Série de Maclaurin & Taylor.

① la Série de Maclaurin de la fonction $\sin(2x)$.

$f(x) = \sin(2x)$
$f'(x) = 2\cos(2x)$
$f''(x) = -4\sin(2x)$
$f'''(x) = -8\cos(2x)$
$f^{(4)}(x) = 16\sin(2x)$
$f^{(5)}(x) = 32\cos(2x)$
$\sin(0) = 0$
$\cos(0) = 1$

$\sin(2x) = ?$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}}{5!}x^5 + \dots$$

$$\sin(2x) = \cancel{\sin(0)}^{0} + 2\cos(0)^{1}.\frac{x}{1!} + (-4)\sin(0)^{0}\frac{x^2}{2!} + (-8)\cancel{\cos(0)}^{1}\frac{x^3}{3!} + 16\cancel{\sin(0)}^{0}\frac{x^4}{4!} + 32\cancel{\cos(0)}^{1}\frac{x^5}{5!}$$

$$\sin(2x) = 2x - 8\frac{x^3}{3!} + 32\frac{x^5}{5!} = 2x - 8\frac{x^3}{6} + \frac{32}{120}x^5 + \dots$$

$$\sin(2x) = 2x - 1{,}333x^3 + 0.26667x^5 + \dots$$

le coefficient de terme x^5 est : 0.26667 #

② $f(3) = 6$; $f'(3) = 8$; $f''(3) = 11$; $f^{(n\geq 3)}(3) = 0$ $\quad$ continue et $f(x) \in [3, 7]$

$x_0 = 3 > 0 \Rightarrow$ formule de Taylor.

$$f(x) = f(x_0) + \frac{(x - x_0)}{1!}f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + \frac{(x-x_0)^3}{3!}f'''(x_0) + \dots$$

$x = 3 \Rightarrow (x - x_0) = (7 - 3) = 4$ (pas).

$$f(3+4) = f(3) + 4f'(3) + \frac{4^2}{2!}f''(3) + \frac{4^3}{3!}\cancel{f'''(3)}^{0} + \dots$$

$$= f(3) + 4f'(3) + 8f''(3) + 0$$

$$= 6 + 4(8) + 8(11) = 126 \ \#$$

③ $\frac{dy}{dx} = y^3 + 2$ et $y(0) = 3$

$$y(x) = y(x_0) + \frac{(x - x_0)}{1!}y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0)$$

$y(0{,}2) = ?$ on a $x_0 = 0 \Rightarrow x - x_0 = 0{,}2 - 0 = 0{,}2$

$$y(0{,}2) = y(0) + (0{,}2)y'(0) + \frac{(0{,}2)^2}{2!}y''(0)$$

donnée $\Rightarrow y(0) = 3$

$y'(x) = \frac{dy}{dx} = y^3 + 2 \Rightarrow y'(0) = [y(0)]^3 + 2 = 3^3 + 2 = 29$ #

~~$y''(x) = \frac{d^2y}{dx^2} = 3y^2\frac{dy}{dx} \Rightarrow y''(0) = 3.[y(0)]^2 = 3\times($~~

$y''(x) = 3y^2.\frac{dy}{dx} = 3y^2[y^3 + 2] \Rightarrow y''(0) = 3[y(0)]^2[(y(0))^3 + 2]$

$= 3(3^2)(3^3 + 2) = 3.9.29 = 783$ #

$$y(0.2) = 3 + 0.2(29) + 0.02(783) = 24.46 \ \#$$

Méthode des différences finies.

① EDP : $A\dfrac{\partial^2 u}{\partial x^2}+B\dfrac{\partial^2 u}{\partial x\partial y}+C\dfrac{\partial^2 u}{\partial y^2}+D=0$

est Elliptique si $B^2-4(A)(C)<0$.

② $\dfrac{d^2y}{dx^2}=6x-0{,}5x^2$; $y(0)=0$; $y(12)=0$ et $\Delta x=h=4$.

$\xleftarrow{\Delta x}\xrightarrow{}\xleftarrow{\Delta x}\xrightarrow{}$

$i-1 \qquad i \qquad i+1$

pour $x_i=x_1$:

$\dfrac{d^2y}{dx^2}$ pour le noeud i, par la méthode des différences finies.

$\dfrac{d^2y}{dx^2}\simeq\dfrac{y_{i+1}-2y_i+y_{i-1}}{\Delta x^2}$, on cherche $y(4)$.

