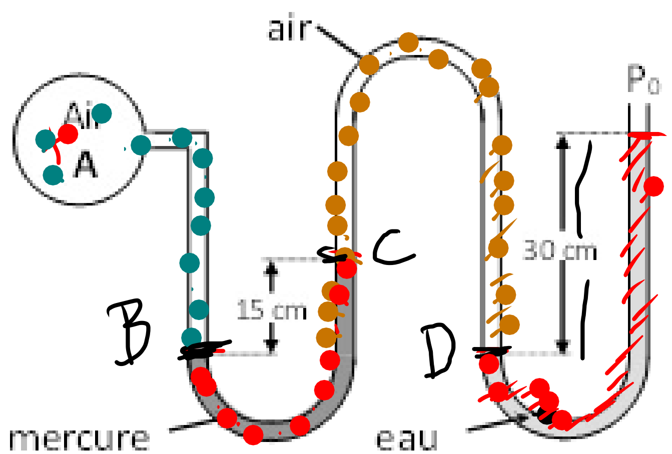
Exercice 01 : Quelle est la pression au point A de la figure 1 ?

Figure 1



$= P_{atm}$ $\quad P_E = P_{atm} = 1\,bar \simeq 10^5\,Pa$

RFH $\Rightarrow P + \rho g z = cte$

$$P_E + \rho_e g z_E = P_D + \rho_e g z_D$$

$$P_D = P_E + \rho_e g [z_E - z_D]$$

$$P_A + \rho g z_A = P_B + \rho g z_B$$

* التجانس

× الاستمرارية

$$P_D + \rho g \cancel{z_D} = P_C + \rho g \cancel{z_C} \quad , \quad \rho_{air} \ll$$

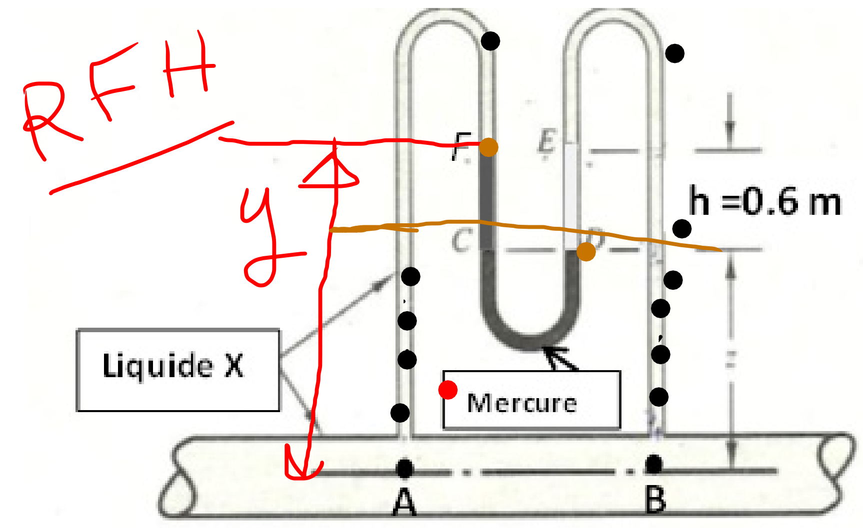
$$P_D \approx P_C \quad / \quad \underline{P_C} + \rho_{Me} \cdot g z_C = P_B + \rho_{Me} \cdot g z_B$$

$$P_B = P_C + \rho_{Me} \cdot g (z_C - z_B) \quad , \quad z_C - z_B = 15\,cm$$

$$P_A \approx P_B = \ldots Pa$$

A N

Exercice 04 : Un manomètre est fixé entre deux point A et B d'un tuyau horizontale ou s'écoule un liquide X de densité d =1. La dénivellation h du mercure dans le manomètre est de 0.6 m. Calculer la différence de pression entre A et B en Pa, sachant que le poids volumique de mercure est On prend g=10 m/s2



$$P_A - P_B = ?$$

$$P_B + \rho_x g z_B = P_D + \rho g z_D \Rightarrow P_B = P_D + \rho_x g (z_D - z_B)$$

$$P_B = P_D + \rho_x g z$$

Poids volumique = الوزن الحجمي = الثقل : $\overline{W} = \rho g$

$$\overline{W} = \frac{Kg}{m^3} \cdot \frac{m}{s^2} = \frac{N}{m^3}$$

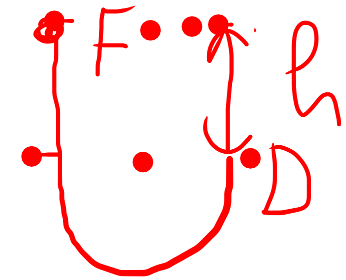
$$\overline{W} = \frac{Kg}{m^2 \cdot s^2}$$

$$P_A + \rho \times g z_A = P_F + \rho \times g z_F$$

$$P_A = P_F + \rho \times g (z_F - z_A)$$

$$P_A = P_F + \rho \times g y$$

$$P_A - P_B = P_F + \rho_x g y - P_D - \rho_x g z$$



$$= (P_F - P_D) + \rho_x g (y - z)$$

$$y - z = h$$

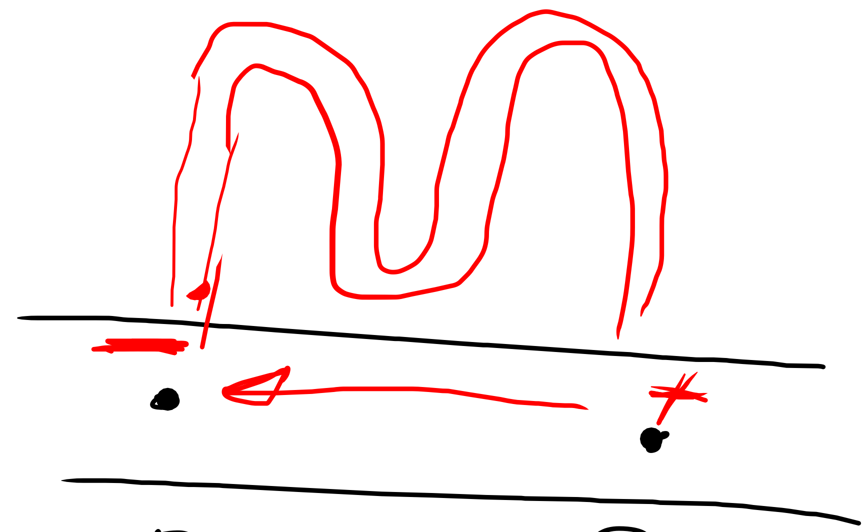
$$P_F + \rho_{Me} g z_F = P_D + \rho_{Me} g z_D \Rightarrow P_F - P_D =$$

$$= \rho_{Me} g (z_D - z_F) = -\underbrace{\rho_{Me} g}_{w_{Me}} h$$

$$\overset{(-)}{P_A - P_B} = \rho_x g h - \rho_{Hg}\, g h = g h(\rho_x - \rho_{Hg})$$

$$\rho_{Hg} \simeq 13570 \ \frac{kg}{m^3}$$

$$\rho_{eau} = 1000 \ \frac{kg}{m^3}$$



P_A P_B

$$P_A - P_B < 0$$

$$P_A < P_B$$

ρ , d , ν , ∂ الخصائص الخاصة بالمائع :

