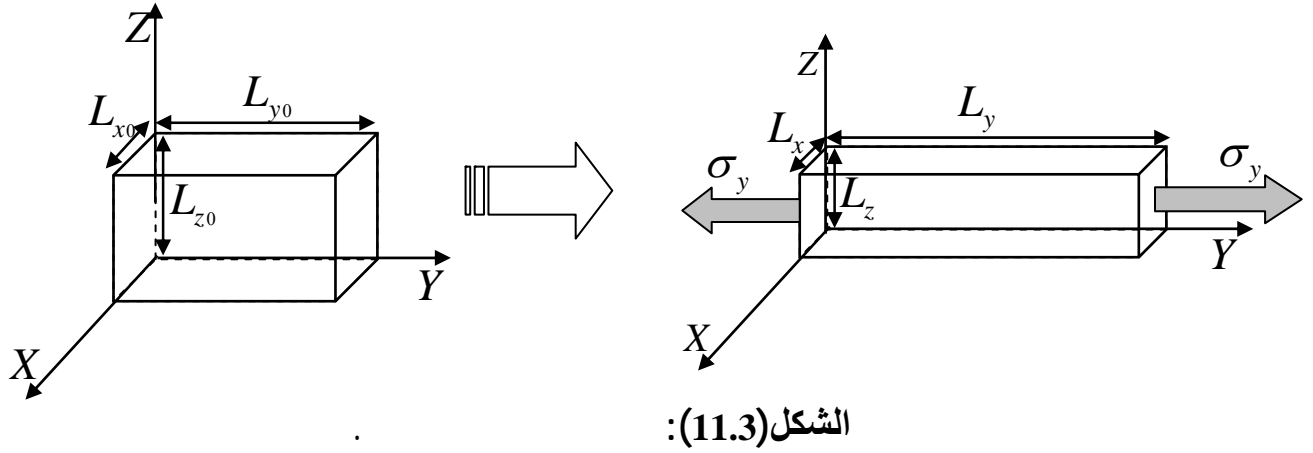
$$(39.3) \quad \varepsilon_x = \frac{\Delta L_x}{L_{x0}} = \frac{L_x - L_{x0}}{L_{x0}} < 0$$

$$(40.3) \quad \varepsilon_z = \frac{\Delta L_z}{L_{z0}} = \frac{L_z - L_{z0}}{L_{z0}} < 0$$

$\nu$

:

$$(41.3) \quad \nu = -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{\varepsilon_z}{\varepsilon_y}$$



(8.3)

المطاط	النحاس	الفولاذ	المادة
0.5	0.36	0.25	معامل بواسون

الجدول (8.3):

$$\frac{\Delta V}{V_0} ( \quad )$$

$$(42.3) \quad V_0 = L_{x0} \times L_{y0} \times L_{z0} \quad V = L_x \times L_y \times L_z$$

$$(43.3) \quad \epsilon_x = \frac{L_x - L_{x0}}{L_{x0}} \Rightarrow L_x = L_{x0}(1 + \epsilon_x)$$

$$(44.3) \quad L_y = L_{y0}(1 + \epsilon_y)$$

$$(45.3) \quad L_z = L_{z0}(1 + \epsilon_z)$$



:

$$V = L_{x0}(1 + \varepsilon_x) \times L_{y0}(1 + \varepsilon_y) \times L_{z0}(1 + \varepsilon_z) = V_0((1 + \varepsilon_x) \times (1 + \varepsilon_y) \times (1 + \varepsilon_z))$$

$$\frac{\Delta V}{V_0} = \frac{V - V_0}{V_0} = ((1 + \varepsilon_x) \times (1 + \varepsilon_y) \times (1 + \varepsilon_z) - 1)$$

$$(46.3) \quad \frac{\Delta V}{V_0} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \sum_{i=1}^3 \varepsilon_i$$

$$\varepsilon_x \varepsilon_y \approx \varepsilon_x \varepsilon_z \approx \varepsilon_y \varepsilon_z \approx \varepsilon_x \varepsilon_y \varepsilon_z \approx 0 \quad :$$

$$: (46.3) \quad (41.3)$$

$$(47.3) \quad \frac{\Delta V}{V_0} = \varepsilon_y(1 - 2\nu)$$

$$: \quad r \quad L$$

$$(48.3) \quad \nu = -\frac{\varepsilon_r}{\varepsilon_L} = -\frac{\Delta r/r_0}{\Delta L/L_0}$$

$$V = L\pi r^2 \Rightarrow \frac{\Delta V}{V_0} = \frac{\Delta L}{L_0} + 2\frac{\Delta r}{r_0} \quad :$$

$$(49.3) \quad \frac{\Delta V}{V_0} = \varepsilon_L(1 - 2\nu) \quad :$$

3-9-3 معامل القص:

$$\tau \propto \theta \quad (12.3)$$

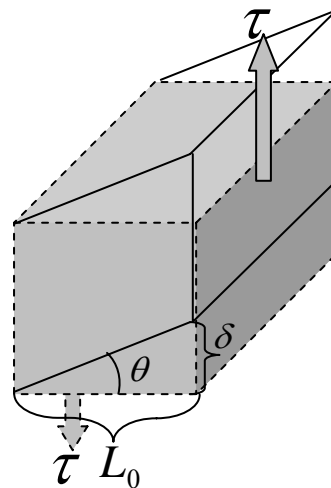
$$(50.3) \quad \gamma = \tan(\theta) = \frac{\delta}{L_0}$$

$$\tau \propto \gamma$$

$$(51.3) \quad \gamma \cong \theta(\text{radian}) \cong \frac{\delta}{L_0}$$

$$(52.3) \quad \tau = G\gamma$$

$$\tau = G\gamma$$



الشكل (12.3): تأثير إجهاد القص

(9.3)

$E (10^9 Nm^2 )$		المادة
النهاية الصغرى	النهاية العظمى	
25	29	Al
31	77	Cu
61	180	Fe
171	184	Mg
155	155	W

الجدول (9.3):

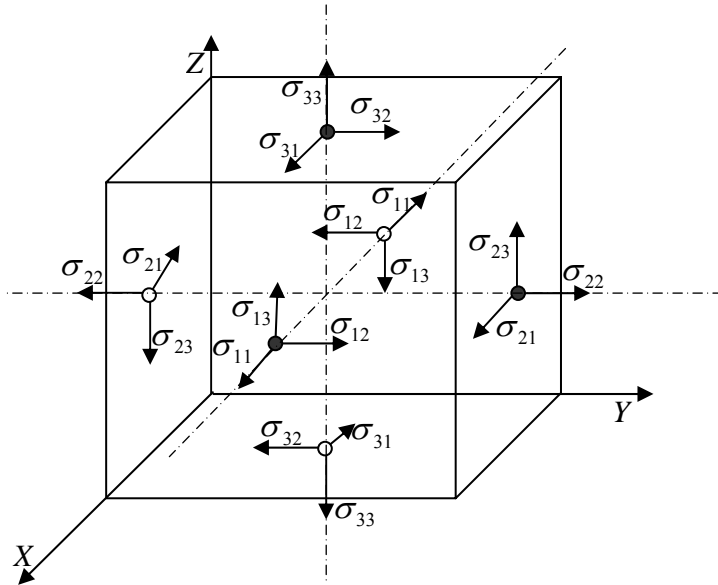
ملاحظة:

$$(53.3) \quad G = \frac{E}{2(1+\nu)}$$

3-9-5 ممتد الإجهاد:

$$i \quad [ \sigma_{ij} ] \quad j \quad , \quad 3^2 = 9 \quad ((13.3))$$

$$(54.3) \quad [ \sigma_{ij} ] = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$



الشكل (13.3):

1. خصائص ممتد الإجهاد:

$$\sigma_{ii} > 0$$

$$(i = j) \quad \sigma_{ii}$$

.i

$$\sigma_{ii} < 0$$

i

9

6

$$(i \neq j)$$

$$\sigma_{ij} = \sigma_{ji}$$

:

(55.3)

$$[\sigma_{ij}] = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

$$(56.3) \quad \det([\sigma_{ij}] - \mu I) = \begin{vmatrix} \sigma_{11} - \mu & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \mu & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} - \mu \end{vmatrix} = 0$$

$\mu_3, \mu_2, \mu_1$

:

$\sigma_3, \sigma_2, \sigma_1$

$$(57.3) \quad [\sigma] = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

$\Sigma^-(O', x'_1, x'_2, x'_3)$

$\Sigma(O, x_1, x_2, x_3)$

:

$$(58.3) \quad \sigma_{ij} = \sum_{k,l=1}^3 a_{ik} a_{jl} \sigma_{kl} \quad i, j = 1, 2, 3$$

$\Sigma(O, x_1, x_2, x_3)$

:  $a_{ik}, a_{jl}$  :

$\cdot \Sigma^-(O', x'_1, x'_2, x'_3)$

2. حساب شعاع الإجهاد الكلي  $\vec{T}(M, \vec{n})$

$\vec{n}$

$M$

$\vec{T}(M, \vec{n})$

$\vec{n}$

$$(59.3) \quad \vec{T}(M, \vec{n}) = [\sigma_{ij}] \cdot \vec{n}$$

$T_t$

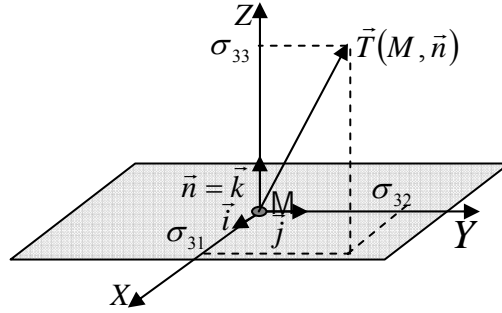
$T_n$

$$(60.3) \quad \begin{aligned} T_n &= \vec{T}(M, \vec{k}) \cdot \vec{n} \\ T_t &= \sqrt{(\vec{T}(M, \vec{k}))^2 - (T_n)^2} \end{aligned}$$

:(14.3)

 $\vec{k}$ 

مثال:

 $\vec{k}$ 

الشكل (14.3):

$$: \quad (i \neq j) \quad \sigma_{ij} = \sigma_{ji}$$

$$\vec{T}(M, \vec{k}) = \sigma_{31} \vec{i} + \sigma_{32} \vec{j} + \sigma_{33} \vec{k} = \sigma_{13} \vec{i} + \sigma_{23} \vec{j} + \sigma_{33} \vec{k}$$

$$\vec{T}(M, \vec{k}) = [\sigma_{ij}] \cdot \vec{k} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sigma_{13} \\ \sigma_{23} \\ \sigma_{33} \end{pmatrix} = \sigma_{13} \vec{i} + \sigma_{23} \vec{j} + \sigma_{33} \vec{k}$$

:  $T_t$        $T_n$ 

$$(61.3) \quad T_n = \vec{T}(M, \vec{k}) \cdot \vec{k} = \sigma_{33}$$

$$(62.3) \quad T_t = \sqrt{(\vec{T}(M, \vec{k}))^2 - (T_n)^2} = \sqrt{(\sigma_{13}^2 + \sigma_{23}^2 + \sigma_{33}^2) - \sigma_{33}^2} = \sqrt{\sigma_{13}^2 + \sigma_{23}^2}$$

