$\boxed{P01}$

سلسلة التمارين حول تحويل فورييه

حلول لبعض التمارين غير المنجزة في حصص الدرس الحضورية

$$* \ F\left(X(1-|t|)\cdot(1-2|t|)\right)(\alpha) =$$

$$= \int_{-\infty}^{+\infty} X(1-|t|)\cdot(1-2|t|)e^{-i\alpha t}dt =$$

$$= \int_{-1}^{1}(1-2|t|)e^{-i\alpha t}dt = 2\int_0^1(1-2t)\cos\alpha t\,dt$$

$$= 2\left[\left[(1-2t)\frac{\sin\alpha t}{\alpha}\right]_0^1 - \int_0^1 -2\,\frac{\sin\alpha t}{\alpha}dt\right]$$

$$= 2\left[-\frac{\sin\alpha}{\alpha} + \frac{2}{\alpha}\left[-\frac{\cos\alpha t}{\alpha}\right]_0^1\right]$$

$$= 2\left[-\frac{\sin\alpha}{\alpha} - \frac{2}{\alpha^2}\cos\alpha + \frac{2}{\alpha^2}\right]$$

$$= 2\left[\frac{2-2\cos\alpha-\alpha\sin\alpha}{\alpha^2}\right]$$

تمرين ⑤ : تطبيق صيغة بلونشار - بارسفال

$$\int_{-\infty}^{+\infty}\frac{t^2}{(t^2+a^2)(t^2+b^2)}dt =$$

$$\int_{-\infty}^{+\infty}\frac{t}{t^2+a^2}\cdot\frac{t}{t^2+b^2}dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}F\left(\frac{t}{t^2+a^2}\right)(\alpha)\cdot\overline{F\left(\frac{t}{t^2+b^2}\right)}d\alpha$$

$$F\left(\frac{t}{t^2+a^2}\right)(\alpha) = F\left(t\cdot\frac{1}{t^2+a^2}\right) = i.F'\left(\frac{1}{t^2+a^2}\right)(\alpha)$$

لكن:

$$F\left(\frac{1}{t^2+a^2}\right)(\alpha) = \frac{\pi}{a}e^{-a|\alpha|} = \frac{\pi}{a}\times\begin{cases}e^{a\alpha} & ;\alpha<0\\ e^{-a\alpha} & ;\alpha\geq 0\end{cases}$$

ومنه،

$$F'\left(\frac{1}{t^2+a^2}\right)(\alpha) = \begin{cases}\frac{\pi}{a}\cdot a\,e^{a\alpha} & ,\alpha<0\\ \frac{\pi}{a}(-a)\cdot e^{-a\alpha} & ;\alpha\geq 0\end{cases}$$

إذا

$$F\left(\frac{t}{t^2+a^2}\right) = \begin{cases}i\pi e^{a\alpha} & ;\alpha<0\\ -i\pi e^{-a\alpha} & ;\alpha\geq 0\end{cases}$$

إذا:

$$\int_{-\infty}^{+\infty}\frac{t}{t^2+a^2}\cdot\frac{t}{t^2+b^2}dt = \frac{1}{2\pi}\int_{-\infty}^{0}(i\pi e^{a\alpha})(-i\pi e^{b\alpha})d\alpha$$

$$+\frac{1}{2\pi}\int_0^{+\infty}(-i\pi)e^{-a\alpha}\cdot(i\pi)e^{-b\alpha}d\alpha$$

$$= \frac{\pi^2}{2\pi}\left[\int_{-\infty}^{+0}e^{(a+b)\alpha}d\alpha + \int_0^{+\infty}e^{-(a+b)\alpha}d\alpha\right]$$

$$= \frac{\pi^2}{2\pi}\left[\frac{1}{a+b} + \frac{1}{a+b}\right] = \frac{\pi}{a+b}$$

$$*F\left(X_{(t-1)}\cdot e^{-3t}\cos t\right)(\alpha) =$$ تمرين ④ :

$$\int_{-\infty}^{+\infty}X(t-1)\cdot e^{-3t}\cos t\cdot e^{-i\alpha t}dt =$$

$$= \int_1^{+\infty}1\cdot e^{-3t}\left[\frac{e^{it}+e^{-it}}{2}\right]\cdot e^{-i\alpha t}dt$$