$i-1 \qquad i \qquad i+1$

$y(0) \qquad y(4) \qquad y(8)$

$\begin{cases} y_{i+1}=y(8)\\ y_i=y(4)\\ y_{i-1}=y(0)\end{cases}$

$$\boxed{\frac{d^2y}{dx^2}=\frac{y(8)-2y(4)+y(0)}{16}}$$

$\dfrac{y(8)-2y(4)+y(0)}{16}=6x_i-0{,}5x_i^2$

$\dfrac{y(8)-2y(4)+y(0)}{16}=6x_i-0{,}5x_i^2$

$y(8)-2y(4)+y(0)=6x_i.16-0{,}5.16\,x_1^2=96x_1-8x_1^2$

$0 \xleftarrow{4}\xrightarrow{} 4 \qquad 8 \qquad 12$

$x_0^0 \quad x_1^1 \quad x_2^2 \quad x_3^3 \quad x_4^4$

$y(8)-2y(4)+y(0)=96(4)-8(4)^2$

$\boxed{y(8)-2y(4)+y(0)=256}$

pour $x_i=x_2$: $i=2$, $x_2=8$

$\dfrac{d^2y}{dx^2}=\dfrac{y(12)-2y(8)+y(4)}{\Delta x^2}=y(12)-2y(8)+y(4)/16$.

$y(12)-2y(8)+y(4)=16.6x_2-0{,}5.16x_2^2=96x_2-8x_2^2$.

$\boxed{y(12)-2y(8)+y(4)=96(8)-8(64)=256}$

$\begin{cases} y(\cancel{0})^{0}-2y(4)+y(8)=256\\ y(4)-2y(8)+y(\cancel{12})^{0}=256\end{cases}$

$\begin{cases} -2y(4)+y(8)=256\\ y(4)-2y(8)=256\end{cases}$

$\begin{bmatrix}-2 & +1\\ 1 & -2\end{bmatrix}\begin{bmatrix}y(4)\\ y(8)\end{bmatrix}=\begin{bmatrix}256\\ 256\end{bmatrix}$

$$\boxed{\begin{aligned} y(4)&=-256\\ y(8)&=-256\end{aligned}}$$

Scanned with CamScanner

(3) $x^3\dfrac{\partial^2 u}{\partial x^2}+27\dfrac{\partial^2 u}{\partial y^2}+3\dfrac{\partial^2 u}{\partial x\partial y}+5u=0$ dans quelle region Elliptique

EDP : $A\dfrac{\partial^2 u}{\partial x^2}+B\dfrac{\partial^2 u}{\partial x\partial y}+C\dfrac{\partial^2 u}{\partial y^2}+D=0$ Elliptiq : $B^2-4(A)(C)<0$

$A=x^3$; $B=3$; $C=27$

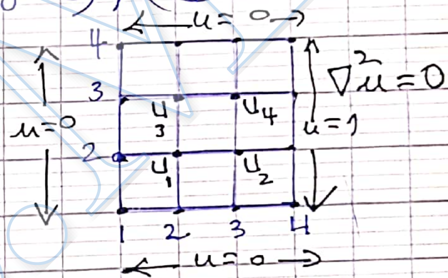
$\Delta = B^2-4(A)(C)=(3)^2-4(x^3)(27)=9-108x^3<0$

$-108x^3<-9 \iff -x^3<-\dfrac{9}{108} \Rightarrow x^3>\dfrac{1}{12} \Rightarrow x>\left[\dfrac{1}{12}\right]^{1/3}$

(4) $\dfrac{\partial^2 u}{\partial x^2}=u(x+\Delta x,y)-2u(x,y)+u(x-\Delta x,y)/(\Delta x)^2$

$\dfrac{\partial^2 u}{\partial y^2}=u(x,y+\Delta y)-2u(x,y)+u(x,y-\Delta y)/(\Delta y)^2$

(5) $u_{i,j}$ par différences finis pour $h=k=1/3$.



$u_{i,j}=\dfrac{1}{4}\left[u_{i+1,j}-u_{i-1,j}+u_{i,j+1}+u_{i,j-1}\right]$

$u_1=[u_2+0+u_3+0]/4$ $u_3=[u_4+0+0+u_1]/4$

$u_2=[u_1+1+u_4+0]/4$ $u_4=[u_2+1+0+u_3]/4$

$u_1=(u_2+u_3)/4$; $u_2=(u_1+u_4+1)4$; $u_3=$

$u_3=(u_4+u_1)/4$; $u_4=(u_2+u_3+1)/4$.