6-9-3 معامل الانضغاط الحجمي:

$$\Delta p \quad A_0 \quad P_0 \quad F$$

$$P_0 + \Delta p$$

$$-\Delta p$$

$$(15.3)$$

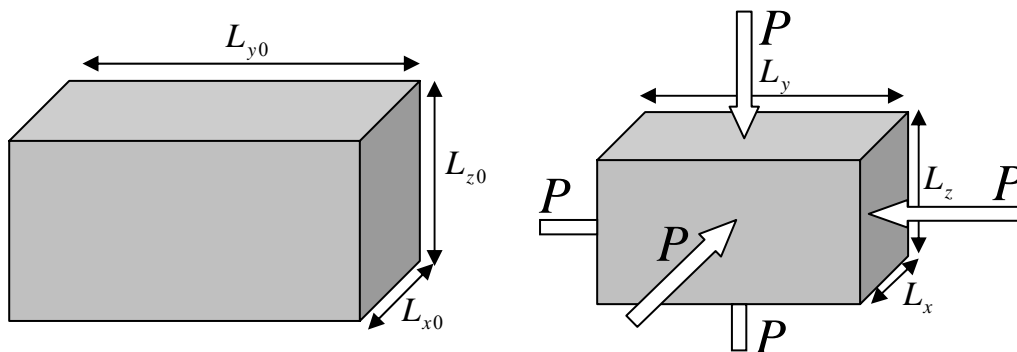
:

$$(63.3) \quad \sigma_{ij} = -\Delta p \delta_{ij}$$

$\delta_{ij}$  :

$$: \quad B \quad \Delta P \quad \frac{\Delta V}{V_0}$$

$$(64.3) \quad B = -\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)}$$



الشكل (15.3):

$\chi$  :

$$(65.3) \quad \chi = \frac{1}{B} = -V_0 \frac{\Delta V}{\Delta P}$$





$$[\zeta_{ij}]$$

:

$$(69.3) \quad \zeta_{ij} = \varpi_{ij} + \varepsilon_{ij} \quad i, j = 1, 2, 3$$

$$[\varpi_{ij}]$$

:

0

$$(70.3) \quad \varpi_{ij} = \frac{1}{2}(\zeta_{ij} - \zeta_{ji}) = -\frac{1}{2}(\zeta_{ji} - \zeta_{ij}) = -\varpi_{ji} \quad i, j = 1, 2, 3$$

:

$$[\varpi_{ij}]$$

$$(71.3) \quad [\varpi_{ij}] = \begin{pmatrix} 0 & \varpi_{12} & \varpi_{13} \\ -\varpi_{12} & 0 & \varpi_{23} \\ -\varpi_{13} & -\varpi_{23} & 0 \end{pmatrix}$$

$$(72.3) \quad [\varpi_{ij}] = \begin{pmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ -\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) \\ -\frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) & -\frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) & 0 \end{pmatrix}$$

6

$$[\varepsilon_{ij}]$$

:

$$(73.3) \quad \varepsilon_{ij} = \frac{1}{2}(\zeta_{ij} + \zeta_{ji}) = \frac{1}{2}(\zeta_{ji} + \zeta_{ij}) = \varepsilon_{ji} \quad i, j = 1, 2, 3$$

:

 $[\varepsilon_{ij}]$ 

$$[\varepsilon_{ij}] = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{pmatrix}$$

$$(74.3) \quad [\varepsilon_{ij}] = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2} \left( \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) & \frac{1}{2} \left( \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) \\ \frac{1}{2} \left( \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2} \left( \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right) \\ \frac{1}{2} \left( \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) & \frac{1}{2} \left( \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right) & \frac{\partial u_3}{\partial x_3} \end{pmatrix}$$

·  $[\zeta_{ij}]$  $[\varepsilon_{ij}]$ 

## 7-9-3 قانون هوك المعمم:

(1) حالة المواد موحدة الخواص (متماثلة المتناحي):

:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{pmatrix} = \begin{pmatrix} E & \nu & \nu & 0 & 0 & 0 \\ \nu & E & \nu & 0 & 0 & 0 \\ \nu & \nu & E & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{pmatrix} \quad (35.3)$$

(75.3)

$$\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E}$$

:

:  $\varepsilon_{11}$

$$(76.3) \quad \varepsilon_{11} = \frac{\sigma_{11}}{E} - \nu \frac{\sigma_{22}}{E} - \nu \frac{\sigma_{33}}{E}$$

$$\varepsilon_{11} = \frac{1}{E} (\sigma_{11} - \nu(\sigma_{22} + \sigma_{33}))$$

:  $\varepsilon_{33}, \varepsilon_{22}$

$$(77.3) \quad \varepsilon_{22} = \frac{1}{E} (\sigma_{22} - \nu(\sigma_{11} + \sigma_{33}))$$

$$(78.3) \quad \varepsilon_{33} = \frac{1}{E} (\sigma_{33} - \nu(\sigma_{11} + \sigma_{22}))$$

: (78.3), (77.3), (76.3)

$$(79.3) \quad \varepsilon_{ii} = \frac{1}{E} (\sigma_{ii} - \nu\sigma_{11} - \nu\sigma_{22} - \nu\sigma_{33} + \nu\sigma_{ii}) \quad i = 1, 2, 3$$

$$(80.3) \quad \varepsilon_{ii} = \frac{1}{E} ((1 + \nu)\sigma_{ii} - \nu \text{trac}[\sigma_{ij}]) \quad i = 1, 2, 3$$

.  $\text{trac}[\sigma_{ij}] = \sigma_{11} + \sigma_{22} + \sigma_{33}$  :

$$\gamma_{ij} = 2\varepsilon_{ij} (i \neq j)$$

$$\tau_{ij} = \sigma_{ij} (i \neq j)$$

:  $(\gamma_{ij})$

: (52.3)

$$(81.3) \quad \sigma_{ij} = G\gamma_{ij} = 2G\varepsilon_{ij} \Rightarrow \varepsilon_{ij} = \frac{1}{2G}\sigma_{ij} \quad i, j = 1, 2, 3$$

$$: (81.3) \quad (53.3)$$

$$(82.3) \quad \varepsilon_{ij} = \frac{1}{2\left(\frac{E}{2(1+\nu)}\right)} \sigma_{ij} = \frac{1+\nu}{E} \sigma_{ij} \quad i, j = 1, 2, 3$$

$$: (82.3) \quad (80.3)$$

$$(83.3) \quad \varepsilon_{ij} = \frac{1}{E} \left( (1+\nu) \sigma_{ij} - \nu \text{trac}[\sigma_{ij}] \delta_{ij} \right) \quad i, j = 1, 2, 3$$

$\delta_{ij} :$

:

$$(84.3) \quad \sigma_{ij} = \frac{E}{1+\nu} \left( \varepsilon_{ij} + \frac{\nu}{1-2\nu} \text{trac}[\sigma_{ij}] \delta_{ij} \right) \quad i = 1, 2, 3$$

$$: \quad \mu, \lambda \quad (38.3)$$

$$(85.3) \quad \sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda \text{trac}[\sigma_{ij}] \delta_{ij} \quad i = 1, 2, 3$$

$$: \quad (85.3) \quad (84.3)$$

$$(86.3) \quad 2\mu = \frac{E}{1+\nu}$$

$$(87.3) \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$E \quad G \quad B \quad : \quad :$$

$$: \quad \nu$$

$$(88.3) \quad B = -\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} = -\frac{\Delta p}{\left(\sum_{i=1}^3 \varepsilon_{ii}\right)}$$

$$.(64.3) \quad (46.3)$$

$$: \quad (84.3) \quad (83.3)$$

$$(89.3) \quad \varepsilon_{ii} = \frac{1}{E} \left( (1+\nu)\sigma_{ii} - \nu\sigma_{11} - \nu\sigma_{22} - \nu\sigma_{33} \right) \quad i = 1,2,3$$

$$(90.3) \quad \sum_{i=1}^3 \varepsilon_{ii} = \sum_{i=1}^3 \frac{1}{E} \left( (1+\nu)\sigma_{ii} - \nu\sigma_{11} - \nu\sigma_{22} - \nu\sigma_{33} \right) \quad i = 1,2,3$$

$$= \frac{1}{E} \left( (1-2\nu)(\sigma_{11} + \sigma_{22} + \sigma_{33}) \right)$$

:

$$(91.3) \quad \begin{aligned} \sigma_{ij} &= -\Delta p \delta_{ij} & i, j &= 1,2,3 \\ \sigma_{ii} &= -\Delta p & i &= 1,2,3 \end{aligned}$$

$$: \quad (90.3) \quad (91.3)$$

$$(92.3) \quad \sum_{i=1}^3 \varepsilon_{ii} = \frac{-3\Delta p}{E} (1-2\nu) \quad i = 1,2,3$$

$$: \quad (88.3)$$

$$(93.3) \quad B = \frac{E}{3(1-2\nu)}$$

$$: \quad (93.3) \quad (53.3) \quad \nu$$

$$(94.3) \quad B = \frac{GE}{3(3G-E)}$$

$$:( \quad ) \quad (2)$$

$$\begin{aligned} & ) \quad 9 \\ & (3^4 = 9 \times 9 = 81) \quad ( \\ & ( \quad ) \end{aligned}$$

:

$$(95.3) \quad \sigma_{ij} = \sum_{k,l=1}^3 C_{ijkl} \epsilon_{kl} \quad i,j=1,2,3$$

$$(96.3) \quad \epsilon_{ij} = \sum_{k,l=1}^3 S_{ijkl} \sigma_{kl} \quad i,j=1,2,3$$

$$(96.3) \quad (95.3)$$

$$81 \quad S_{ijkl} \quad C_{ijkl}$$

$$(9 \quad 6 \quad )$$

$$, (6 \times 6 = 36)$$

$$S_{ijkl} = S_{klij} \quad C_{ijkl} = C_{klij} :$$

$$) \quad 21$$

$$)) \quad ($$

$$(($$

$$180^\circ$$

:

$$(97.3) \quad C'_{ijkl} = \sum_{m,n,p,q=1}^3 a_{im} a_{jn} a_{kp} a_{lq} C_{mnpq} \quad i,j,k,l=1,2,3$$

$$(98.3) \quad S'_{ijkl} = \sum_{m,n,p,q=1}^3 a_{im} a_{jn} a_{kp} a_{lq} S_{mnpq} \quad i,j,k,l=1,2,3$$

$$\begin{matrix} & & x_r & x'_h & a_{hr} \\ \cdot y = S'_{3333} & x'_3 & E & & \end{matrix} \quad (98.3)$$

$$: \quad (k \leftrightarrow l) \quad (i \leftrightarrow j)$$

$$(11 \rightarrow 1) \quad (23, 32 \rightarrow 4)$$

$$(22 \rightarrow 2) \quad (13, 31 \rightarrow 5)$$

$$(33 \rightarrow 3) \quad (12, 21 \rightarrow 6)$$

:

$$(99.3) \quad \sigma_p = \sum_{g=1}^6 C_{pg} \varepsilon_g \quad g=1,\dots,6$$

$$(100.3) \quad \varepsilon_p = \sum_{g=1}^6 S_{pg} \sigma_g \quad g=1,\dots,6$$

$$C_{ijkl} = C_{klij} \quad C_{pg} = C_{gp} \quad ((100.3) \quad ) (99.3)$$

:

$$(101.3) \quad \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{pmatrix} \times \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix}$$

B

تطبيق:

$$(102.3) \quad B = -\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} = -\frac{\Delta p}{\left(\sum_{i=1}^3 \varepsilon_{ii}\right)}$$

$$(103.3) \quad \sum_{i=1}^3 \varepsilon_{ii} = \sum_{i=1}^3 \sum_{k,l=1}^3 S_{iikl} \sigma_{kl}$$

: (103.3) (91.3)

$$(104.3) \quad \begin{aligned} \sum_{i=1}^3 \varepsilon_{ii} &= \sum_{i=1}^3 \sum_{k=1}^3 S_{iikk} \sigma_{kk} \\ &= -\Delta p \sum_{i,k=1}^3 S_{iikk} \end{aligned}$$

: (102.3) (104.3)

$$(105.3) \quad B = \left( \sum_{i,k=1}^3 S_{iikk} \right)^{-1}$$



- تحديد عناصر ممتد ثوابت أو معاملات اطرونة:

21

( )

