Gas Dynamics

Series N°1: Generalities & Sound physics (**Manual Solutions**)

<u>ExO1.</u>*Solution*

On a: $T = 15°C = (15 + 273.15) = 288.15\ K$

$P = 760\ mmHg = 1\ atm = 1.013 \times 10^5\ Pa,$

Pour l'air considéré comme un gaz parfait ; $\boldsymbol{\rho} = {P}\Big/{\left(\frac{\mathcal{R}}{\mathcal{M}}\right) T} = \dfrac{1.013\times10^5\ Pa}{\left(\frac{8314\ \frac{J}{kmol.K}}{28.97\ \frac{kg}{kmol}}\right)(288.15\ K)} = \mathbf{1.225\ kg/m^3.}$

<u>ExO2.</u>*Solution*

On a : $T = 6°C = 279.15\ K,\ \gamma_{H_2} = 1.4$

Aussi, pour l'hydrogène ; $r = \frac{\mathcal{R}}{\mathcal{M}_{H2}} = \frac{8314}{2} = 4\ 157\ \frac{J}{kg.K}$,

La célérité du son se propageant dans le gaz d'hydrogène est : $\boldsymbol{a} = \sqrt{\gamma_{H_2} rT} = \mathbf{1274.60\ m/s.}$

<u>ExO3.</u>*Solution*

On a : $P = 500 \times 10^3\ Pa,\ \ T = 60°C = 333.15\ K,$

a) Pour l'air ; $r = \frac{\mathcal{R}}{\mathcal{M}_{air}} = \frac{8314}{28.97} = 286.99\ \frac{J}{kg.K}$,

La masse volumique est : $\boldsymbol{\rho} = {P}/{rT} = \frac{500\times10^3}{(287)(333.15)} = \mathbf{5.229\,\frac{kg}{m^3}}.$

b) Pour l'hydrogène ; $r = \frac{\mathcal{R}}{\mathcal{M}_{H2}} = \frac{8314}{2} = 4\ 157\ \frac{J}{kg.K}$,

La masse volumique est : $\boldsymbol{\rho} = {P}/{rT} = \frac{500\times10^3}{(4157)(333.15)} = \mathbf{0.361\,\frac{kg}{m^3}}.$

<u>ExO4.</u>*Solution*

On a: $V = 200\frac{m}{s};\ \ r = 287\ J/kg.K\ ;\ \gamma = 1.4.$

Quand l'air est à la température $T = 30°C$.

a) la vitesse du son sera :

$$\boldsymbol{a} = \sqrt{\gamma\ r\ T} = \sqrt{(1.4)(287)(30 + 273.15)} = \mathbf{348.92\,\frac{m}{s}},$$

b) le nombre de Mach à l'entrée du diffuseur est :

$$\boldsymbol{M} = \frac{V}{a} = \frac{200}{348.92} = \mathbf{0.573}\ \ (subsonique).$$

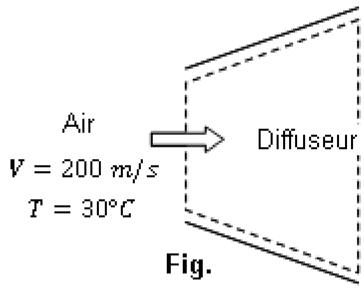


Fig.

<u>ExO5.</u>*Solution*

On a: $T = 288.15\ K;\ \ \alpha = 50° \Longrightarrow \sin(\alpha) = \frac{1}{M} \Leftrightarrow M = \frac{1}{\sin(\alpha)} = \frac{1}{0.7660} = 1.3054$,

La vitesse de cette balle sera : $\boldsymbol{V} = M \times a = M\sqrt{\gamma\ r\ T} = 1.3054\sqrt{(1.4)(287)(288.15)} = \mathbf{444.18\,\frac{m}{s}}.$