$$= \frac{1}{2}\left[\int_1^{+\infty}e^{-(3+(\alpha+1)i)t}dt + \int_1^{+\infty}e^{-(3+(\alpha-1)i)t}dt\right]$$

$$= \frac{1}{2}\left[\frac{e^{-(3+(\alpha+1)i)t}}{-(3+(\alpha+1)i)} + \frac{e^{-(3+(\alpha-1)i)t}}{-(3+(\alpha-1)i)}\right]_1^{+\infty}$$

$$= \frac{1}{2}\left[\frac{e^{-(3+(\alpha+1)i)}}{3+(\alpha+1)i} + \frac{e^{-(3+(\alpha-1)i)}}{3+(\alpha-1)i}\right]$$

$$= \frac{e^{-(3+\alpha i)}}{2}\cdot\left[\frac{e^{i}+e^{-i}}{9-(\alpha^2-1)+6\alpha i}\right.$$

$$= \frac{e^{-(3+\alpha i)}\cdot\cos 1}{10-\alpha^2+6\alpha i} = \frac{e^{-3}\cdot\cos 1\,(\cos\alpha - i\sin\alpha)}{10-\alpha^2+6\alpha i}$$

$$*F\left(X(t+1)\cdot e^{-|t|}\right)(\alpha) = \int_{-1}^{+\infty}e^{-|t|}\cdot e^{-i\alpha t}dt$$

$$= \int_{-1}^{0}e^{t}\cdot e^{-i\alpha t}dt + \int_0^{+\infty}e^{-t}e^{-i\alpha t}dt$$

$$= \left[\frac{e^{(1-i\alpha)t}}{1-i\alpha}\right]_{-1}^{0} + \left[\frac{e^{-(1+i\alpha)t}}{-(1+i\alpha)}\right]_0^{+\infty}$$

$$= \frac{1}{1-i\alpha} - \frac{e^{-(1-i\alpha)}}{1-i\alpha} + \frac{1}{1+i\alpha}$$

$$= \frac{2 - e^{-1}(\cos\alpha+i\sin\alpha)(1+i\alpha)}{1+\alpha^2}$$

$$= \frac{2 - e^{-1}(\cos\alpha - \alpha\sin\alpha + i(\alpha\cos\alpha+\sin\alpha))}{1+\alpha^2}$$

P2

إذا $\hat{\hat{f}}(\alpha) = 2\pi \cdot f(-\alpha)$

ومنه

$$F\left(F(f) * F(g)\right)(\alpha) = F\left(\hat{f} * \hat{g}\right)(\alpha)$$

$$= F(\hat{f})_{(\alpha)} \cdot F(\hat{g})_{(\alpha)} = \hat{\hat{f}}(\alpha) \cdot \hat{\hat{g}}(\alpha)$$

$$= 2\pi \cdot f(-\alpha) \cdot 2\pi \cdot g(-\alpha)$$

$$= 2\pi\left(2\pi \cdot f(-\alpha) \cdot g(-\alpha)\right)$$

$$= 2\pi\left(2\pi \cdot (f \cdot g)(-\alpha)\right)$$

$$= 2\pi \left\{ F\left[F(f\cdot g)\right](\alpha)\right.$$

$$= F\left[2\pi \cdot F(f\cdot g)\right](\alpha)$$

إذا : $F(f) * F(g) = 2\pi \cdot F(f\cdot g)$

التمرين 8 : حل معادلة لابلاس $\frac{\partial^2 \varphi(x,y)}{\partial x^2} + \frac{\partial^2 \varphi(x,y)}{\partial y^2} = 0$

نطبق تحويل فوريه بالنسبة للمتغير x على الطرفين

$$F\left(\frac{\partial^2 \varphi}{\partial x^2}(x,y)\right)(\alpha) = \int_{-\infty}^{+\infty} \frac{\partial^2 \varphi(x,y)}{\partial x^2} e^{-i\alpha x} dx$$

$$= \left[\frac{\partial \varphi}{\partial x} \cdot e^{-i\alpha x}\right]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} \frac{\partial \varphi}{\partial x} (-i\alpha) e^{-i\alpha x} dx$$

$$= (i\alpha) \int_{-\infty}^{+\infty} \frac{\partial \varphi}{\partial x} e^{-i\alpha x} dx$$