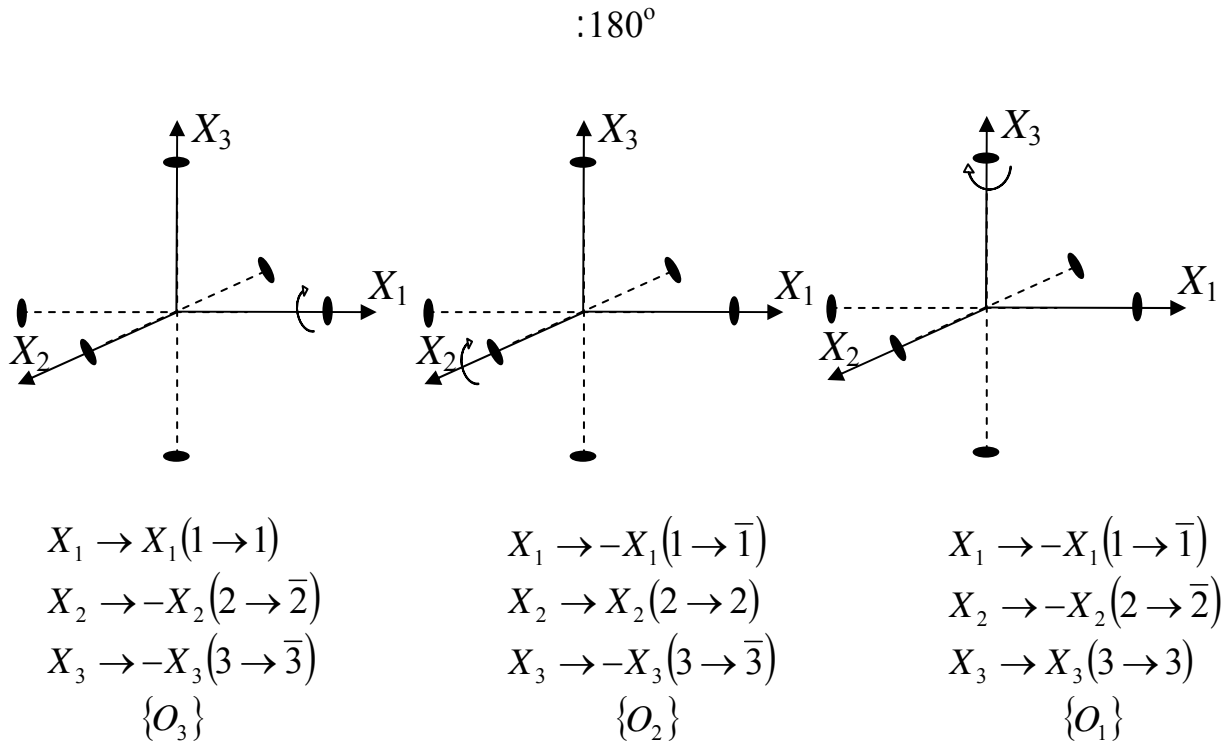
(10.3).

عدد العناصر المستقلة $S_{pg}$ أو $C_{pg}$	الفتة البلورية
21	ثلاثية الميل
31	أحادية الميل
09	المعينية المستقيمة
6	ثلاثية متساوية الأحرف
5	السداسية
3	المكعبة
2	المواد موحدة الخواص

لمتدات

الشكل (10.3):

$$\frac{2}{m} \frac{2}{m} \frac{2}{m}$$



:

$$ijkl \quad [C_{ijkl}]$$

$$[A] = \begin{pmatrix} 1111 & 1122 & 1133 & 1123 & 1131 & 1112 \\ & 2222 & 2233 & 2223 & 2231 & 2212 \\ & & 3333 & 3323 & 3331 & 3312 \\ & & & 2323 & 2331 & 2312 \\ & & & & 3131 & 3112 \\ & & & & & 1212 \end{pmatrix}$$

$$[B] \quad \{O_1\} \quad [A]$$

: [B]

$$3323 \xrightarrow{O_1} 33\bar{2}3 = -3323$$

$$3131 \xrightarrow{O_1} 3\bar{1}3\bar{1} = 3131$$

$$2223 \xrightarrow{O_1} \bar{2}\bar{2}\bar{2}3 = -2223$$

$$1112 \xrightarrow{O_1} \bar{1}\bar{1}\bar{1}\bar{2} = 1112$$

$$3333 \xrightarrow{O_1} 3333$$

$$[B] = \begin{pmatrix} 1111 & 1122 & 1133 & -1123 & -1131 & 1112 \\ & 2222 & 2233 & -2223 & -2231 & 2212 \\ & & 3333 & -3323 & -3331 & 3312 \\ & & & 2323 & 2331 & -2312 \\ & & & & 3131 & -3112 \\ & & & & & 1212 \end{pmatrix}$$

$$[B] = [A]$$

$$[B], [A]$$

$$(3331 \rightarrow -3331 \Leftrightarrow C_{3331} \rightarrow -C_{3331} \Rightarrow C_{3331} = 0)$$

$$[D] \quad [B]$$

$$[D] = \begin{pmatrix} 1111 & 1122 & 1133 & 0 & 0 & 1112 \\ & 2222 & 2233 & 0 & 0 & 2212 \\ & & 3333 & 0 & 0 & 3312 \\ & & & 2323 & 2331 & 0 \\ & & & & 3131 & 0 \\ & & & & & 1212 \end{pmatrix}$$

$$: [E] \quad O_2 \quad [D]$$

$$[E] = \begin{pmatrix} 1111 & 1122 & 1133 & 0 & 0 & -1112 \\ & 2222 & 2233 & 0 & 0 & -2212 \\ & & 3333 & 0 & 0 & -3312 \\ & & & 2323 & -2331 & 0 \\ & & & & 3131 & 0 \\ & & & & & 1212 \end{pmatrix}$$

$$O_3 \quad \cdot$$

[E]

:

$$(106.3) \quad [C_{ij}] = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix}$$

:

$$. C_{44}, C_{12}, C_{11} \quad : \quad (1)$$

$$(107.3) \quad [C_{ij}] = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{pmatrix}$$

$$. C_{44}, C_{12}, C_{13}, C_{33}, C_{11} \quad : \quad (2)$$

$$(108.3) \quad [C_{ij}] = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) \end{pmatrix}$$

$$. C_{12}, C_{11} \quad : \quad (3)$$

$$(109.3) \quad [C_{ij}] = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) \end{pmatrix}$$

الفصل الرابع

# اهتزازات الشبكة البلورية والخصائص الحرارية

1-4 مقدمة

)

,

(

( )

2-4 الخط الذري المتجانس أو الوتر المشدود

$a$

$$\lambda > a$$

(1.4)

$$(\omega_{\min} = 2\pi\nu_S / \lambda_{\max})$$

(1.4)

1

2

$$(\omega_{\max} = 2\pi\nu_S / \lambda_{\min} = \pi\nu_S / a) :$$

$$(\lambda_{\min} = 2a)$$

:  $v_s$  u

(1.4) 
$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v_s^2} \frac{\partial^2 u}{\partial t^2}$$

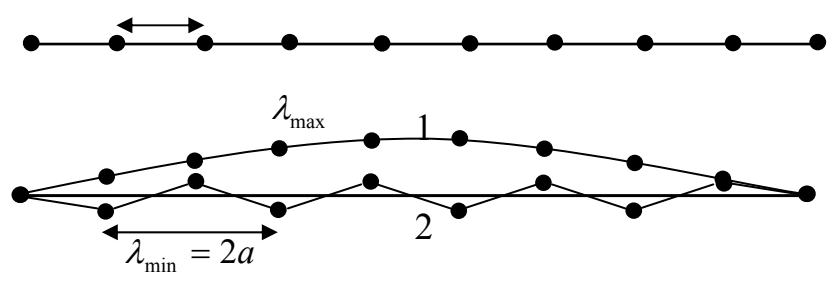
:  $\rho$  E  $(v_s = \sqrt{E/\rho})$  :

(2.4) 
$$u = u_0 \exp(i(\omega t \pm k x))$$

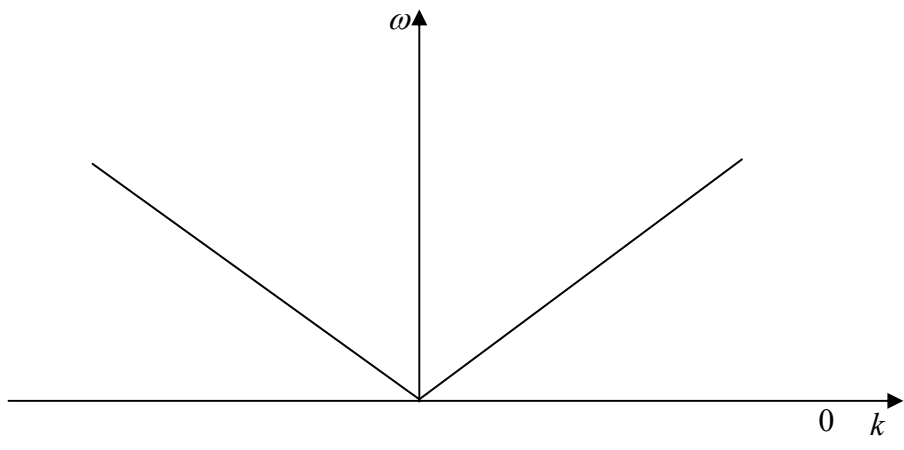
$k$   $\omega$  ( , )  $\omega = v_s k$  :

$v_p$   $v_p = v_s$  .(2.4)

·  $v_g$



الشكل (1.4):

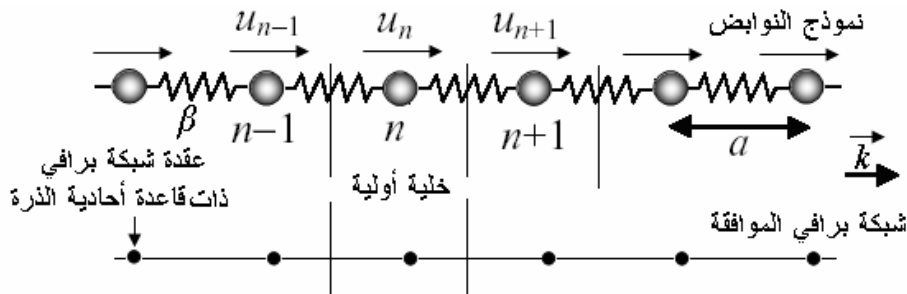


الشكل (3.4):



3-4 أنماط الاهتزاز الطبيعية للشبكة البلورية الخطية المولفة من ذرة واحدة في الخلية لأولية (شبكة براغي الخطية أحادية الذرة)

(3.4) (a) (m) ,  
 $\beta$   
 (n) .....