SI

ρ = $\frac{\text{Masse}}{\text{volumique}}$ = $\frac{Kg}{m^3}$;

$d = \frac{\rho_x}{\rho_{ref}}$, ρ_{ref} → d (liquide) سائل → water ماء

→ d(gaz) ; غاز → Air, هواء ,

d_L : $\frac{\rho_L}{\rho_{water}}$, $d_g = \frac{\rho_g}{\rho_{air}}$ (بدون وحدة)

$\mu =$ viscosité dynamique

$\mu = kg/(m \cdot s) = \frac{N \cdot S}{m^2} = Pa \cdot S$

1 Poiseuille = 1 PI = 1 Pa·S = 1 $\frac{Kg}{m \cdot s}$;

1 PI = 10 P (Poise)

ν = viscosité cinématique, $\mu = \rho \cdot \nu$

$\nu : \left[\frac{m^2}{s} \right]$, 1 stoks = 1 $\frac{cm^2}{s}$ = $10^{-4} \frac{m^2}{s}$

$\vec{a} = \vec{g} = \left[\frac{m}{s^2}\right]$

$m =$ وحدة الطول

وحدة الحجم

$1m = 10^2\ cm = 10^3\ mm$ (D, r) القطر

$1mm = 10^{-3}\ m = 0,1\ cm$

$V =$ الحجم $\Rightarrow [m^3] \Rightarrow 1m^3 = 10^3\ L$

$\vec{q}(\overset{x}{u}, \overset{y}{v}, \overset{t}{w}) = \frac{m}{s}$ $/\ (t = s)$

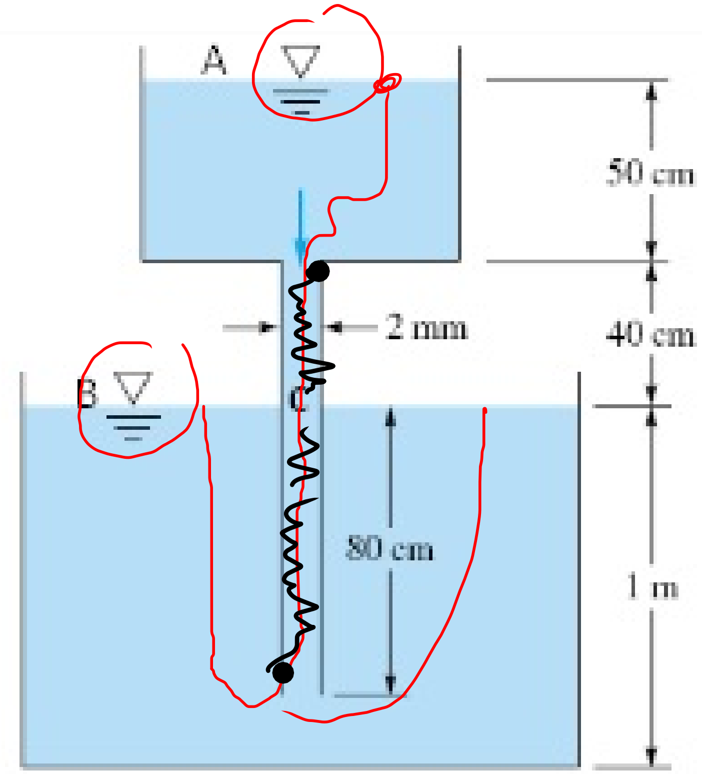
Q_V = التدفق الحجمي $\left[\frac{m^3}{s}\right] \rightarrow \left[\frac{L}{s}\right]$

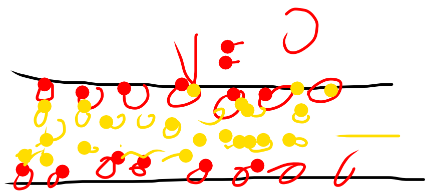
$1\,m^3 = 10^3\,L$

$\dot{Q} = Q_m = \dot{m}$ = التدفق الكتلي $\left[\frac{kg}{s}\right]$

$$Q_m = \rho \cdot Q_V = \frac{kg}{\cancel{m^3}} \cdot \frac{\cancel{m^3}}{s} = \frac{kg}{s}$$

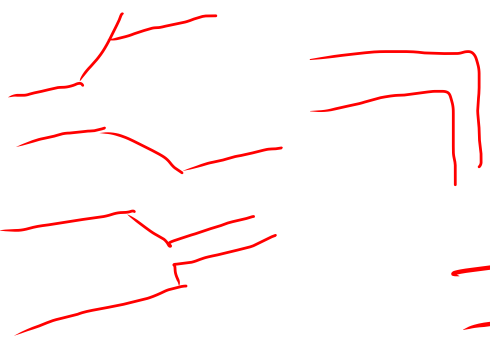
Exercice 06 : Soit l'écoulement laminaire d’Éthanol entre deux réservoirs très larges (voir figure 2). On suppose que les pertes de charge singulières est négligeable. 1-Calculer la vitesse d'écoulement. 2- Déduire le débit volumique 3- Calculer la valeur de la pression au point C.