(لأن : $\frac{\partial \varphi}{\partial x} \xrightarrow[|x| \to +\infty]{} 0$)

$$= i\alpha \left[\underbrace{\left[\varphi(x,y) \cdot e^{-i\alpha x}\right]_{-\infty}^{+\infty}}_{=0} - \int_{-\infty}^{+\infty} \varphi(x,y)(-i\alpha) e^{-i\alpha x} dx \right]$$

$$= (i\alpha)^2 \int_{-\infty}^{+\infty} \varphi(x,y) e^{-i\alpha x} dx = (i\alpha)^2 \cdot F(\varphi(x,y))(\alpha)$$

(لأن : $\varphi(x,y) \xrightarrow[|x| \to +\infty]{} 0$)

$$F\left(\frac{\partial^2 \varphi}{\partial y^2}\right)(\alpha) = \frac{\partial}{\partial y^2}\left[\int_{-\infty}^{+\infty} \varphi(x,y) e^{-i\alpha x} dx\right]$$

$$= \frac{\partial}{\partial y^2} F\left(\varphi(x,y)\right)(\alpha)$$

ومنه نأخذ معادلة تفاضلية من الشكل التالي

$$\phi'' - \alpha^2 \phi = 0$$

حيث $\phi = F(\varphi(x,y))(\alpha)$ (دالة بمتغيرين α و y)

$$\int_{-\infty}^{+\infty} \frac{dt}{(1+it)^2(1-it)^2} = \int_{-\infty}^{+\infty} \frac{dt}{(1+t^2)^2}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left|F\left(\frac{1}{1+t^2}\right)(\alpha)\right|^2 d\alpha = \frac{1}{2\pi}\int_{-\infty}^{+\infty} (\pi e^{-|\alpha|})^2 d\alpha$$

$$= \frac{\pi^2}{2\pi}\int_{-\infty}^{+\infty} e^{-2|\alpha|} d\alpha = \frac{2\pi^2}{2\pi}\int_{0}^{+\infty} e^{-2\alpha} d\alpha$$

$$= \pi\left[\frac{1}{-2} \cdot e^{-2\alpha}\right]_0^{+\infty} = \frac{\pi}{2}$$

التمرين 6 : إثبات أن : $f_a * f_b = f_{a+b}$

تحويل فوريه متباين ولذلك يكفي إثبات أن

$$F(f_a * f_b) = F(f_{a+b})$$

لدينا :

$$F(f_a)(\alpha) = F\left(\frac{a}{\pi(t^2+a^2)}\right)(\alpha)$$

$$= \frac{a}{\pi} F\left(\frac{1}{a^2\left[\left(\frac{t}{a}\right)^2+1\right]}\right)(\alpha)$$

$$= \frac{a}{a^2\pi} F\left(\frac{1}{\left(\frac{t}{a}\right)^2+1}\right)(\alpha) = \frac{a^2}{a\pi} \cdot F\left(\frac{1}{t^2+1}\right)(a\alpha)$$

$$= \frac{1}{\pi} \cdot \pi \cdot e^{-a|\alpha|} = e^{-a|\alpha|}$$

إذا

$$F(f_a * f_b) = F(f_a) \cdot F(f_b) =$$

$$e^{-a|\alpha|} \cdot e^{-b|\alpha|} = e^{-(a+b)|\alpha|}$$

$$= F(f_{a+b})$$

ومنه

$$f_a * f_b = f_{a+b}$$

تمرين 07

$$F(f * g)(\alpha) = F(f)(\alpha) \cdot F(g)(\alpha)$$

(مبرهنة في الدرس)

ومن جهة أخرى لدينا :

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(\alpha) e^{i\alpha t} d\alpha$$

إذا

$$f(-t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(\alpha) e^{-i\alpha t} d\alpha$$

وباستبدال t بـ α

$$f(-\alpha) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(t) e^{-i\alpha t} dt$$

$$= \frac{1}{2\pi} \cdot \hat{\hat{f}}(\alpha)$$

P3

$\boxed{\text{شرح } g}$

$$f_T(t)=\int_{-T}^{T}\left(1-\frac{|x|}{T}\right)e^{ixt}\cdot\hat{f}(x)\,dx$$

$$=\int_{-\infty}^{+\infty}\chi\left(1-\frac{|x|}{T}\right)\cdot\left(1-\frac{|x|}{T}\right)\cdot\hat{f}(x)e^{ixt}\,dx$$