الشكل (3.4):

( )  
 (3.4) (n)  
 : ... n-1 , n, n+1... ( ... u\_{n-1}, u\_n, u\_{n+1} ... )

: n+1 n  
 (3.4)  $F_1 = -\beta(u_n - u_{n+1})$

: n-1 n  
 (4.4)  $F_2 = \beta(u_n - u_{n-1})$   
 : n

$$(5.4) \quad F_n = F_1 - F_2 = -\beta(2u_n - u_{n+1} - u_{n-1})$$

$$:(3)$$

$$F_n = m \frac{d^2 u_n}{dt^2} = m \ddot{u}_n = -\beta(2u_n - u_{n+1} - u_{n-1})$$

$$(6.4) \quad m \ddot{u}_n + \beta(2u_n - u_{n+1} - u_{n-1}) = 0$$

(6.4)

$$\begin{matrix} \cdot & N & N & (6.4) \\ x_n = na & & & \\ ) & ( & ) & a \\ & (3.4) & & \end{matrix}$$

$$k \quad u \quad \omega \quad ($$

$$(7.4) \quad u_n = u \exp(i(kx_n - \omega t)) = u \exp(i(nka - \omega t))$$

$$\begin{matrix} x_n- & x_{n+1}=a(n+1) & (6.4) & (7.4) \\ & & : & x_{n-1}=a(n-1) \end{matrix}$$

$$m \omega^2 = \beta(2 - e^{ika} - e^{-ika})$$

$$: \quad \cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) \quad \theta = ka$$

$$\omega^2 = \frac{2\beta}{m} (1 - \cos ka) = \frac{4\beta}{m} \sin^2 \frac{ka}{2}$$

$$\omega = \pm 2 \sqrt{\frac{\beta}{m}} \left| \sin \frac{ka}{2} \right|$$

$$\omega = \pm \omega_{\max} \left| \sin \frac{ka}{2} \right|$$

$$\omega_{\max} = 2\sqrt{\beta/m}$$

(8.4)

### 1-3-4 خصائص علاقة التبدد

$$\left| \sin \frac{ka}{2} \right| \quad \omega(-k) = \omega(k) \quad \omega(k)$$

$$: \quad \omega(k) \quad k' \quad , \quad n' \quad (n'\pi)$$

$$\omega(k) = \omega(k + k') \Rightarrow$$

$$\left| \sin \left( \frac{ka}{2} \right) \right| = \left| \sin \left( \frac{(k + k')a}{2} \right) \right| = \left| \sin \left( \frac{ka}{2} + n'\pi \right) \right| \Rightarrow$$

$$\frac{k'a}{2} = n'\pi \Rightarrow k' = \frac{2\pi n'}{a}$$

(9.4)

$$(8.4) \quad (7.4) \quad k+2\pi n/a \quad k$$

$$k+2\pi n/a$$

$$+\pi/a \quad -\pi/a \quad k \quad . \quad n=1$$

$$k \quad k \quad .(4.4)$$

$$(k_{\max} = \pi/a)$$

$$\omega_{\max}$$

$$2d \sin$$

$$.2a$$

$$\lambda_{\min} = 2\pi/k_{\max} = 2a$$

$$. \lambda=2a \quad n_1=1 \quad k=2\pi/\lambda \quad d=a \quad \theta = \pi/2 \quad \theta = n_1\lambda$$

$$(7.4) \quad (k_{\max} = \pm\pi/a)$$

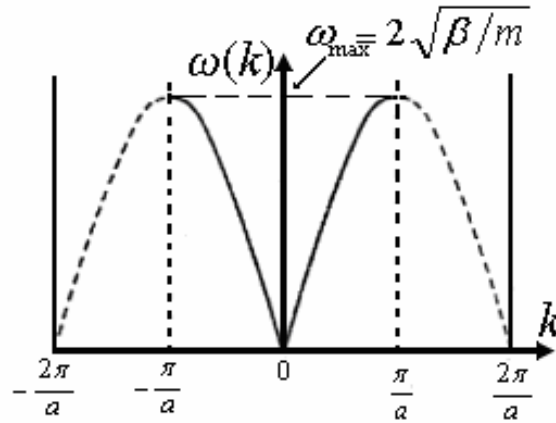
:

$$(10.4) \quad u_n = u \exp(\pm in\pi - i\omega t) = (-1)^n u \exp(-i\omega t)$$

$n$

( )

( )



الشكل (4.4) :

سرعة الطور وسرعة المجموعة

:

$$(11.4) \quad V_p = \frac{\omega}{k}$$

:

2

$a$

$$(11.4) \quad (8.4)$$

$$(12.4)$$

$$V_p = \frac{\omega}{k} = \frac{2\sqrt{\frac{\beta}{m}} \left| \sin\left(\frac{ka}{2}\right) \right|}{k} = \sqrt{\frac{\beta a^2}{m}} \left| \frac{\sin\left(\frac{ka}{2}\right)}{\frac{ka}{2}} \right|$$

:

$$(13.4) \quad V_g = \frac{\partial \omega}{\partial k}$$

$$: \quad k \quad (8.4)$$

$$(14.4) \quad V_g = \frac{\partial \omega}{\partial k} = \sqrt{\frac{\beta a^2}{m}} \left| \cos\left(\frac{ka}{2}\right) \right|$$

$$k = \pm \pi/a \quad (14.4)$$

$$.((2\pi/a)\sqrt{\beta/m}) \quad (12.4)$$

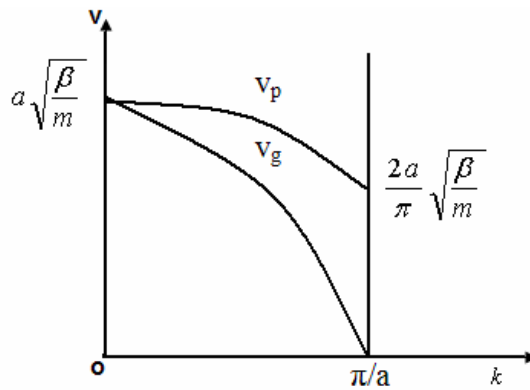
$v_p$

$$(k=(2\pi/\lambda) \rightarrow 0)$$

$$.(5.4)$$

$$(a\sqrt{\beta/m})$$

$v_g$



:الشكل (5.4)

$$: \quad (ka \ll 1) \quad (\lambda \gg a)$$

$$(15.4) \quad \sin\left(\frac{ka}{2}\right) = \frac{ka}{2} - \frac{(ka)^3}{3!} + \frac{(ka)^5}{5!} - \dots \approx \frac{ka}{2}$$

$$: \quad (8.4) \quad (15.4)$$

$$(16.4) \quad \omega = 2\sqrt{\frac{\beta}{m}} \sin\left(\frac{ka}{2}\right) \approx 2\sqrt{\frac{\beta}{m}} \frac{ka}{2} = \sqrt{\frac{\beta a^2}{m}} k$$

$$(16.4)$$

$$(17.4) \quad v_p = v_g = a \sqrt{\frac{\beta}{m}} = v_s$$

• الشروط الحدية الدورية لبورن- فون كارمن (Born-von Karmann)

$N$

( )

:

$N$

$$(17.4) \quad u_{n \pm N} = u_n$$

-

: (17.4) (7.4) (discrete)

$$u e^{i(kan \pm kaN - \omega t)} = u e^{i(kan - \omega t)} \Rightarrow$$

$$\exp(\pm ikaN) = 1 \Rightarrow kaN = 2\pi h \Rightarrow$$

$$(18.4) \quad k = \frac{2\pi}{aN} h = 0, \pm \frac{2\pi}{aN}, \pm \frac{4\pi}{aN}, \pm \frac{6\pi}{aN} \dots, \pm \frac{N\pi}{aN} = \pm \frac{\pi}{a}$$

$$(19.4) \quad -\frac{\pi}{a} \leq k \leq +\frac{\pi}{a}, \quad -\frac{N}{2} \leq h \leq +\frac{N}{2}$$

$h$

$(\pm\pi/a)$

$N$

(19.4) (18.4)

.a

( )

) ( $G=(2\pi/a)n_g$ )  $k$

: (a)

$$(20.4) \quad k' = k + G = k + \frac{2\pi}{a} n_g$$

$$(18.4) \quad (7.4) \quad (20.4)$$

2-3-4 كثافة الأنماط الاهتزازية

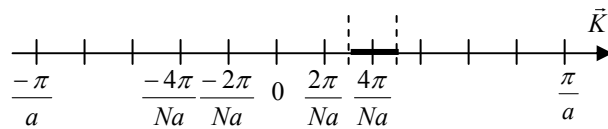
$\vec{k}$

$k$

( )  $k$

: (  $k$  )  $\vec{k}$   $k$   $(2\pi/aN)$

$$(21.4) \quad g(k) = \frac{1}{\left(\frac{2\pi}{aN}\right)} = \frac{aN}{2\pi}, \quad -\frac{\pi}{a} \leq k \leq +\frac{\pi}{a}$$



الشكل (6.4):  $\vec{k}$

:  $k + dk$   $k$   $k$

$$(22.4) \quad g(k)dk = \frac{aN}{2\pi} dk, \quad -\frac{\pi}{a} \leq k \leq +\frac{\pi}{a}$$

:  $k$

$$(23.4) \quad g(|k|)dk = 2 \frac{aN}{2\pi} dk$$

:  $|k + dk|$   $|k|$   $k$   $\omega + d\omega$   $\omega$

$$(24.4) \quad D(\omega)d\omega = g(|k|)dk$$

$$D(\omega)d\omega = g(|k|)dk = 2 \frac{aN}{2\pi} dk$$

$$:$$

$$D(\omega)$$

$$(25.4) \quad D(\omega) = \frac{aN}{\pi} \frac{dk}{d\omega}$$

:

$$\omega = \omega_{\max} \left| \sin \left( \frac{|k|a}{2} \right) \right|$$

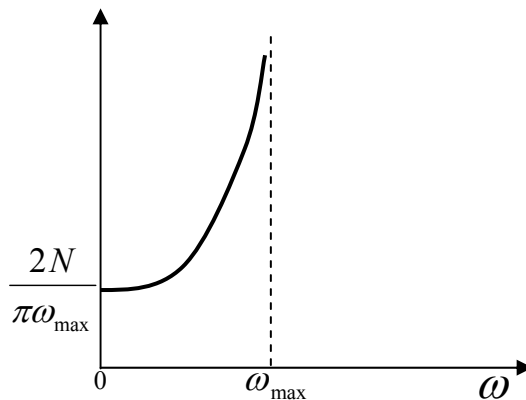
$$\frac{d\omega}{dk} = \frac{a\omega_{\max}}{2} \left| \cos \left( \frac{ka}{2} \right) \right| = \frac{a\omega_{\max}}{2} \left( 1 - \sin^2 \left( \frac{ka}{2} \right) \right)^{\frac{1}{2}}$$

$$= \frac{a}{2} (\omega_{\max}^2 - \omega^2)^{\frac{1}{2}}$$

:

$$(26.4) \quad D(\omega) = \frac{2N}{\pi} (\omega_{\max}^2 - \omega^2)^{-\frac{1}{2}} = \frac{2N}{\pi\omega_{\max}} \left( 1 - \frac{\omega^2}{\omega_{\max}^2} \right)^{-\frac{1}{2}}$$

(7.4)



الشكل (7.4):



$$k \quad [\omega, \omega_{\max}] \quad \omega$$

:

$$\int_0^{\omega_{\max}} D(\omega) d\omega = \frac{2N}{\pi} \int_0^{\omega_{\max}} (\omega_{\max}^2 - \omega^2)^{\frac{1}{2}} d\omega = \frac{2N}{\pi} \left[ \arcsin\left(\frac{\omega}{\omega_{\max}}\right) \right]_0^{\omega_{\max}} = \frac{2N}{\pi} \left[ \frac{\pi}{2} \right] = N$$

$$(27.4) \quad \int_0^{\omega_{\max}} D(\omega) d\omega = \int_0^{\pi/a} g(|k|) dk = \int_0^{\pi/a} \frac{aN}{\pi} dk = \frac{aN}{\pi} \left[ \frac{\pi}{a} \right] = N$$

. N

:N

#### 4-4 أنماط الاهتزاز الطبيعية للشبكة البلورية الخطية المولفة من ذرتين في الخلية الأولية (شبكة براهي الخلية ثنائية الذرة)

....CsCl , NaCl

....Ge, Si

M m

$\beta_1$

$$\beta_2 \leq \beta_1 \quad \beta_2$$

(... $u_{n-1}, u_n, u_{n+1}$ ...)