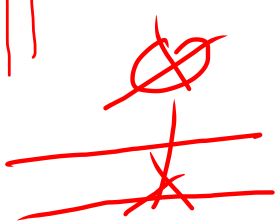




$$J_L = \lambda \frac{V^2}{2} \frac{L}{D}$$

$$J_S = \xi \frac{V^2}{2} \simeq 0$$





$$[PER]_A = [PER]_B + J_{AB}$$

$$\cancel{\frac{P_A}{\rho}} + \cancel{\frac{V_A^2}{2}} + g z_A = \cancel{\frac{P_B}{\rho}} + \cancel{\frac{V_B^2}{2}} + g z_B + \lambda \frac{V^2}{2} \frac{L}{D}$$

$$\begin{cases} P_A = P_B = P_{atm} \\ V_A = V_B << V \end{cases}$$

* L'écoulement laminaire

$$g z_A = g z_B + \lambda \frac{V^2}{2} \cdot \frac{L}{D} \rightarrow 1$$

$$\lambda = \frac{64}{Re} \rightarrow 2$$

$$Re = \frac{\rho V D}{\mu}$$

$$\lambda = \frac{64}{\frac{\rho V D}{\mu}} = \frac{\mu \cdot 64}{\rho V D}$$

$$g z_A = g z_B + \frac{\cancel{64}\,\nu}{\rho \cancel{v} D} \frac{\cancel{v^2}}{\cancel{2}} \frac{L}{D}$$

$$g(z_A - z_B) = \frac{32\,\nu}{\rho D^2} L v$$

$$v = \frac{\rho D^2 \cdot g(z_A - z_B)}{32 \cdot \nu \cdot L}$$

$\cdot A \nu \cdot$

$z_A - z_B = 90\,cm = 0{,}9\,m$

$L = 120\,cm = 1{,}2\,m$

2) $Q_V = S \cdot v = \frac{\pi d^2}{4} \cdot v \quad [m^3/s]$

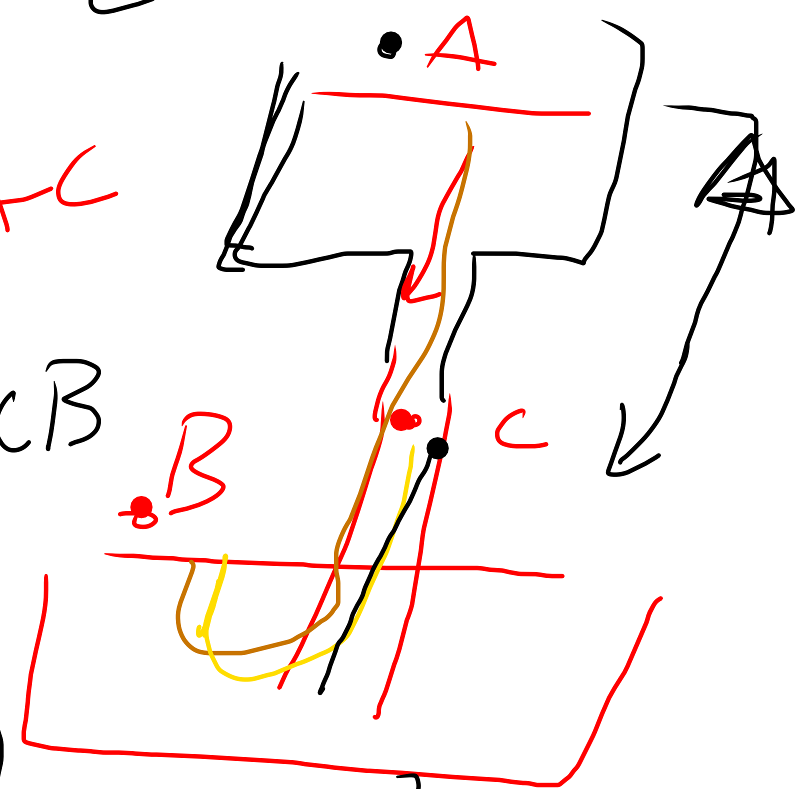
B) $[PER]_A = [RER]_C + J_{AC}$

$[PER]_C = [PER]_B + J_{CB}$

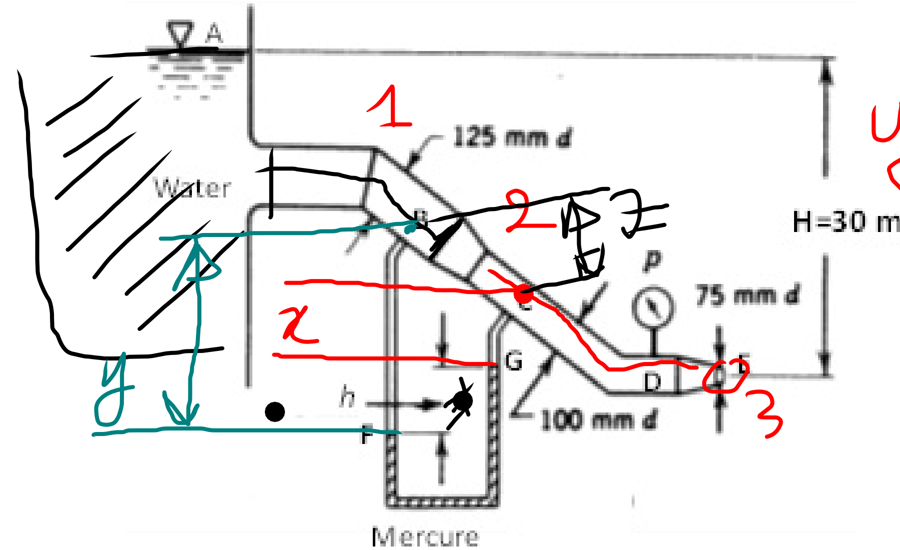
?

$\frac{P_A}{\rho} + \cancel{\frac{v_A^2}{2}}^{\,0} + g z_A = \frac{P_C}{\rho} + \frac{v_C^2}{2} + g z_C + \lambda \frac{v^2}{2} \frac{L}{D}$

$v_C = v$



Exercice 7 En analysant la figure suivante :
1-Calculer la vitesse d'écoulement dans chaque partie de la conduite. 2- En déduire le débit volumique. 3- Calculer la valeur de la pression au point D. 4- Déduire la dénivellation du manomètre h. On suppose la viscosité négligeable. ρHg=13600 Kg/m3



$y - x = h + z$

1) PER → A → E

LLVE

$$\cancel{\frac{P_A}{\rho}} + \cancel{\frac{V_A^2}{2}} + g z_A = \cancel{\frac{P_E}{\rho}} + \frac{V_E^2}{2} + g z_E$$

$$g z_A = \frac{V_E^2}{2} + g z_E$$

$$Q_1 = Q_2 = Q_3$$

$$V_1 S_1 = V_2 S_2 = V_3 S_3$$

$$V_1 \frac{\pi d_1^2}{4} = V_2 \frac{\pi d_2^2}{4} = V_3 \frac{\pi d_3^2}{4}$$

$$\frac{v_E^2}{2} = g(z_A - z_F) \Rightarrow v_E = \sqrt{2gH}$$

$$\Rightarrow v_1 \sim v_2$$

---

3) P_D : ? → PEP → A – D

D D → E

$$\frac{P_D}{\rho} + \frac{v_D^2}{2} + g\cancel{z_D} = \frac{P_E}{\rho} + \frac{v_E^2}{2} + g\cancel{z_E}$$