بفرض:
$$\hat{g}(x)=\chi\left(1-\frac{|x|}{T}\right)\cdot\left(1-\frac{|x|}{T}\right)$$

حيث
$$\chi(t)=\begin{cases}1, & t\geq 0\\ 0, & t<0\end{cases}$$

اذا:
$$g(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{g}(x)e^{ixt}\,dx$$

$$=\frac{1}{2\pi}\int_{-T}^{T}\left(1-\frac{|x|}{T}\right)e^{ixt}\,dx$$

$$=\frac{1}{\pi}\int_{0}^{T}\left(1-\frac{x}{T}\right)\cos xt\,dt$$

(لأن $x\mapsto 1-\frac{|x|}{T}$ زوجي)

$$=\frac{1}{\pi}\left[\left[\left(1-\frac{x}{T}\right)\frac{\sin xt}{t}\right]_0^T-\int_0^T-\frac{1}{T}\cdot\frac{\sin xt}{t}\,dx\right]$$

$$=\frac{1}{\pi T t}\left[\frac{-\cos xt}{t}\right]_0^T=\frac{1}{\pi T}\cdot\frac{1-\cos Tt}{t^2}$$

$$=\frac{1}{\pi T}\cdot\frac{2\sin^2\left(\frac{T}{2}t\right)}{t^2}$$

اذا:
$$f_T(t)=\int_{-\infty}^{+\infty}\hat{g}(x)\cdot\hat{f}(x)e^{ixt}\,dx$$

$$=\int_{-\infty}^{+\infty}F\left[(g*f)\right](x)e^{ixt}\,dx$$

اذا: تمت اعادة كتابتها على الصفحة الموالية

$$\frac{1}{2\pi}f_T(x)=(g*f)(x)$$

$$=\int_{-\infty}^{+\infty}\Big(\underbrace{\frac{2}{\pi T}\cdot\frac{\sin^2\left(\frac{T t}{2}\right)}{t^2}}_{=g(t)}\Big)\cdot f(x-t)\,dt$$

$$=\frac{2}{\pi T}\left[\int_0^{+\infty}\frac{\sin^2\left(\frac{T}{2}t\right)}{t^2}\cdot f(x-t)\,dt+\int_{-\infty}^{0}\frac{\sin^2\left(\frac{T}{2}t\right)}{t^2}\cdot f(x-t)\,dt\right]$$

$$=\frac{2}{\pi T}\left[\int_0^{+\infty}\frac{\sin^2\left(\frac{T}{2}t\right)}{t^2}\cdot f(x-t)\,ds+\int_{+\infty}^{0}\frac{\sin^2\left(\frac{T}{2}t\right)}{(-t)^2}f(x+t)(-dt)\right]$$

$$f_T(x)=\frac{2\pi\times 2}{\pi T}\times 2\int_0^{+\infty}\frac{\sin^2\left(\frac{T}{2}t\right)}{t^2}\left[f(x-t)+f(x+t)\right]dt$$

كيف الحدود المميزة لهذه المعادلة التفاضلية ذات المتغير y ومن الرتبة الثانية معاملات ثابتة هو

$$\lambda^2-\alpha^2=0\implies\lambda=\pm\alpha$$

وبالتالي حلولها من الشكل $\Phi(\alpha,y)=C(\alpha)e^{-|\alpha|y}$

(لأن: $\varphi(x,y)\xrightarrow[y\to+\infty]{}0$).

$$\Phi(\alpha,0)=F\left[\varphi(x,0)\right](\alpha)$$

$$=\int_{-\infty}^{+\infty}\varphi(x,0)e^{-i\alpha x}\,dx=\int_{-1}^{1}e^{-i\alpha x}\,dx$$

$$=\left[-\frac{1}{i\alpha}e^{-i\alpha x}\right]_{-1}^{1}=\left[\frac{e^{i\alpha}-e^{-i\alpha}}{i\alpha}\right]$$

$$=\frac{2}{\alpha}\sin\alpha$$

$$\Phi(\alpha,y)=F\left(\varphi(x,y)\right)(\alpha)=\frac{2\sin\alpha}{\alpha}\cdot e^{-y|\alpha|}$$

$$=F\left(\varphi(x,0)\right)\cdot F\left(\frac{1}{\pi}\frac{y}{x^2+y^2}\right)$$