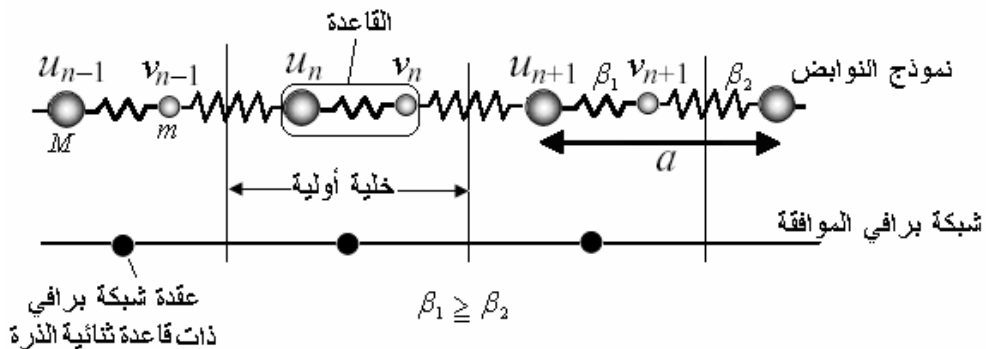
M

a

$$(8.4)$$

(... $v_{n-1}, v_n, v_{n+1}$ ...)

m



الشكل (8.4):

$$\begin{aligned}
 (28.4) \quad & M \ddot{u}_n = -\beta_1(u_n - v_n) - \beta_2(u_n - v_{n-1}) \\
 (29.4) \quad & m \ddot{v}_n = -\beta_1(v_n - u_n) - \beta_2(v_n - u_{n+1}) \\
 (8.4) \quad &
 \end{aligned}$$

$$\begin{aligned}
 (30.4) \quad & u_n = u \exp(i(nka - \omega t)) \\
 (31.4) \quad & v_n = v \exp(i(nka - \omega t))
 \end{aligned}$$

$$\begin{aligned}
 & \exp(iNka) = 1 : \quad v_n = v_{n+N} \quad u_n = u_{n+N} \\
 k = \frac{2\pi}{aN} h = 0, \pm \frac{2\pi}{aN}, \pm \frac{4\pi}{aN}, \pm \frac{6\pi}{aN}, \dots, \pm \frac{N\pi}{aN} = \pm \frac{\pi}{a} \\
 -\frac{\pi}{a} \leq k \leq +\frac{\pi}{a}, \quad -\frac{N}{2} \leq h \leq +\frac{N}{2}
 \end{aligned}$$

$$\begin{aligned}
 (29.4) \quad & (28.4) \quad (31.4) \quad (30.4) \quad (29.4) \quad (28.4)
 \end{aligned}$$

$$\begin{aligned}
 (32.4) \quad & (M\omega^2 - (\beta_1 + \beta_2))u + (\beta_1 + \beta_2 \exp(ika))v \\
 (33.4) \quad & (\beta_1 + \beta_2 \exp(ika))u + (m\omega^2 - (\beta_1 + \beta_2))v \\
 & \quad \quad \quad : \quad v, u
 \end{aligned}$$

$$(34.4) \quad \begin{vmatrix} M\omega^2 - (\beta_1 + \beta_2) & \beta_1 + \beta_2 \exp(ika) \\ \beta_1 + \beta_2 \exp(ika) & m\omega^2 - (\beta_1 + \beta_2) \end{vmatrix} = 0$$

:

$$(35.4) \quad \omega^4 - \frac{\beta_1 + \beta_2}{\mu} \omega^2 + \frac{4\beta_1\beta_2}{Mm} \sin^2\left(\frac{ka}{2}\right) = 0$$

. m M

$\mu = \frac{Mm}{M+m}$  :

:  $\omega^2$

(35.4)

$$(36.4) \quad \omega_1^2 = \frac{\beta_1 + \beta_2}{2\mu} \left( 1 - \sqrt{1 - \alpha \sin^2\left(\frac{ka}{2}\right)} \right)$$

$$(37.4) \quad \omega_2^2 = \frac{\beta_1 + \beta_2}{2\mu} \left( 1 + \sqrt{1 - \alpha \sin^2\left(\frac{ka}{2}\right)} \right)$$

:

$$(38.4) \quad \alpha = 16 \frac{\beta_1\beta_2}{(\beta_1 + \beta_2)^2} \left( \frac{\mu}{M+m} \right) \leq 1$$

$$1 - \alpha \sin^2\left(\frac{ka}{2}\right) :$$

$$M = m \quad \beta_1 = \beta_2$$

$\alpha$

.  $\omega_2, \omega_1$

2N

N

k

$\omega_2, \omega_1$

k

(37.4)

2N

(38.4)

:

$$(ka \ll 1) \quad (\lambda \gg a)$$

(38.4) (37.4)

1

:  $(\sin(ka/2) \approx (ka/2))$

$$\omega_1^2 = \frac{\beta_1 + \beta_2}{2\mu} \left( 1 - \sqrt{1 - \alpha \left(\frac{ka}{2}\right)^2} \right) \approx \frac{\beta_1 + \beta_2}{2\mu} \left( 1 - \left( 1 - \alpha \left(\frac{1}{2}\right) \frac{k^2 a^2}{4} \right) \right) \Rightarrow$$

$$(39.4) \quad \omega_1 = \frac{\sqrt{\alpha(\beta_1 + \beta_2)}}{4\sqrt{\mu}} ak$$

$$(40.4) \quad \omega_2^2 = \frac{\beta_1 + \beta_2}{2\mu} \left( 1 + \sqrt{1 - \alpha \left( \frac{ka}{2} \right)^2} \right) \approx \frac{\beta_1 + \beta_2}{2\mu} \left( 1 + \left( 1 - \alpha \frac{k^2 a^2}{8} \right) \right) \Rightarrow$$

$$\omega_2 = \frac{\sqrt{\beta_1 + \beta_2}}{\sqrt{\mu}} \left( 1 - \frac{\alpha a^2}{32} k^2 \right)$$

$$\omega = C k \quad k \quad \omega_1(k) \quad (39.4)$$

$$\omega_{ac} \quad \omega_1 \quad k \quad \omega_2(k) \quad (40.4)$$

$$\omega_{op} \quad \omega_2$$

$$: k = \pm \frac{\pi}{a} \quad .2$$

$$(41.4) \quad \omega_{ac} \left( \pm \frac{\pi}{a} \right) = \omega_{ac}^{\max} = \frac{\beta_1 + \beta_2}{2\mu} (1 - \sqrt{1 - \alpha})$$

$$(42.4) \quad \omega_{op} \left( \pm \frac{\pi}{a} \right) = \omega_{op}^{\min} = \sqrt{\frac{\beta_1 + \beta_2}{2\mu}} (1 + \sqrt{1 - \alpha})$$

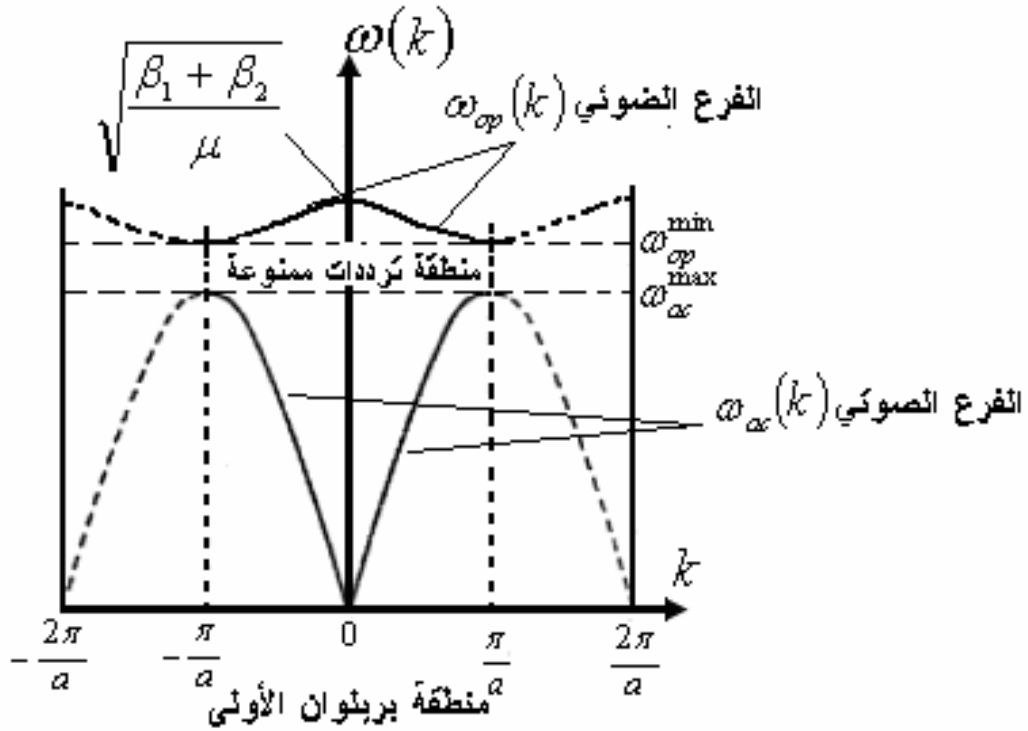
$$(\beta_1 = \beta_2 \quad m = M : ) \alpha = 1 \quad \omega_{ac}^{\max} = \omega_{op}^{\min}$$

$$\omega_{ac}^{\max} = \omega_{op}^{\min} = 2\sqrt{\frac{\beta}{m}}$$

$$. \omega_{ac} \neq \omega_{op} \quad (\beta_1 \neq \beta_2 \quad m \neq M : ) \alpha \neq 1$$

$$(43.4) \quad \omega_{ac}(k=0) = 0$$

$$(44.4) \quad \omega_{op}(k=0) = \sqrt{\frac{\beta_1 + \beta_2}{\mu}}$$



الشكل (9.4):

$$\omega_{op}^{\min}, \omega_{ac}^{\max}$$

• طبيعة اهتزاز الذرات في الفرعين الصوتي و البصري

$$. (k \approx 0)$$

$$(45.4) \quad \omega_{ac}(k=0) = 0$$

$$(46.4) \quad \omega_{op}(k=0) = \sqrt{\frac{\beta_1 + \beta_2}{\mu}}$$

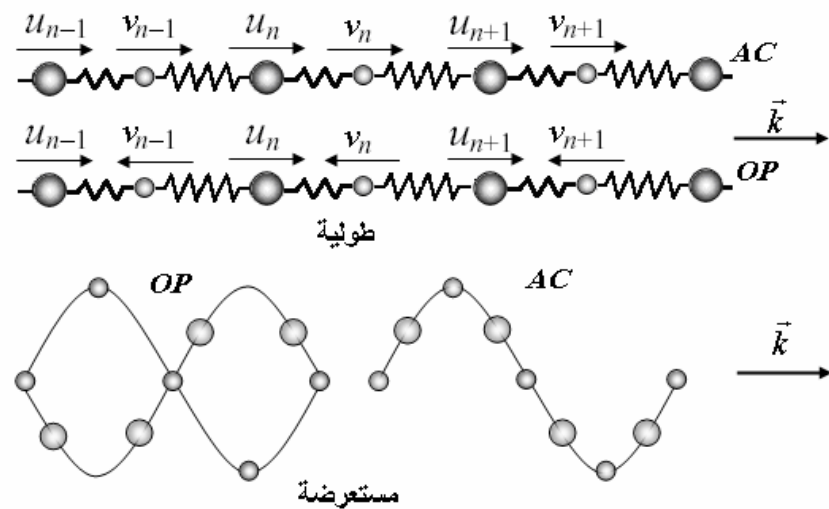
$$: \quad (45.4) \quad (33.4) \quad (32.4) \quad (31.4) \quad (30.4)$$

$$(47.4) \quad \frac{u_n}{v_n} = \frac{u}{v} = \frac{\beta_1 + \beta_2 \exp(ika)}{\beta_1 + \beta_2 - M\omega_{ac}^2(k=0)} = \frac{\beta_1 + \beta_2}{\beta_1 + \beta_2} = 1$$

$$(48.4) \quad \frac{u_n}{v_n} = \frac{u}{v} = \frac{\beta_1 + \beta_2 \exp(ika)}{\beta_1 + \beta_2 - M\omega_{op}^2(k=0)} = \frac{\beta_1 + \beta_2}{\beta_1 + \beta_2 - M\sqrt{\frac{\beta_1 + \beta_2}{\mu}}} = -\frac{M}{m}$$

$$M u_n + m u_n = 0 \quad (10.4)$$

$$M u_n + m u_n = 0 \quad (10.4)$$



الشكل (10.4) :

$$\left( k = \pm \frac{\pi}{a} \right)$$

$$(49.4) \quad \omega_{ac} \left( \pm \frac{\pi}{a} \right) = \omega_{ac}^{\max} = \frac{\beta_1 + \beta_2}{2\mu} (1 - \sqrt{1 - \alpha})$$

$$(50.4) \quad \omega_{op} \left( \pm \frac{\pi}{a} \right) = \omega_{op}^{\min} = \sqrt{\frac{\beta_1 + \beta_2}{2\mu}} (1 + \sqrt{1 - \alpha})$$