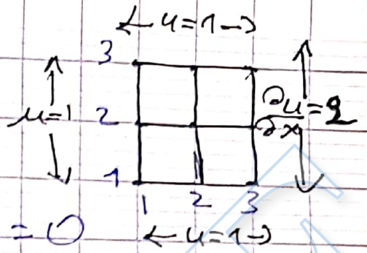
$4u_1-u_2-u_3=0$; $4u_2-u_1-u_4-1=0$

$4u_3-u_4-u_1=0$; $4u_4-u_2-u_3-1=0$

$$\begin{cases}4u_1-u_2-u_3=0\\-u_1+4u_2-u_4=1\\-u_1+4u_3-u_4=0\\-u_2-u_3+4u_4=1\end{cases}\qquad \begin{bmatrix}4&-1&-1&0\\-1&4&0&-1\\-1&0&4&-1\\0&-1&-1&4\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\\u_4\end{bmatrix}=\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$$

$u_1=1/8$; $u_2=3/8$, $u_3=1/8$ et $u_4=3/8$

(6) Résoudre $\nabla^2 u = 0$

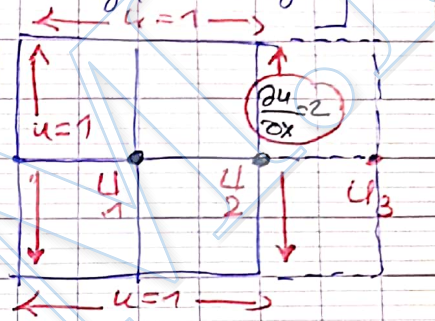


$$\nabla^2 u = 0 \Leftrightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0$$

$$\frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{ij} + u_{i,j-1}}{\Delta y^2} = 0$$

$$\Delta x = \Delta y \Rightarrow u_{i,j} = \frac{1}{4}\left[u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}\right]$$

$$\boxed{\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2 \times \Delta x}} \quad \boxed{\frac{\partial u}{\partial x} = \frac{u_3 - u_1}{2 \times 0{,}5} = 2}$$



$$u_1 = [u_2 + 1 + 1 + 1]/4 \Rightarrow 4u_1 - u_2 = 3$$

$$u_2 = [u_3 + u_1 + 1 + 1]/4 \Rightarrow 4u_2 - u_3 - u_1 = 2$$

$$\frac{\partial u}{\partial x} = \frac{u_3 - u_1}{1} = 2 \Rightarrow \boxed{u_3 = 2 + u_1}$$

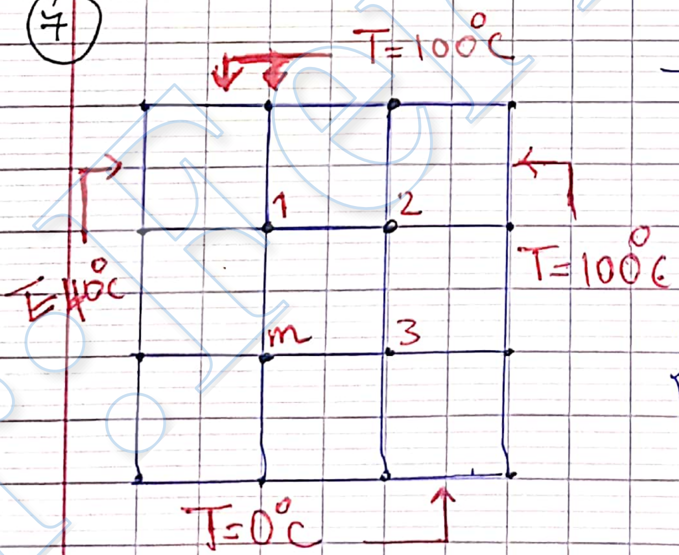
$$4u_1 - u_2 = 3$$

$$4u_2 - (u_1 + 2) - u_1 = 2 \Leftrightarrow 4u_2 - 2u_1 = 4$$

$$\begin{cases} 4u_1 - u_2 = 3 \\ -1u_1 + 2u_2 = 2 \end{cases}$$

$$\begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad u_1 = \frac{8}{7} \qquad u_2 = \frac{11}{7}$$

(7)



$T_m = 50°C$; $\Delta x = \Delta y$, $\nabla^2 u = 0$

$$\begin{cases} 4T_1 = T_2 + 100 + 40 + T_m \\ 4T_2 = 100 + 100 + T_1 + T_3 \\ 4T_3 = 100 + T_2 + 0 + T_m \end{cases}$$

$$\begin{cases} 4T_1 - T_2 = 190 \\ -T_1 + 4T_2 - T_3 = 200 \\ -T_2 + 4T_3 = 150 \end{cases}$$

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 190 \\ 200 \\ 150 \end{bmatrix}$$

$T_1 = 475/7$

$T_2 = 570/7$

$T_3 = 405/7$