$\left\{ \begin{array}{l} z_E = z_D \\ P_D ? \\ P_E = P_{atm} \end{array} \right.$

$$\frac{P_D}{\rho} = \frac{P_E}{\rho} + \frac{V_E^2 - V_D^2}{2}$$

$$P_D = P_E + \frac{\rho}{2}\left(V_E^2 - V_D^2\right) \qquad \cdot A \cdot N$$

$$P_C + \rho g z_C = P_G + \rho g z_G \Rightarrow P_C = P_G + \rho_w g (z_G - z_C)$$

$$P_B + \rho g z_B = P_F + \rho g z_F \Rightarrow P_B = P_F + \rho_w g (z_F - z_B)$$

$$P_G + \rho_{me} g z_G = P_F + \rho_{me} g z_F$$

$$P_F = P_G + \rho_{Me} g (z_G - z_F) = P_G + \rho_{Me} g h$$

$$\begin{cases} P_F - P_G = \rho_{Me} g h \\ P_G = P_C + \rho_w g (\overbrace{z_C - z_G}^{x}) \\ P_F = P_B + \rho_w g (\overbrace{z_B - z_F}^{y}) \end{cases}$$

$$P_F - P_G = P_B - P_C + \rho_w g [y - x]$$

$$= P_B - P_C + \rho_w g [h + z]$$

$$\rho_{Me} g h = P_B - P_C + \rho_w g (h + z) \qquad \rho_w g \cancel{h} + \rho_w g \cancel{z}$$

$$\frac{P_B}{\rho_w} + \frac{V_B^2}{2} + g z_B = \frac{P_C}{\rho_w} + \frac{V_C^2}{2} + g z_C$$

$$\frac{P_B - P_C}{\rho_w} + g(z_B - z_C) = \frac{V_C^2 - V_B^2}{2}$$

$$P_B - P_C + \rho_w g (z) = \frac{\rho_w}{2}\left(V_C^2 - V_B^2\right)$$

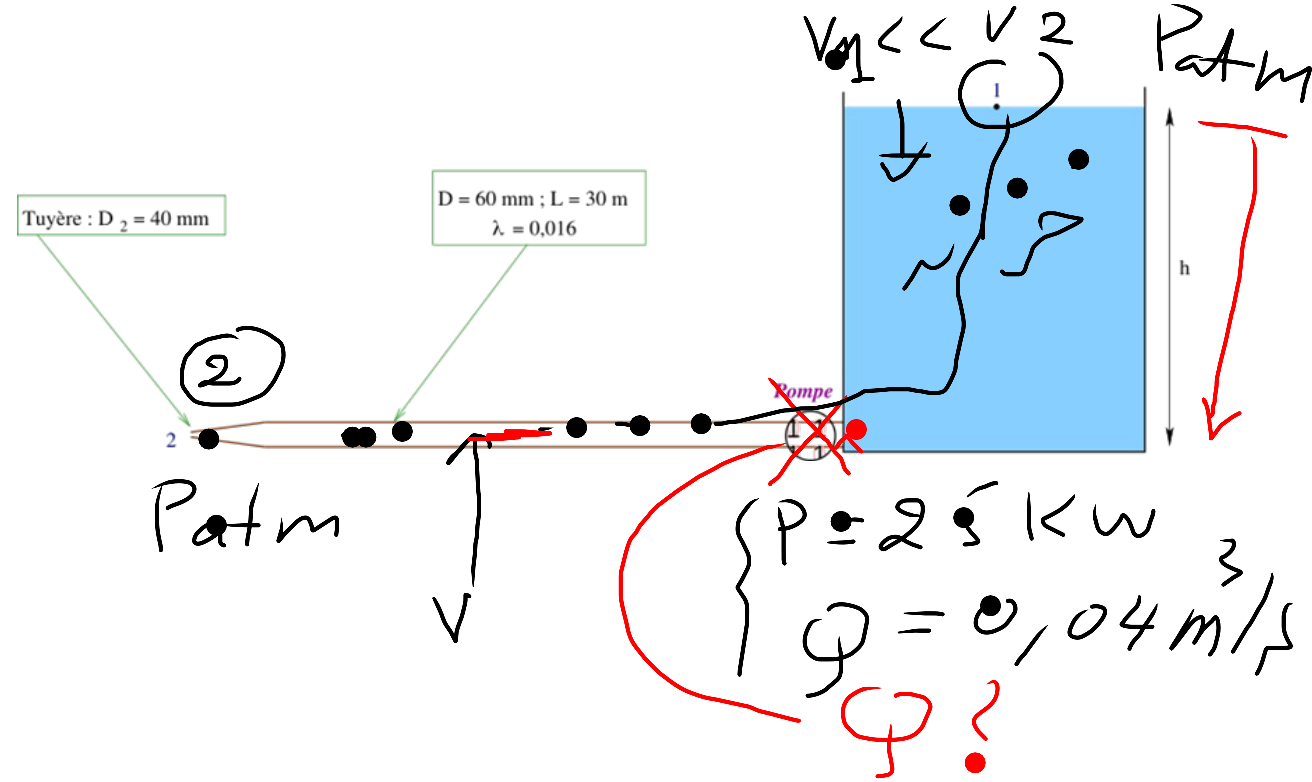
$$\rho_{Me} g h = \frac{\rho_w}{2}\left(V_C^2 - V_B^2\right) + \rho_w g h$$

$$h\left(\rho_{Me} g - \rho_w g\right) = \frac{\rho_w}{2}\left(V_C^2 - V_B^2\right)$$

$$h = \frac{\rho_w\left(V_C^2 - V_B^2\right)}{2g\left(\rho_{Me} - \rho_w\right)}, \quad A \cdot N$$

Exercice N 8

Une pompe de 25 kW permet de distribuer un débit d'eau de 0.04 m3/s depuis un grand réservoir à travers une conduite dont les caractéristiques sont montrées sur la figure ci-dessous. Déterminer le débit qui sera véhiculé si on enlève la pompe du système. On suppose que dans les deux cas (avec ou sans pompe) le coefficient de perte de charge linéaire est 0,016 et que les pertes de charge singulières sont négligeables dans tout le système.