: (49.4) (33.4) (32.4) (31.4) (30.4)

$$(51.4) \quad \frac{u_n}{v_n} = \frac{u}{v} = \frac{\beta_1 + \beta_2 \exp(ika)}{\beta_1 + \beta_2 - M\omega_{ac}^2 (k = \pm \pi/a)} = \frac{\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}}{1 - \frac{M+m}{2m} (1 - \sqrt{1 - \alpha})}$$

: (50.4) (33.4) (32.4) (31.4) (30.4)

$$(52.4) \quad \frac{u_n}{v_n} = \frac{u}{v} = \frac{\beta_1 + \beta_2 \exp(ika)}{\beta_1 + \beta_2 - M\omega_{op}^2 (k = \pm \pi/a)} = \frac{\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}}{1 - \frac{M+m}{2m} (1 + \sqrt{1 - \alpha})}$$

:

:  $m \neq M$   $\beta_1 = \beta_2$  .1

: (51.4)  $\alpha$

$$(53.4) \quad \frac{u_n}{v_n} = \frac{\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}}{1 - \frac{M+m}{2m} \left( 1 - \frac{|M-m|}{M+m} \right)}$$

$$v_n \neq 0 \quad u_n = 0 : \quad \frac{u_n}{v_n} = \frac{0}{(m-M)/m} = 0 \quad m > M$$

. m M

: (52.4)  $\alpha$

$$(54.4) \quad \frac{u_n}{v_n} = \frac{\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}}{1 - \frac{M+m}{2m} \left(1 + \frac{|M-m|}{M+m}\right)}$$

$$v_n \neq 0 \quad u_n = 0 \quad \frac{u_n}{v_n} = \frac{0}{\frac{(m-M)}{m}} = 0 \quad M > m$$

$$: \beta_1 > \beta_2 \quad m = M \quad .2$$

$$: (51.4) \quad \alpha \quad -$$

$$(55.4) \quad \frac{u_n}{v_n} = \frac{\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}}{1 - \left(1 - \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}\right)} = 1$$

$$m \quad M$$

$$: (52.4) \quad \alpha \quad -$$

$$(56.4) \quad \frac{u_n}{v_n} = \frac{\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}}{1 - \left(1 + \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}\right)} = -1$$

$$m \quad M$$

#### 5-4 الأنماط الطبيعية لشبكة براغي ثلاثية الأبعاد

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$$(57.4) \quad \vec{u}(\vec{r}, t) = \vec{\varepsilon} \exp(i(\vec{k} \cdot \vec{r} - \omega t))$$

$\vec{r}$  :  $\vec{u}(\vec{r}, t)$   
 $\vec{k}$  :  $\omega = f(k)$

$$(58.4) \quad \vec{u}(\vec{r}, t) = \vec{u}(\vec{r} + N_i \vec{a}_i, t)$$

$N_i (i=1,2,3)$  :  $\vec{a}_i (i=1,2,3)$   
 $N = N_1 N_2 N_3$

$$(59.4) \quad \exp(iN_i \vec{k} \cdot \vec{a}_i) = 1 \quad i=1,2,3$$

$$(60.4) \quad \vec{k} = \sum_{i=1}^3 \frac{n_i}{N_i} \vec{A}_i \quad i=1,2,3$$

$A_i = \frac{2\pi}{a_i} (i=1,2,3)$  .  $n_i (i=1,2,3)$

$$(61.4) \quad \vec{a}_i \cdot \vec{A}_j = 2\pi \delta_{ij} \quad i, j=1,2,3$$

$\vec{G}$

$$\vec{u}(\vec{r}, t) = \sum_{\vec{k}} \vec{u}(\vec{k}) \exp(i\vec{k} \cdot \vec{R}) = 1$$

$\vec{k}$  :  $\vec{k}' = \vec{k} + \vec{G}$

$$(62.4) \quad \vec{k}_{A_i} = \frac{n_i}{N_i} \vec{A}_i$$

$$\vec{A}_i \quad \vec{k} \quad \vec{k}_{A_i}$$

$$\vec{k}$$

$$\vec{k}$$

:

$$(63.4) \quad \Delta \vec{k}_{A_1} \cdot (\Delta \vec{k}_{A_2} \times \Delta \vec{k}_{A_3}) = \frac{\vec{A}_1}{N_1} \cdot \left( \frac{\vec{A}_2}{N_2} \times \frac{\vec{A}_3}{N_3} \right) = \frac{V_e^*}{N}$$

$$V_e^*$$

:

$$\vec{k}$$

$$(64.4) \quad \frac{V_e^*}{\left( \frac{V_e^*}{N} \right)} = N$$

$$\vec{\varepsilon}_p(\vec{k}) (p=1,2,3)$$

$$\vec{k}$$

$$N$$

$$3N$$

$$\omega_p(\vec{k}) (p=1,2,3)$$

$$(\omega_p(\vec{k} \rightarrow 0)) \rightarrow 0$$

2

)

: $\zeta$

$$N$$

$$3\zeta N$$

موزعة على  $3\zeta$  فرعا

(

$$N$$

$$\omega_p(\vec{k}) (p=1,2,3,\dots, 3\zeta)$$

$$3\zeta$$

$$\vec{k}$$

$$\vec{k}$$

$$(\omega_p(\vec{k} \rightarrow 0)) \rightarrow \omega_{\max} \neq 0$$

:

$$3(\zeta - 1)$$



$$(65.4) \quad g(k)dk = \frac{N}{V_e^*} 4\pi k^2 dk$$

$$) N \quad : V_e \quad V_e^* = (2\pi)^3 / V_e :$$

$$: \quad V = NV_e \quad ($$

$$(66.4) \quad g(k)dk = \frac{V}{2\pi^2} k^2 dk$$

$k$

$$: \quad \omega + d\omega \quad \omega \quad d\omega$$

$$(67.4) \quad D(\omega)d\omega = 3g(k)dk$$

$$: \quad D(\omega)$$

$$(68.4) \quad D(\omega)d\omega = 3 \frac{V}{2\pi^2} k^2 dk$$

:

$$(69.4) \quad \omega = v_g k = v_p k = v_s k$$

$$v_g, v_p, v_s :$$

:

$$(70.4) \quad D(\omega)d\omega = 3 \frac{V}{2\pi^2} \frac{\omega^2}{v_s^3} d\omega$$

$$(\omega_{\max} = \omega_D)$$

$$(\omega_{\min} = 0)$$

$k_D$

$\vec{k}$

$k$

$\omega_D$

)

:

.(

$$(71.4) \quad 3N = \int_0^{\omega_D} D(\omega)d\omega$$

$$3N = \int_0^{\omega_D} 3 \frac{V}{2\pi^2} \frac{\omega^2}{v_s^3} d\omega$$

:

$$(72.4) \quad \omega_D = \sqrt[3]{\left(\frac{6N\pi^2}{V}\right)} v_s = \sqrt[3]{6\pi^2 n_a} v_s$$

$$(73.4) \quad k_D = \sqrt[3]{\left(\frac{6N\pi^2}{V}\right)} = \sqrt[3]{6\pi^2 n_a}$$

$$\cdot \quad k_D \quad n_a$$

:

$$(74.4) \quad D_D(\omega) = \frac{9N}{\omega_D^3} \omega^2$$

#### 6-4 تكميم اهتزازات الشبكة البلورية

( )

: ( )

$$(75.4) \quad E_{n_{\vec{k},p}} = \left( n_{\vec{k},p} + \frac{1}{2} \right) \hbar \omega_p(\vec{k})$$

$p$  ( )  $\vec{k}$  ( الموافق له ) :  $\omega_p(\vec{k})$



(77.4)

$$\vec{K}' = \vec{K} + \vec{G}$$

$\vec{K}'$     $\vec{K}$     $\vec{G}$

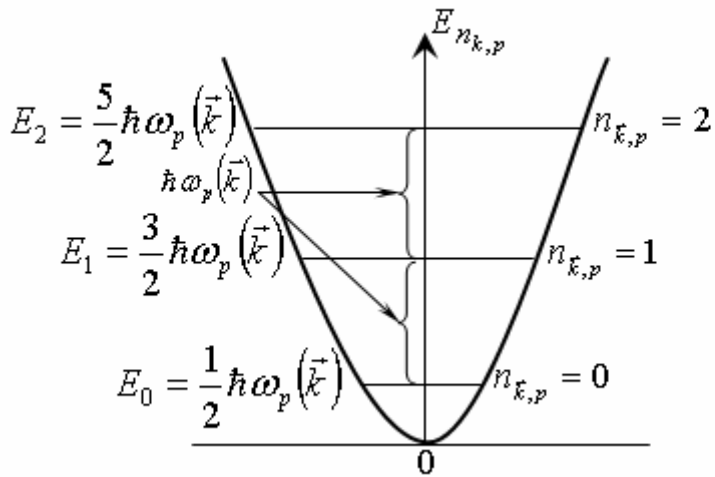
$(-\hbar\vec{G})$

(absorption)

(creation)

(78.4)

$$\vec{K}' = \vec{K} + \vec{G} \pm \vec{k}_p$$



:(12.4)

7-4 الخصائص الحرارية

1-7-4 السعة الحرارية

:

(79.4)  $\Delta Q = C_s m \Delta T = C \Delta T$

( $C_s$ )

( $C_s m = C$ )

$C_p$

$C_v$

( )

:

(80.4)  $\Delta Q = \Delta U - W \Rightarrow \Delta Q = \Delta U \quad (W = 0)$

$$C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T}$$

(13.4)

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$R \quad 3R = 25 J / mole \overset{\circ}{k} = 6 cal / mol \overset{\circ}{K}$

2

$20 \overset{\circ}{k}$



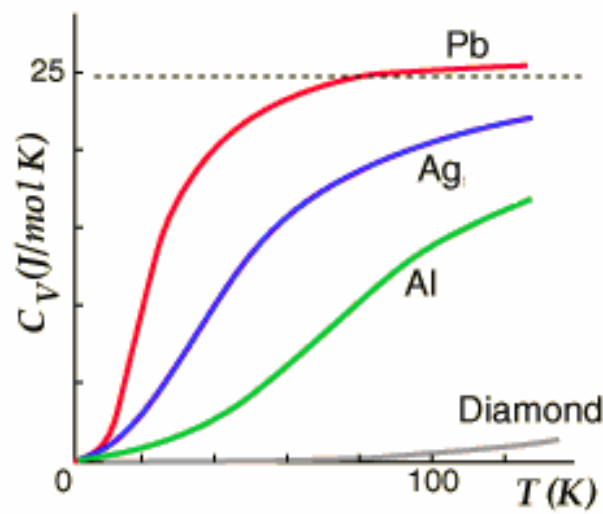
:

$$C = aT^3 + \gamma T$$

$$C = aT^3$$

$$C = aT^2$$

سنحاول



:(13.4)