<u>ExO6.</u>*Solution*

$A\ z = 33000\ ft = 10.06\ km$; on a : $T = 223\ K,$

Donc, $a = \sqrt{\gamma\ r\ T} = [(1.4)(287)(223)]^{1/2} = 229.33\,\frac{m}{s},$

$V = 520\,\frac{mile}{hr} = \left(520\,\frac{mile}{hr}\right) \times \frac{1609.344\ m}{3600\ s} = 232.46\ m/s,$

a) le nombre de Mach du vol en croisière ; $\boldsymbol{M} = \frac{V}{a} = \frac{232.46}{229.33} = \mathbf{1.014}$.

b) la vitesse du vol correspondant à $M_{max} = 0.9$; $\boldsymbol{V} = M_{max} \times a = 0.9 \times 229.33 = \mathbf{206.40}\,\frac{\boldsymbol{m}}{\boldsymbol{s}}$.

<u>ExO7.</u>*Solution*

A l'altitude de $z = 85000\,ft = 85000\,ft \times 0.3048\frac{m}{ft} = 25908\,m,\ T = 222\,K.$

a) Pour ces conditions, la vitesse du son ; $\boldsymbol{a} = \sqrt{\gamma\, r\, T} = [(1.4)(287)(222)]^{1/2} = \mathbf{298.66}\,\frac{\boldsymbol{m}}{\boldsymbol{s}}$,

et la vitesse du vol **;** $\boldsymbol{V} = M.a = 3.3 \times 298.66 = \mathbf{985.59}\,\frac{\boldsymbol{m}}{\boldsymbol{s}}$.

b) Comparaison ; $\frac{\boldsymbol{V}}{\boldsymbol{V_{balle}}} = \frac{985.59}{700} = \mathbf{1.41}.$ l'avion est plus rapide que la balle d'environ 41%.

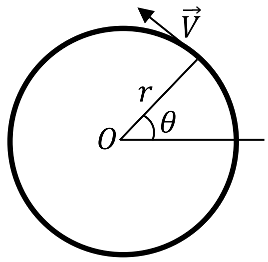
<u>ExO8.</u>*Solution*

A l'altitude de $z = 30000\,ft\ = 9144\,m,\ T = 229\,K.$

$a = \sqrt{\gamma\, r\, T} = \sqrt{[(1.4)(287)(229)]} = 303.34\ \frac{m}{s}$,

Pour un mouvement circulaire :

$V = r\omega = r\frac{d\theta}{dt} \qquad \therefore \quad r = V\Big/\frac{d\theta}{dt}$



et $\gamma_n = \frac{V^2}{r} = r\omega^2$

Cas **a)** $\boldsymbol{r} = V\Big/\frac{d\theta}{dt} = \frac{M\times a}{\frac{d\theta}{dt}} = \frac{0.7\times 303.34}{\frac{6°}{s}\times\frac{\pi\, rad}{180°}} = \mathbf{2027.68}\,\boldsymbol{m}$,

$\boldsymbol{\gamma_n} = \frac{V^2}{r} = \frac{M^2a^2}{r} = \frac{\{0.7\times 303.34\}^2}{2027.68} = \mathbf{22.24}\,\frac{\boldsymbol{m}}{\boldsymbol{s^2}}$.

Cas **b)** $\boldsymbol{r} = V\Big/\frac{d\theta}{dt} = \frac{M\times a}{\frac{d\theta}{dt}} = \frac{1.6\times 303.34}{\frac{3.5°}{s}\times\frac{\pi\, rad}{180°}} = \mathbf{7945.20}\,\boldsymbol{m}$,

$\boldsymbol{\gamma_n} = \frac{V^2}{r} = \frac{M^2a^2}{r} = \frac{\{1.6\times 303.34\}^2}{7945.20} = \mathbf{29.65}\,\frac{\boldsymbol{m}}{\boldsymbol{s^2}}$.