1)* Sans pomp :

$$[PER]_1 = [PER]_2 + J_{12}$$

$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + g z_1 = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + g z_2 + \lambda \frac{V^2}{2} \frac{L}{D}$$

$$gz_1 = \frac{V_2^2}{2} + gz_2 + \lambda \frac{V^2}{2} \frac{L}{D}$$

$$\begin{cases} Q = Q_2 \\ V \cdot \frac{\cancel{\pi} D^2}{\cancel{4}} = V_2 \frac{\cancel{\pi} d^2}{\cancel{4}} \end{cases}$$

$$g(z_1 - z_2) = \frac{1}{2} V^2 \left(\frac{D}{d}\right)^4 + \lambda \frac{V^2}{2} \frac{L}{D}$$

$$gh = \frac{V^2}{2} \left[\left(\frac{D}{d}\right)^4 + \lambda \frac{L}{D} \right]$$

$$\left\{ \begin{array}{l} VD^2 = V_2 d^2 \\ V = V_2 \left(\frac{d}{D}\right)^2 \\ V_2 = V \left(\frac{D}{d}\right)^2 \end{array} \right.$$

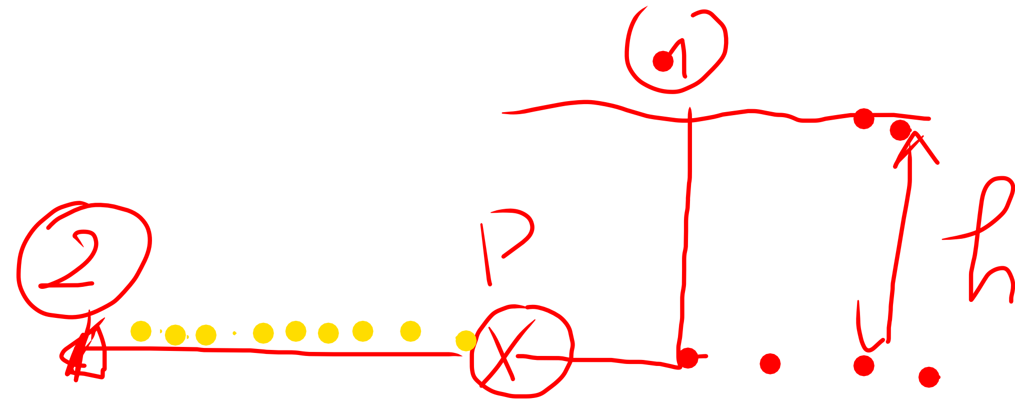
$$V = \sqrt{\frac{2gh}{\left[\left(\frac{D}{d}\right)^4 + \lambda \frac{L}{D} \right]}}$$

?

Sans pomp

$$\rightarrow Q = V \frac{\pi D^2}{4}$$

$$[PER]_1 + \frac{P}{Q_m} = [PER]_2 + J_{12}$$



$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + g z_1 + \frac{P}{Q_m} = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + g z_2 + \lambda \frac{V^2}{2} \frac{L}{D}$$

$$\begin{cases} P = 25\,kW = 25 \cdot 10^3\,W \\ Q_m = \rho \cdot Q_V = 10^3 \cdot 0{,}041 = 40\,\frac{kg}{s} \end{cases}$$

$$g z_1 + \frac{P}{Q_m} = \frac{V_2^2}{2} + g z_2 + \lambda \frac{V^2}{2} \frac{L}{D}$$

$$\frac{g(z_1 - z_2)}{h} = \frac{V_2^2}{2} + \lambda \frac{V^2}{2}\frac{L}{D} - \frac{P}{\rho m}, \quad \begin{cases} Q_V = 0{,}04\ m^3/s \\ Q_V = V \cdot \frac{\pi D^2}{4} \\ \quad = V_2 \frac{\pi}{4} d^2 \end{cases}$$

$$g h = \frac{1}{2} V^2 \left(\frac{D}{d}\right)^4 + \lambda \frac{V^2}{2}\frac{L}{D} - \frac{P}{\rho m}$$

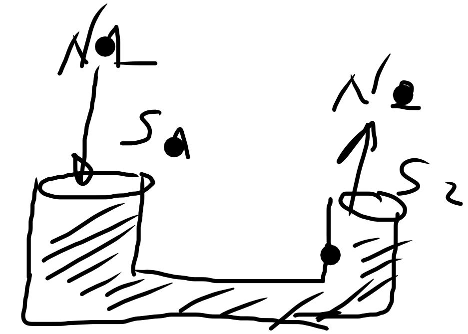
$$V_2 = V\left(\frac{D}{d}\right)^2$$

$$\underline{\underline{h}} = \frac{V^2}{2g}\left[\left(\frac{D}{d}\right)^4 + \lambda \frac{L}{D}\right] - \frac{P}{g \cdot \rho m}$$

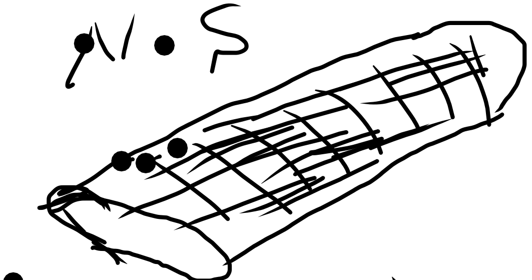
TD 2 Ecoulement des fluides visqueux
Navier-Stokes Equation

MDF

- Statique des fluides



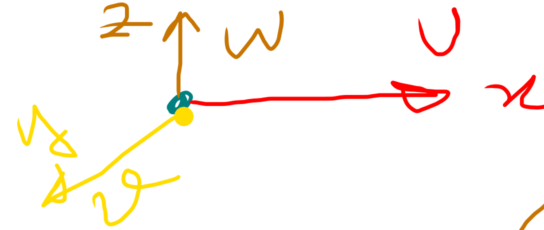
- Dynamique des fluides
  - parfait et réel
    BERNOULLI
  - visqueux
    N.S



- Cinématique des fluides
- Couche limite الطبقة الحدية
- Ecoulement Turbulent

* E.N.S : $\vec{q}\,(u, v, w)$

T. Pression



$\Delta = \nabla^2$

$$\frac{D\vec{q}}{Dt} = \vec{f} - \frac{1}{\rho}\vec{\nabla}.P + \frac{1}{\rho}\nabla\left[(\varepsilon+\nu)\nabla\vec{q}\right] + \partial\Delta\vec{q}$$