أ- السعة الحرارية وفق النموذج الكلاسيكي

—

$$(K_B T/2) \quad (N) \quad (K_B T)$$

:

$$(81.4) \quad \langle U_{tot} \rangle = 3NK_B T$$

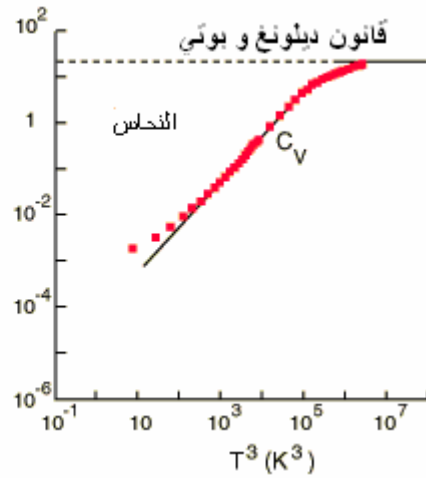
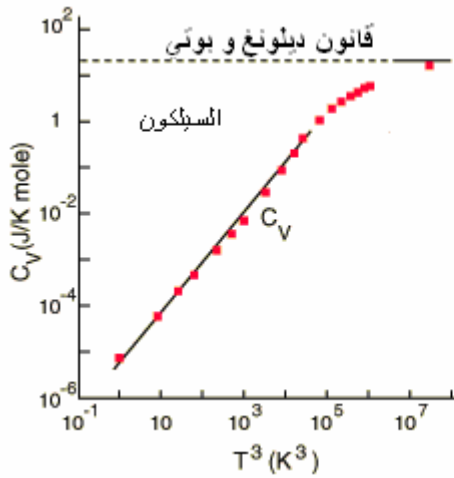
$$: \quad N_A = 6.022 \cdot 10^{23} \quad N$$

$$(82.4) \quad \langle U_{tot} \rangle = 3N_A K T = 3RT$$

$$: \quad R = N_A K_B \approx 2 \text{ cal/mol} \cdot \text{K} \quad R$$

$$(83.4) \quad C = \frac{d\langle U_{tot} \rangle}{dT} = 3R \approx 6 \text{ cal/mol} \cdot \text{K} = 25 \text{ J/mol} \cdot \text{K}$$

((14.4) ) (Dulong-Petit) -



:(14.4)

## ب- نموذج أينشتاين للسعة الحرارية

 $\omega_E$ 

:

$$(84.4) \quad E_n = n\hbar\omega \quad n=0,1,2,3,\dots$$

((12.4) )

:

$$(85.4) \quad E_n = \left(n + \frac{1}{2}\right)\hbar\omega \quad n = 0,1,2,3,\dots$$

 $n=0$  $n=0$  $E_n$  $\langle E \rangle$ :  $N$ 

$$(86.4) \quad \begin{aligned} N &= \sum_n N(E_n) \\ E &= \sum_n N(E_n)E_n \end{aligned} \Rightarrow \langle E \rangle = \frac{E}{N} = \frac{\sum_n N(E_n)E_n}{\sum_n N(E_n)}$$

:

(86.4)

$$(87.4) \quad \langle E \rangle = \frac{\int_0^{\infty} N(E)EdE}{\int_0^{\infty} N(E)dE}$$

$$E_n \quad \left( e^{\frac{-E}{K_B T}} \right) \quad : (86.4)$$

$$(88.4) \quad \langle E \rangle = \frac{\sum_{n=0}^{\infty} n \hbar \omega e^{\frac{-n \hbar \omega}{K_B T}}}{\sum_{n=0}^{\infty} e^{\frac{-n \hbar \omega}{K_B T}}}$$

$$\langle E \rangle = \frac{0 + \hbar \omega e^{\frac{-\hbar \omega}{K_B T}} + 2 \hbar \omega e^{\frac{-2 \hbar \omega}{K_B T}} + \dots}{1 + e^{\frac{-\hbar \omega}{K_B T}} + e^{\frac{-2 \hbar \omega}{K_B T}} + \dots}$$

$$: \quad x = \frac{-\hbar \omega}{K_B T}$$

$$(89.4) \quad \langle E \rangle = \frac{\hbar \omega e^x (1 + 2e^x + 3e^{2x} + \dots)}{1 + e^x + e^{2x} + \dots}$$

$$\left( \frac{1}{(1 - e^x)^2} \right)$$

$$: (89.4)$$

$$\left( \frac{1}{1 - e^x} \right)$$

$$(90.4) \quad \langle E \rangle = \frac{\hbar \omega e^x}{1 - e^x} = \frac{\hbar \omega}{e^{-x} - 1} = \frac{\hbar \omega}{e^{\frac{\hbar \omega}{K_B T}} - 1}$$

$$: \quad 3N_A$$

$$(91.4) \quad \langle U_{tot} \rangle = 3N \frac{\hbar \omega}{e^{\frac{\hbar \omega}{K_B T}} - 1} = 3N_A \frac{\hbar \omega}{e^{\frac{\theta_E}{T}} - 1}$$

$$\theta_E = \frac{\hbar \omega}{K_B}$$

:

$$(92.4) \quad C_v = \frac{d \langle U_{tot} \rangle}{dT} = \frac{3 N_A K_B \left( \frac{\hbar \omega}{K_B T} \right)^2 e^{\frac{\hbar \omega}{K_B T}}}{\left( e^{\frac{\hbar \omega}{K_B T}} - 1 \right)^2}$$

$$: \quad (92.4)$$

(1) الدراسة عند المجالات الحرارية العالية

$$: \quad (K_B T \gg \hbar \omega)$$

$$(93.4) \quad e^{\frac{\hbar \omega}{K_B T}} - 1 = 1 + \frac{\hbar \omega}{K_B T} + \left( \frac{\hbar \omega}{K_B T} \right)^2 + \dots - 1 \approx \frac{\hbar \omega}{K_B T}$$

$$: \quad (92.4) \quad (93.4)$$

$$(94.4) \quad C_v = 3 N_A K_B \left( 1 + \frac{\hbar \omega}{K_B T} \right) = 3 N_A K_B + \frac{\hbar \omega}{T} \approx 3 N_A K_B = 3R$$

$$(94.4)$$

(2) الدراسة عند المجالات الحرارية المنخفضة

$$: \quad (92.4) \quad (K_B T \ll \hbar \omega)$$

$$C_v = \frac{d \langle E \rangle_{tot}}{dT} = \frac{3 N_A K_B \left( \frac{\hbar \omega}{K_B T} \right)^2 e^{\frac{\hbar \omega}{K_B T}}}{\left( e^{\frac{\hbar \omega}{K_B T}} \right)^2} = 3 N_A K_B \left( \frac{\hbar \omega}{K_B T} \right)^2 e^{\frac{-\hbar \omega}{K_B T}}$$

$$(95.4) \quad C_v = 3R \left( \frac{\hbar \omega}{K_B T} \right)^2 e^{\frac{-\hbar \omega}{K_B T}} = 3R \left( \frac{\theta_E}{T} \right)^2 e^{\frac{-\theta_E}{T}}$$

$$(T=0) \quad (95.4)$$

ج- نموذج ديبي للسعة الحرارية

$$(11) \quad (\omega_{\min} \leq \omega \leq \omega_{\max})$$

$$(\omega_{\min} = 0)$$

$$(\omega_{\max} = \omega_D)$$

$$\langle U_{tot} \rangle = \int_0^{E_{\max}} \langle E \rangle dN(E) = \int_0^{\omega_{\max}} \frac{\hbar \omega}{e^{\frac{\hbar \omega}{K_B T}} - 1} D(\omega) d\omega$$

$$(95.4) \quad \langle U_{tot} \rangle = \int_0^{\omega_D} \frac{\hbar \omega}{e^{\frac{\hbar \omega}{K_B T}} - 1} D_D(\omega) d\omega$$

:(74.4)

$$(96.4) \quad D_D(\omega) = \frac{9N}{\omega_D^3} \omega^2$$

: (95.4) (96.4)

$$(97.4) \quad \langle U_{tot} \rangle = \frac{9N}{\omega_D^3} \int_0^{\omega_D} \frac{\hbar \omega^3}{e^{\frac{\hbar \omega}{K_B T}} - 1} . d\omega$$

$$: \quad x = \frac{\hbar\omega}{K_B T}$$

$$x = \frac{\hbar\omega}{K_B T} \Rightarrow \omega = \frac{K_B T}{\hbar} x \Rightarrow d\omega = \frac{K_B T}{\hbar} dx$$

$$(98.4) \quad \omega^3 d\omega = \frac{K_B^3 T^3 x^3}{\hbar^3} \cdot \frac{K_B T}{\hbar} dx \Rightarrow \frac{K_B^4 T^4 x^3}{\hbar^4} dx$$

$$(99.4) \quad x = 0 \Rightarrow \omega = 0$$

$$(100.4) \quad \omega_{\max} = \omega_D = \frac{K_B T}{\hbar} x_{\max} \Rightarrow x_{\max} = \frac{\hbar\omega_D}{K_B T} = \frac{\theta_D}{T}$$

$$\theta_D = \frac{\hbar\omega_D}{K_B}$$

$$: \quad (97.4) \quad (100.4) \quad (99.4) \quad (98.4)$$

$$(101.4) \quad \langle U_{tot} \rangle = \frac{9NK_B T^4}{\theta_D^3} \int_0^{x_{\max}} \frac{x^3}{e^x - 1} dx$$

$$(101.4) \quad \cdot$$

$$(102.4) \quad C_v = 9NK_B \left( \frac{T}{\theta_D} \right)^3 \int_0^{\frac{\theta_D}{T}} \frac{x^4 e^x}{(e^x - 1)^2} dx$$

$$: \quad (102.4) \quad (101.4)$$

أ- الدراسة عند الدرجات الحرارية العالية

$$: \quad (101.4) \quad K_B T \gg \hbar\omega$$

$$(103.4) \quad \frac{x^3}{e^x - 1} = \frac{x^3}{1 + x + x^2 + \dots} \approx \frac{x^3}{x} = x^2$$

$$: \quad (101.4) \quad (103.4)$$

$$\langle U_{tot} \rangle = \frac{9NK_B T^4}{\theta_D^3} \int_0^{x_{max}} \frac{x^3}{e^x - 1} dx = \frac{9NK_B T^4}{\theta_D^3} \int_0^{x_{max}} x^2 dx$$

$$\langle U_{tot} \rangle = \frac{9NK_B T^4}{\theta_D^3} \cdot \frac{x_{max}^3}{3} = \frac{9NK_B T^4}{\theta_D^3} \cdot \frac{\theta_D^3}{3T^3} = 3NK_B T$$

$$(104.4) \quad C_v = \frac{d\langle U \rangle}{dT} = 3NK_B$$

$$: \quad N = N_A$$

$$(105.4) \quad C_v = 3N_A K_B = 3R$$

$$- \quad (105.4)$$

ب- الدراسة عند الدرجات الحرارية المنخفضة

$$(0 \mapsto (x_{max} \rightarrow \infty))$$

$$K_B T \ll \hbar \omega$$

:

$$(106.4) \quad \int_0^{\infty} \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

$$: \quad (101.4)$$

$$\langle U_{tot} \rangle = \frac{9NK_B T^4}{\theta_D^3} \int_0^{x_{max} \rightarrow \infty} \frac{x^3}{e^x - 1} dx = \frac{9NK_B T^4}{\theta_D^3} \cdot \frac{\pi^4}{15} = \frac{3\pi^4 NK_B T^4}{5\theta_D^3}$$



:

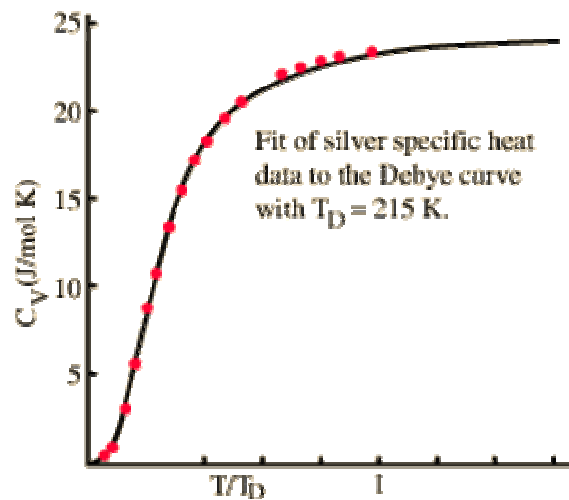
$$(107.4) \quad C_v = \frac{d\langle U \rangle}{dT} = \frac{12\pi^4 N_A K_B T^3}{5\theta_D^3} = \frac{12}{5} \pi^4 R \left( \frac{T}{\theta_D} \right)^3$$

$T^3$

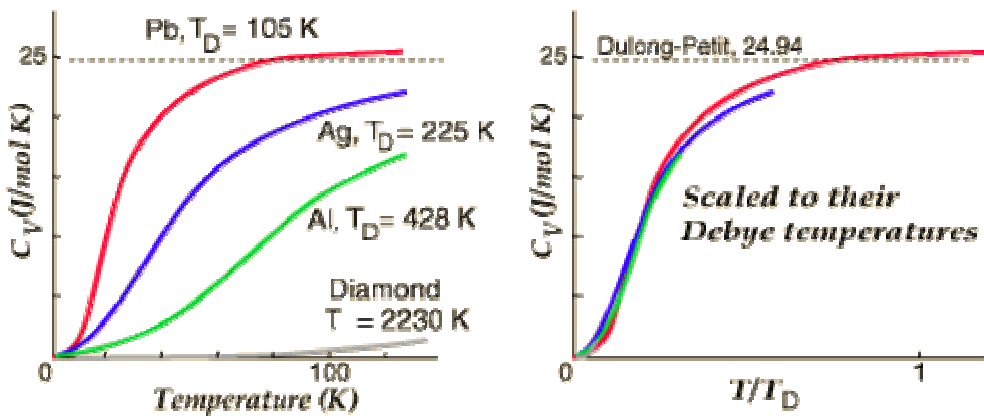
(107.4)

$$(15.4) \quad \dots \quad \left( \dots \right) T^3$$

.(16.4)



( ) :(15.4)



:(16.4)

:

(1.4)

العنصر	درجة حرارة ديباي $\theta_D$ (°K)	العنصر	درجة حرارة ديباي $\theta_D$
Al	428	Ca	230
Pb	110	Cr	630
Na	158	Mn	450
Li	370	Fe	467
Be	1160	Cu	343
Au	164	Zn	310
Si	640	Ge	370
SiO <sub>2</sub>	470	LiF	732
NaCl	321	CaF <sub>2</sub>	510

(1.4):

#### 2-7-4 الاهتزازات اللاتوافقية

:  $x$

$$(108.4) \quad F(x) = -\beta x + \gamma x^2 - \alpha x^3$$

:  $x$

$$(109.4) \quad F(x) = -\beta x$$

$x$

:

$$(110.4) \quad U(x) = -f x^2 - g x^3 + h x^4$$

$x^3$  ,  $f, g, h$  :  
 $g$

... ( )

### 3-7-4 التمدد الحراري

عد

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T X

:

x

(111.4)

$$\langle x \rangle = \frac{\int_{-\infty}^{+\infty} x \exp\left(-\frac{U(x)}{K_B T}\right) dx}{\int_{-\infty}^{+\infty} \exp\left(-\frac{U(x)}{K_B T}\right) dx}$$

(110.4)

: (111.4)

$$\begin{aligned}
 \int_{-\infty}^{+\infty} x \exp\left(-\frac{U(x)}{K_B T}\right) dx &= \int_{-\infty}^{+\infty} x \exp\left(-\frac{f x^2}{K_B T}\right) \cdot \int_{-\infty}^{+\infty} \exp\left(-\frac{g x^3 + h x^4}{K_B T}\right) dx \\
 &\cong \int_{-\infty}^{+\infty} x \exp\left(-\frac{f x^2}{K_B T}\right) \cdot \left(1 + \frac{g}{K_B T} x^3 + \frac{h}{K_B T} x^4 + \dots\right) dx \\
 &= \int_{-\infty}^{+\infty} \exp\left(-\frac{f x^2}{K_B T}\right) \cdot \left(x + \frac{g}{K_B T} x^4 + \frac{h}{K_B T} x^5 + \dots\right) dx \\
 &= \int_{-\infty}^{+\infty} x \exp\left(-\frac{f x^2}{K_B T}\right) dx + \frac{g}{K_B T} \int_{-\infty}^{+\infty} x^4 \exp\left(-\frac{f x^2}{K_B T}\right) dx + \frac{h}{K_B T} \int_{-\infty}^{+\infty} x^5 \exp\left(-\frac{f x^2}{K_B T}\right) dx + \dots
 \end{aligned}$$

: .

(112.4)  $\int_{-\infty}^{+\infty} x \exp\left(-\frac{U(x)}{K_B T}\right) dx \cong \frac{g}{K_B T} \int_{-\infty}^{+\infty} x^4 \exp\left(-\frac{f x^2}{K_B T}\right) dx = dx \frac{3g\sqrt{\pi}}{4K_B T} \left(\frac{K_B T}{f}\right)^{\frac{5}{2}}$

:

(113.4)  $\int_{-\infty}^{+\infty} \exp\left(-\frac{U(x)}{K_B T}\right) dx \cong \int_{-\infty}^{+\infty} \exp\left(-\frac{f x^2}{K_B T}\right) dx = \left(\frac{\pi K_B T}{f}\right)^{\frac{1}{2}}$

:

(114.4)  $\langle x \rangle = \frac{3K_B T}{4f^2} g$

:  $\langle x \rangle$

(115.4)  $\alpha = \frac{\langle x \rangle}{aT} = \frac{3K_B T}{4a f^2} g$

$g = 0$  .  $a$

## 4-7-4 التوصيل الحراري في العوازل

$$Q = K \frac{dT}{dX} \quad :$$

$$(116.4) \quad Q = K \left( \frac{dT}{dX} \right)$$

K

:

:

$$(117.4) \quad K = \frac{1}{3} C \langle v \rangle \lambda$$

	$\lambda$	$C$	$\langle v \rangle$
"	"		
	$T^3$		
	$K$	'	
:		( )	
			- 1
			- 2
		.( )	-3

 $K_p$  $K_b$  $K_i$ 

:

(118.4)

$$K = K_p + K_i + K_b$$

أ- تفاعلات فونون مع فونون

( )

:

1. قانون حفظ الطاقة

$$(119.4) \quad \begin{aligned} \hbar\omega_1 + \hbar\omega_2 &= \hbar\omega_3 \\ \omega_1 + \omega_2 &= \omega_3 \end{aligned}$$

2. قانون حفظ كمية الحركة

$$(120.4) \quad \begin{aligned} \hbar\vec{k}_1 + \hbar\vec{k}_2 &= \hbar\vec{k}_3 \\ \vec{k}_1 + \vec{k}_2 &= \vec{k}_3 \end{aligned}$$

$\vec{k}_2, \vec{k}_1 :$

:

(Normal)

N

- العملية العادية:  
 $\vec{k}_3$

( )

(Umklapp)

U

- عملية الانقلاب:

$$(\vec{k}'_3 = \vec{k}_1 + \vec{k}_2) \vec{k}'_3$$

$K_p$

$\lambda'_3$

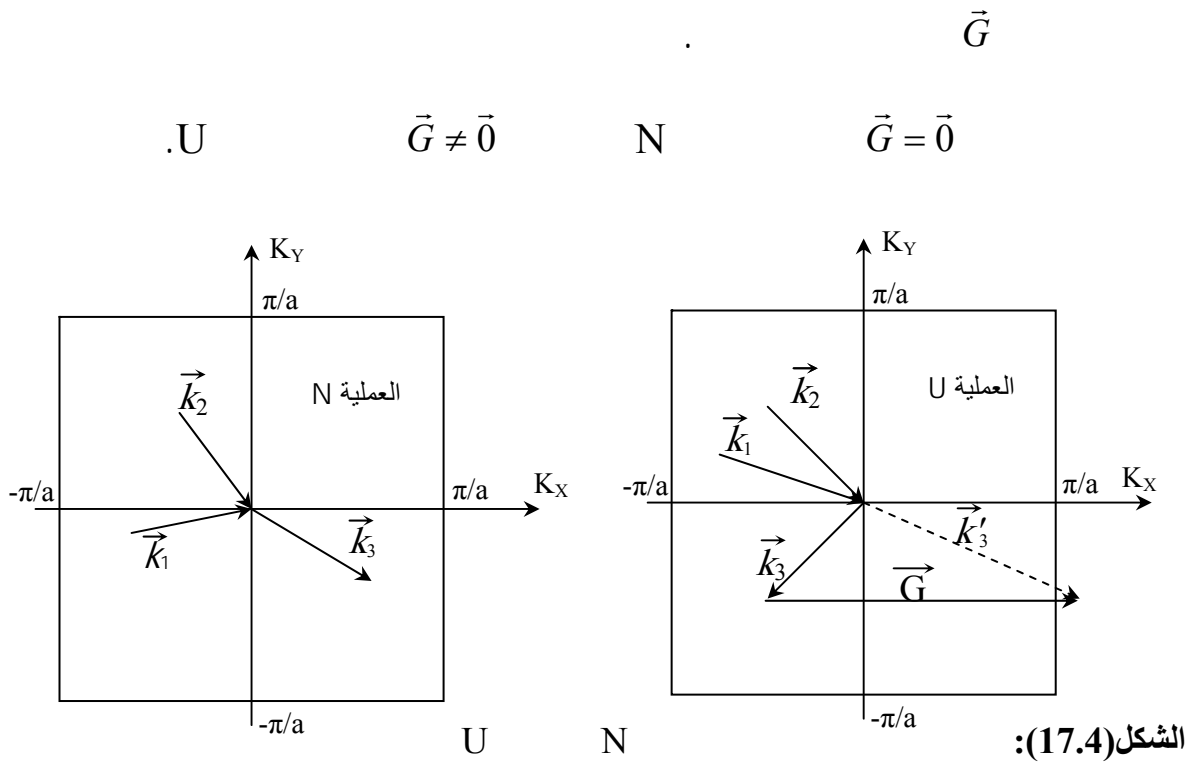
:  $\vec{k}_3$

$\vec{k}_2, \vec{k}_1$

$$k_3 = k_2 - \frac{\pi}{2} : \vec{k}_3$$

(Peierls)  $\vec{G}$   
 (120.4)

(121.4)  $\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{G}$



ب- التشتت بالعيوب البلورية

1 - العيوب النقطية



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.(...

2 - العيوب الخطية

3 - أو كليهما.

ت- التشتت عند حواف العينة

U

D

:

$$(122.4) \quad K = CVD$$

$$(T \ll \theta_D) \quad T^3$$

$$(T > \theta_D)$$

$$(K_B \theta_D / 2)$$

$$: \exp(\theta_D / 2T)$$

(123.4)

$$\lambda \propto \exp(\theta_D / 2T)$$

$$K_p \propto \exp(\theta_D / 2T)$$

(2.4)

$$. T = 20 \overset{\circ}{K}, T = 273 \overset{\circ}{K}$$

$T = 20 \overset{\circ}{K}$		$T = 273 \overset{\circ}{K}$		
$\lambda [\overset{\circ}{A}]$	$\kappa [W/m.\overset{\circ}{K}]$	$\lambda [\overset{\circ}{A}]$	$\kappa [W/m.\overset{\circ}{K}]$	
0.0075	760	97	14	SiO <sub>2</sub>
0.001	85	72	11	CaF <sub>2</sub>
0.00023	45	67	6.4	NaCl
0.041	4200	430	150	Si
0.0045	1300	330	70	Ge

الجدول (2.4):



المراجع

# المراجع

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