<u>ExO9.</u>*Solution*

a) Pour une variation linéaire de la température, $T = T_0\ +\ mz$

$$\frac{\boldsymbol{dT}}{\boldsymbol{dz}} = m = \frac{T - T_0}{z} = \frac{(223.3 - 288.2)}{1\times 10^4} = \mathbf{-6.49\times 10^{-3}}\,\frac{\boldsymbol{K}}{\boldsymbol{m}}.$$

b) Pour un gaz parfait, $a = \sqrt{\gamma\, r\, T} = \{\gamma\, r(T_0\ +\ mz)\}^{1/2}$

$$\frac{\boldsymbol{da}}{\boldsymbol{dz}} = \frac{m\,\gamma\, r}{2[\gamma\, r(T_0\ +\ mz)]^{1/2}} = \frac{\boldsymbol{m\,\gamma\, r}}{\boldsymbol{2\,a}}.$$

c) - Au niveau de la mer, $T = 288.2\,K$

$$a = \sqrt{\gamma\, r\, T} = \{1.4\times 287\times 288.2\}^{\frac{1}{2}} = 340.29\frac{m}{s},$$

$$\frac{\boldsymbol{da}}{\boldsymbol{dz}} = \frac{m\,\gamma\, r}{2\,a} = \frac{-6.49\times 10^{-3}\times 1.4\times 287}{2\times 340.29} = \mathbf{-3.832\times 10^{-3}}\,\boldsymbol{s^{-1}}.$$

- A $z = 10\,km, T = 223.3\,K$

$$a = \sqrt{\gamma\, r\, T} = \{1.4\times 287\times 223.3\}^{\frac{1}{2}} = 299.54\frac{m}{s},$$

$$\frac{\boldsymbol{da}}{\boldsymbol{dz}} = \frac{m\,\gamma\, r}{2\,a} = \frac{-6.49\times 10^{-3}\times 1.4\times 287}{2\times 299.54} = \mathbf{-4.353\times 10^{-3}}\,\boldsymbol{s^{-1}}.$$

ExO10. ***Solution***

D'après l'équation : $sin\ \alpha = \frac{1}{M}$; et en supposant que l'air est comme un gaz parfait où les chaleurs spécifiques sont constantes ;

$$M = \frac{1}{\sin\alpha} \implies \frac{V}{a} = \frac{1}{\sin\alpha} \implies \boldsymbol{V} = \frac{a}{\sin\alpha} = \frac{\sqrt{\gamma\ r\ T}}{\sin\alpha} = \frac{\{1.4\times287\times288\}^{\frac{1}{2}}}{\sin 28°} = \boldsymbol{724.59\,\frac{m}{s}}.$$

ExO11. ***Solution***

a) $\boldsymbol{V} = M\ a = M\sqrt{\gamma r T} = \{1.9\}\{1.4\times287\times298\}^{\frac{1}{2}} = \boldsymbol{657.46\ m/s}.$

b) $sin\ \alpha = \frac{1}{M} = \frac{1}{1.9} = 0.526 \implies \boldsymbol{\alpha} = \arcsin 0.526 = \boldsymbol{31.76}°.$

ExO12. ***Solution***

On a: $P = 50\ psia = 50\times6894.757\ Pa = 344737.85\ Pa,$

$$\rho = 0.27\frac{lbm}{ft^3} = 0.27\times\frac{0.4536\ kg}{(0.3048\ m)^3} = 4.325\ kg/m^3$$

$$sin\ \alpha = \frac{1}{M} \Rightarrow M = \frac{1}{\sin\alpha} = \frac{1}{\sin 10°} = 5.759,$$

$$a = \sqrt{\gamma\ r\ T} = \sqrt{\gamma\ \frac{P}{\rho}} = \sqrt{1.4\times\frac{344737.85}{4.325}} = 334.05\,\frac{m}{s},$$

$$M = \frac{V}{a} \Rightarrow \boldsymbol{V} = M.a = 5.759\times334.05 = \boldsymbol{1923.79\ m/s}.$$