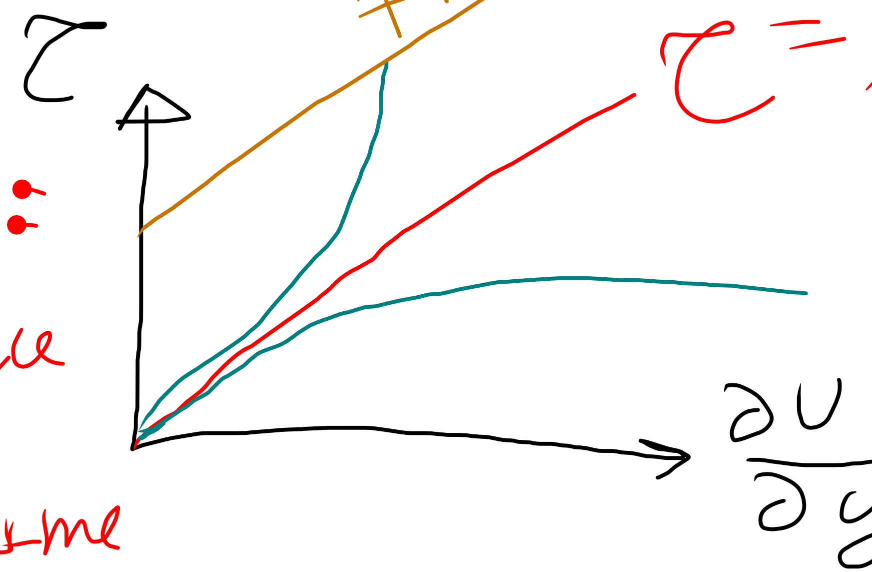
terme d'inertie convective

force volumique

* terme de diffusion

* dissipation

$$\tau = \mu\frac{\partial u}{\partial y}$$



* fluides Newtoniens :

* 2e loi de Newton

$$\frac{DI}{Dt} = \sum F$$

→ surface

→ volume

ε : coefficient de dilatation

$\varepsilon = f(N)$

Equation de continuité :

$$\frac{D\rho}{Dt} + \rho\, \underline{\underline{\nabla}} \cdot \vec{q} = 0$$

- ∇ = nabla
- $\vec{\nabla} \cdot T = \overrightarrow{grad}\, T$
- $\vec{\nabla} \cdot \vec{q} = div\, \vec{q}$

* Fluide Newtonien + incompressible ($\rho = cte$)

Pour un éc. incom. — l'eqn de continuité :

* $\nabla \cdot \vec{q} = div\, \vec{q} = 0$ (continuité)

* $$\frac{D\vec{q}}{Dt} = \vec{f} - \frac{1}{\rho} \vec{\nabla} \cdot P + \nu\, \Delta \cdot \vec{q} \quad (E.N.S)$$

• Selon l'axe $\underline{\underline{x}}$ = "U" | * Non-linear Term

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = f_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \quad (1)$$

• Selon l'axe y = "v"

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = f_y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \quad (2)$$

• selon l'axe "z" = "w"

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = f_z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] \quad (3)$$

l'équation de continuité :

pour un fluide incompressible :

$$\operatorname{div}\vec{q} = 0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

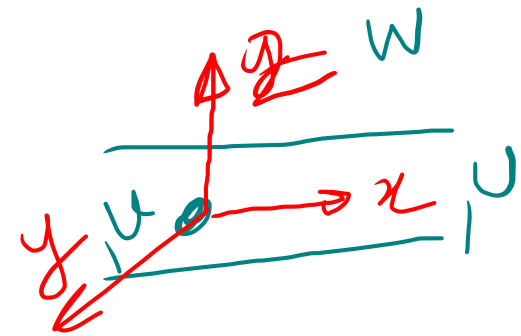
$\rightarrow$ (4)

* inconnus / 4 /

u, v, w, P

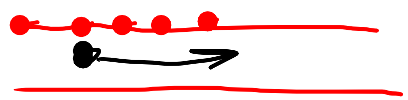
E. Navier-Stokes → Bilan quantité de Mouvement

$$\frac{D\vec{I}}{Dt} = \sum \vec{F}_{\rho} \begin{cases} x \\ y \\ z \end{cases}$$



E. BERNOULLI → E. N. S

S, N, V



P, T ... quantité moyenne

→ $\rho(t, x, y, z)$

$N(t, x, y, z)$

$\vec{q}(u, v, w) = f(t, x, y, z)$

Exercice 13 On considère un écoulement stationnaire, incompressible et visqueux d'un fluide newtonien entre deux plaques horizontales de longueurs l. La plaque inférieure est animée d'une vitesse constante U0. L'écoulement étant parallèle aux plaques de grandes étendues dans le plan xz (plan perpendiculaire à la feuille) et en négligeant les forces de pesanteurs:

1- Ecrire les équations de Navier-Stokes pour un écoulement incompressible et visqueux en coordonnées cartésiennes et définir chacun des termes présents dans ces équations.

2- Simplifier ces équations pour l'écoulement étudié ci-dessus. Justifier toutes vos simplifications.

3- Trouver le profil des vitesses de l'écoulement entre les plaques en fonction de μ, U0, l, d et la perte de charge ΔP (avec ΔP = P(x=0) - P(x=l) > 0).

4- Déterminer la vitesse maximale dans l'écoulement.

5- Déterminer les contraintes tangentielles aux parois inférieure et supérieure. τ

6- Déterminer le débit volumique passant entre les plaques (la largeur des plaques suivant la direction z étant égale à l'unité).

7- Déterminer la vitesse débitante de l'écoulement entre les plaques $\bar{U}$

8. le nombre de Reynolds.

9 . Coefficient de pertes de charge

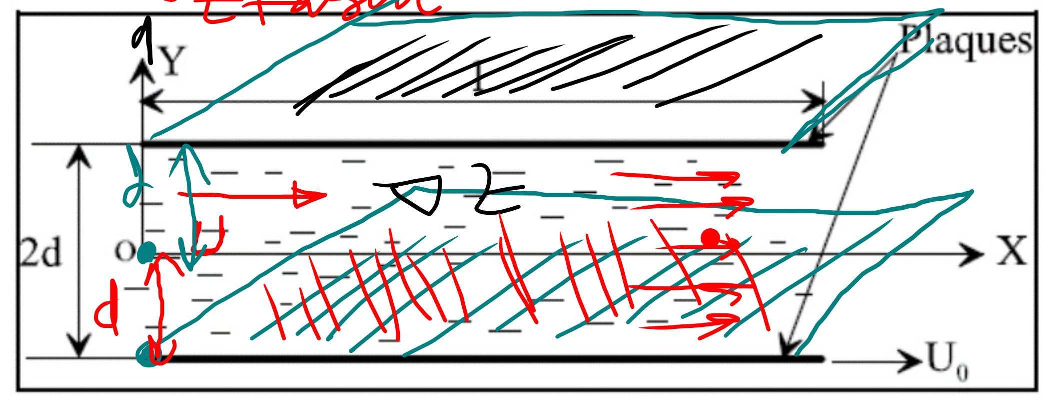
10. Coefficient de frottement à la paroi