$$=F\left(\frac{1}{\pi}\varphi(x,0)*\frac{y}{x^2+y^2}\right)(\alpha)$$

ومنه حل معادلة لابلاس هو

$$\varphi(x,y)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\varphi(t,0)\cdot\frac{y}{(x-t)^2+y^2}\,dt$$

$$=\frac{1}{\pi}\int_{-1}^{1}\frac{y}{(x-t)^2+y^2}\,dt$$

$$=\frac{1}{y}\frac{1}{\pi}\cdot y\int_{-1}^{1}\frac{dt}{\left(\frac{x-t}{y}\right)^2+1}=\frac{y}{y\pi}\int_{\frac{x+1}{y}}^{\frac{x-1}{y}}\frac{-y\,ds}{s^2+1}$$

$$=\frac{1}{\pi}\cdot\left[\arctan(s)\right]_{\frac{x-1}{y}}^{\frac{x+1}{y}}=$$

$$=\frac{1}{\pi}\left[\arctan\left(\frac{x+1}{y}\right)-\arctan\left(\frac{x-1}{y}\right)\right]$$

$$=\frac{1}{\pi}\arctan\frac{\frac{x+1}{y}-\frac{x-1}{y}}{1+\frac{x+1}{y}\cdot\frac{x-1}{y}}=\frac{\frac{2}{y}}{\left(\frac{x^2+y^2-1}{y^2}\right)}\frac{1}{\pi}$$

$$=\frac{1}{\pi}\arctan\left(\frac{2y}{x^2+y^2-1}\right).$$

P4

ومنه : $f*g$ زوجية . نفس الطريقة لتثبت
ان : $f*g$ فردية في حالة f,g فرديتان معا

③ f فردية ، g زوجية

$$f*g(-x) = \int_{-\infty}^{+\infty} f(t)\cdot g(-x-t)\,dt$$
$$= \int_{-\infty}^{+\infty} f(t)\cdot g(x+t)\,dt =$$
$$-\int_{-\infty}^{+\infty} f(-t)\, g(x-(-t))\,dt = -f*g(x)$$

ومنه $f*g$ فردية .

تمرين 12

① $$f(t) = \int_{-\infty}^{+\infty} e^{-x^2}\cos(tx)\,dx$$

وبما ان $x \mapsto e^{-x^2}$ زوجية فإن .

$$f(t) = \int_{-\infty}^{+\infty} e^{-x^2} e^{-itx}\,dx = F(e^{-x^2})$$

ومنه f دالة مستمرة و $\lim_{t\to+\infty} |f(t)| = 0$

تحقق قابلية الاشتقاق على $\mathbb{R}$.

$$f'(t) = \int_{-\infty}^{+\infty} -x\, e^{-x^2}\sin(tx)\,dx$$

وبمكاملة بالتجزئة نجد

$$= \underbrace{\left[\frac{1}{2} e^{-x^2}\sin tx\right]_{-\infty}^{+\infty}}_{0} - \int_{-\infty}^{+\infty} \frac{1}{2} e^{-x^2}(t\cos tx)\,dx$$
$$= -\frac{1}{2} t \int_{-\infty}^{+\infty} e^{-x^2}\cos(tx)\,dx = -\frac{1}{2} t f(t)$$

ومنه

$$\boxed{2f'(t) + t f(t) = 0}$$

② $$f(t) = C\, e^{-\frac{t^2}{4}}$$

وبما ان $f(0) = \int_{-\infty}^{+\infty} e^{-x^2}\,dx = \sqrt{\pi} = C$

اذا $f(t) = \sqrt{\pi}\, e^{-\frac{t^2}{4}} = F(e^{-x^2})(t)$.