11. Force de traînée sur la plaque

• stationnaire

• permanant $\frac{\partial}{\partial t} = 0$

• Établit



$\vec{f} = 0$

* incompressible ⇒ $\rho = cte$

* E. con. $\nabla \cdot \vec{v} = 0$

* grandes étendues : (xoz)

$\frac{\partial}{\partial z} = 0$ | $w = 0$

1) E.N.S $\Longrightarrow \dfrac{D\vec{q}}{Dt} = \vec{f} - \dfrac{1}{\rho}\vec{\nabla}\cdot p + \nu \Delta \vec{q}$

T Convective

T. Volume

Terme de pression

T. diffusion

* E. Con $\Longrightarrow \vec{\nabla}\cdot\vec{q} = \mathrm{div}\,\vec{q} = 0$

2) $* \quad \frac{\partial \cdot}{\partial t} = 0 \;,\; \frac{\partial \cdot}{\partial t} = 0 \;,\; v = 0 \;,\; w = 0$

$$x \Rightarrow \cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial x}} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}} = \cancel{f_x} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$y \Rightarrow \cancel{\frac{\partial v}{\partial t}} + \cancel{u\frac{\partial v}{\partial x}} + \cancel{v\frac{\partial v}{\partial y}} + \cancel{w\frac{\partial v}{\partial z}} = \cancel{f_y} - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\cancel{\frac{\partial^2 v}{\partial x^2}} + \cancel{\frac{\partial^2 v}{\partial y^2}} + \cancel{\frac{\partial^2 v}{\partial z^2}}\right)$$

$$z \Rightarrow \cancel{\frac{\partial w}{\partial t}} + \cancel{u\frac{\partial w}{\partial x}} + \cancel{v\frac{\partial w}{\partial y}} + \cancel{w\frac{\partial w}{\partial z}} = \cancel{f_z} - \cancel{\frac{1}{\rho}\frac{\partial p}{\partial z}} + \nu\left(\cancel{\frac{\partial^2 w}{\partial x^2}} + \cancel{\frac{\partial^2 w}{\partial y^2}} + \cancel{\frac{\partial^2 w}{\partial z^2}}\right)$$

continuity equation $\Rightarrow \frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial w}{\partial z}} = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$

$x \Rightarrow \quad 0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \longrightarrow *$

$y = \quad 0 = -\frac{1}{\rho}\frac{\partial P}{\partial y} \Rightarrow \frac{\partial P}{\partial y} = 0 \Rightarrow P = cte$

$\Rightarrow P = P(x)$

$z \Rightarrow \quad 0 = 0$

E. Continuity $\Rightarrow \frac{\partial u}{\partial x} = 0$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}=0 \Rightarrow \frac{\partial^2 u}{\partial y^2}=\frac{1}{\nu\cdot\rho}\frac{\partial p}{\partial x}$$

$$\Rightarrow \int\frac{\partial^2 u}{\partial y^2}=\frac{1}{\mu}\frac{\partial p}{\partial x}=\int A$$

$$\int\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)=\int A \Rightarrow \int\partial\left(\frac{\partial u}{\partial y}\right)=\int A\,\partial y$$

$$\int\frac{\partial u}{\partial y}=\int Ay+\int C_1$$

$$u=A\frac{y^2}{2}+C_1y+C_2$$

الشروط الحدية

$$\begin{cases} y = -d \Rightarrow U = U_0 \\ y = d \Rightarrow U = 0 \end{cases}$$

$$U = \frac{Ay^2}{2} + C_1 y + C_2$$

$$\begin{cases} U_0 = \frac{A}{2} d^2 - C_1 d + C_2 \\ + \quad\quad + \quad\quad + \\ 0 = \frac{A}{2} d^2 + C_1 d + C_2 \end{cases}$$

$$U_0 = A d^2 + 2C_2$$

$$C_2 = \frac{1}{2}(U_0 - Ad^2)$$

$$C_1 = \frac{1}{d}\left(-\frac{Ad^2}{2} - C_2\right) = \frac{1}{d}\left[-\cancel{\frac{Ad^2}{2}} - \frac{U_0}{2} + \cancel{\frac{Ad^2}{2}}\right]$$

$$C_1 = \frac{-U_0}{2d}$$

$$\boxed{\underset{\overset{\|}{0}}{u(d)} = \cancel{\frac{A}{2}d^2} - \cancel{\frac{U_0}{2\cancel{d}}}\cdot\cancel{d} + \cancel{\frac{1}{2}U_0} - \cancel{\frac{Ad^2}{2}}}$$

$$U = \frac{A}{2}y^2 - \frac{U_0}{2d}y + \frac{1}{2}\left[U_0 - Ad^2\right]$$

$$U(-d) = \cancel{\frac{Ad^2}{2}} + \frac{U_0}{2\cancel{d}}\cdot\cancel{d} + \frac{1}{2}U_0 - \cancel{\frac{Ad^2}{2}} = U_0$$

* 4) U_{max} ? $\quad y :$? $\rightarrow U : U_{max}$

$$u = \frac{A}{2} y^2 - \frac{U_0}{2d} y + \frac{1}{2}\left[U_0 - Ad^2\right]$$

$$* \left.\frac{\partial u}{\partial y}\right|_{y_0} = 0 \Rightarrow U(y_0) = U_{max}$$

$$\frac{\partial u}{\partial y} = Ay - \frac{U_0}{2d} = 0 \Rightarrow y = \frac{U_0}{2Ad} = y_0$$

$$U_{max} = \frac{A}{2}\left(\frac{U_0}{2Ad}\right)^2 - \frac{U_0}{2d}\left(\frac{U_0}{2Ad}\right) + \frac{1}{2}\left[U_0 - Ad^2\right] = -\frac{1}{8}\frac{U_0^2}{Ad^2} + \frac{1}{2}\left[U_0 - Ad^2\right]$$

4) $U_{max} = -\frac{\Lambda}{8}\frac{U_0^2}{Ad^2} + \frac{1}{2}\left[U_0 - Ad^2\right]$