③ $$F(h*h) = \left[F(h)(x)\right]^2$$
$$F[h](x) = \frac{1}{\,}F\left(e^{-\left(\frac{t}{\sqrt{2}}\right)^2}\right)\ldots$$

$$\frac{1}{2\pi} f_T(t) = (g*f)(t) =$$
$$\int_{-\infty}^{+\infty} g(s)\cdot f(t-s)\,ds =$$
$$\frac{2}{\pi T}\int_{-\infty}^{+\infty} \frac{\sin^2\left(\frac{Ts}{2}\right)}{s^2}\cdot f(t-s)\,ds =$$
$$\frac{2}{\pi T}\left[\int_{0}^{+\infty} \frac{\sin^2\left(\frac{Ts}{2}\right)}{s^2}\cdot f(t-s)\,ds + \int_{-\infty}^{0} \frac{\sin^2\left(\frac{Ts}{2}\right)}{s^2}\cdot f(t-s)\,ds\right]$$

نستبدل في التكامل الثاني s بـ $(-s)$

فنجد : $$\int_{+\infty}^{0} \frac{\sin^2\left(\frac{T(-s)}{2}\right)}{(-s)^2}\cdot f(t+s)(-ds)$$
$$= \int_{0}^{+\infty} \frac{\sin^2\left(\frac{Ts}{2}\right)}{s^2}\cdot f(t+s)\,ds$$

اذا

$$f_T(t) = 2\pi \times \frac{2}{\pi T}\left[\int_0^{+\infty} \frac{\sin^2\left(\frac{Ts}{2}\right)}{s^2}\times [f(t-s)+f(t+s)]\,ds\right]$$
$$= \frac{4}{T}\left[\int_0^{+\infty} \frac{\sin^2\left(\frac{Ts}{2}\right)}{s^2}\cdot (f(t-s)+f(t+s))\,ds\right]$$

تمرين 10 :

① $$f*g(x) = \int_{-\infty}^{+\infty} f(t)\cdot g(x-t)\,dt$$

نضع $u = x-t$ ومنه $(du = -dt)$

$$f*g(x) = \int_{+\infty}^{-\infty} g(u)\cdot f(x-u)(-du)$$
$$= \int_{-\infty}^{+\infty} g(u)\, f(x-u)\,du = g*f(u)$$

② نفرض f,g زوجيتان .

$$f*g(-x) = \int_{-\infty}^{+\infty} f(t)\cdot g((-x)-t)\,dt$$
$$= \int_{-\infty}^{+\infty} f(t)\cdot g(x+t)\,dt$$

لأن g زوجية

$$= \int_{-\infty}^{+\infty} f(-t)\cdot g(x-(-t))\,dt$$

(لأن f زوجية) (نضع $-t = u$

$$= \int_{+\infty}^{-\infty} f(u)\, g(x-u)(-du)$$
$$= \int_{-\infty}^{+\infty} f(u)\, g(x-u)\,du = f*g(x)$$

[P5]

$= \frac{\sqrt{2}}{\sqrt{\pi}} \cdot F\left(e^{-t^2}\right)\left(\frac{\alpha}{\frac{1}{\sqrt{2}}}\right) = \sqrt{\frac{2}{\pi}} \cdot \sqrt{\pi} \cdot e^{-\frac{(\alpha\sqrt{2})^2}{4}}$

$= \sqrt{2} \cdot e^{-\frac{\alpha^2}{2}}.$

اذا

$F(h*h)(\alpha) = 2e^{-\alpha^2}$

و منه

$h*h(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} 2e^{-\alpha^2} \cdot e^{i\alpha t}\, d\alpha$

$= \frac{1}{\pi}\int_{-\infty}^{+\infty} e^{-\alpha^2} e^{i\alpha t}\, d\alpha$

$= \frac{1}{\pi}\int_{-\infty}^{+\infty} e^{-\alpha^2} \cos\alpha t\, d\alpha$

$= \frac{1}{\pi} f(t) = \frac{1}{\sqrt{\pi}} e^{-\frac{t^2}{4}}$

تمرين 13 :

$F\left(e^{-|t|}\right)(\alpha) = \frac{2}{\alpha^2+1}$ (1)

$e^{-|t|} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{2}{\alpha^2+1} e^{i\alpha t}\, d\alpha$ (2)

وبما أن الدالة

$\alpha \mapsto \frac{2}{\alpha^2+1}$ زوجية

$e^{-|t|} = \frac{2}{2\pi}\int_{0}^{+\infty} \frac{2\cos\alpha t}{\alpha^2+1}\, d\alpha$

اذا

$\int_{0}^{+\infty} \frac{\cos\alpha t}{\alpha^2+1}\, d\alpha = \frac{\pi}{2} e^{-|t|}$

من اجل $t=1$ نجد

$\int_{0}^{+\infty} \frac{\cos\alpha}{\alpha^2+1}\, d\alpha = \frac{\pi}{2e}.$