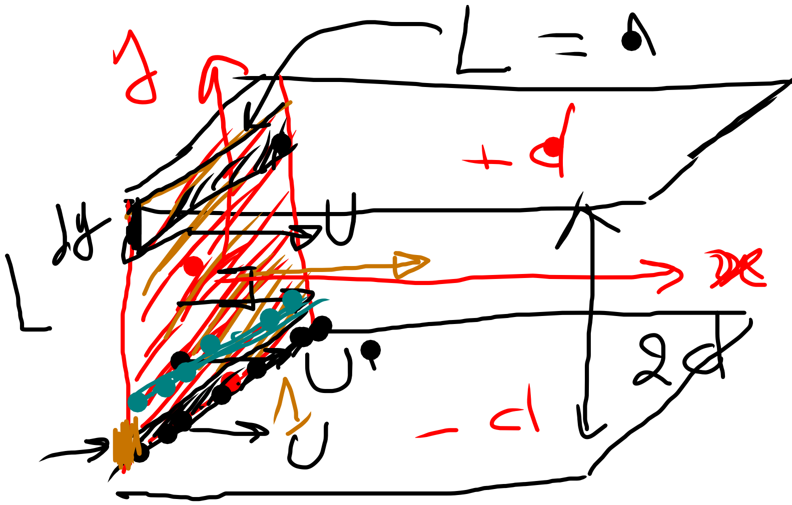
5) $\tau \begin{cases} y = d \\ y = -d \end{cases}$ // $\tau = \mu \frac{\partial U}{\partial y}$

$\tau = \mu\left[Ay - \frac{U_0}{2d}\right] \begin{cases} \tau(d) = \mu\left[Ad - \frac{U_0}{2d}\right] \\ \tau(-d) = \mu\left[-Ad - \frac{U_0}{2d}\right] \end{cases}$

b) φ_V ?

$U = f(y)$

$\varphi_V = \int_{-d}^{+d} U \cdot ds$, $ds = dy \cdot L$



$\varphi_V = S \cdot V$

$\varphi_V = \int_{-d}^{+d} U \cdot L\,dy$

$\varphi_V = L \int_{-d}^{+d} U\,dy$

$$\left\{\begin{array}{l} \varphi_{V_1} = ds \cdot U_1 \\ \varphi_{V_2} = ds \cdot U_2 \\ \vdots \\ \varphi_{V_n} = ds \cdot U_n \end{array}\right\} \quad \varphi_V = \varphi_{V_1} + \varphi_{V_2} + \dots + \varphi_{V_n}$$

$$Q_V = L\int_{-d}^{+d} u\,dy = L\int_{-d}^{+d}\left[\frac{Ay^2}{2} - \frac{U_0}{2d}y + \frac{1}{2}\left[U_0 - Ad^2\right]\right]dy$$

$$Q_V = L\left[\frac{Ay^3}{6} - \frac{U_0}{4d}y^2 + \frac{1}{2}\left[U_0 - Ad^2\right]y\right]_{-d}^{+d}$$

$$Q_V = \left\{\left[\frac{Ad^3}{6} - \frac{U_0}{\cancel{4d}}d^2 + \frac{1}{2}\left[U_0 - Ad^2\right]d\right] - \left[-\frac{Ad^3}{6} - \frac{U_0}{\cancel{4d}}d^2 - \frac{1}{2}\left[U_0 - Ad^2\right]d\right]\right\} =$$

6) $\varphi_v = \frac{Ad^3}{3} + U_0 d - Ad^3 = -\frac{2}{3}Ad^3 + U_0 d$

7) $\bar{U} = ?$ $\quad \varphi_V = L\int_{-d}^{+d} u\,dy = \bar{U}\cdot S = \bar{U}\cdot \underbrace{L\cdot 2d}_{=S}$

$\bar{U} = \frac{\varphi_v}{S} = \frac{1}{S} L \int_{-d}^{+d} u\,dy = \frac{1}{2d}\int_{-d}^{+d} u\,dy$

$\bar{U} = \frac{1}{2d}\left[-\frac{2Ad^3}{3} + U_0 d\right] = \frac{-Ad^2}{3} + \frac{U_0}{2}$

8) Re?

$$Re = \frac{\rho \bar{U} D_H}{\mu} = \frac{\bar{U} D_H}{\nu}$$

$$D_H = \frac{4S}{P}$$

$$S = \frac{\pi d^2}{4}$$

$$P = \pi d$$

$$D_H = \frac{\cancel{4} \cdot \frac{\cancel{\pi} d^2}{\cancel{4}}}{\cancel{\pi} d} = d \text{ ,}$$

$$\Rightarrow Re = \frac{\bar{U} D}{\nu}$$

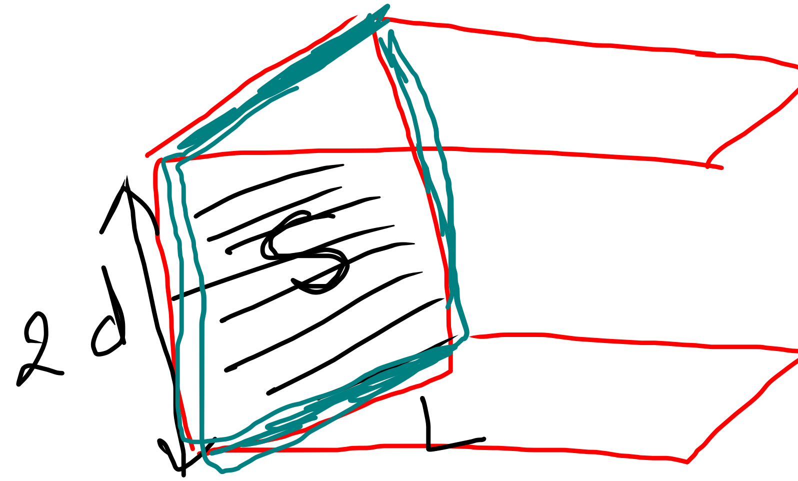
$$S = 2d \cdot L$$

$$P = 2L + 2(2d)$$

$$P = 2[L + 2d]$$

$$D_H = \frac{4 \cdot \cancel{2} d \cdot L}{\cancel{2}[L + 2d]} = \frac{4 d \cdot L}{L + 2d}$$

$$Re = \frac{\overline{U} D_H}{\nu} = \frac{1}{\nu}\left[-\frac{Ad^2}{3} + \frac{U_0}{2}\right]\left[\frac{4 \cdot d \cdot L}{L + 2d}\right]$$



9) Coefficient de perte de charge "λ"

$$J = \Delta P = \lambda \frac{1}{2} \bar{U}^2 \cdot \rho \Rightarrow \lambda = \frac{\Delta P}{\frac{1}{2} \bar{U}^2 \cdot \rho}$$

10) Coefficient de frottement = "C_f"

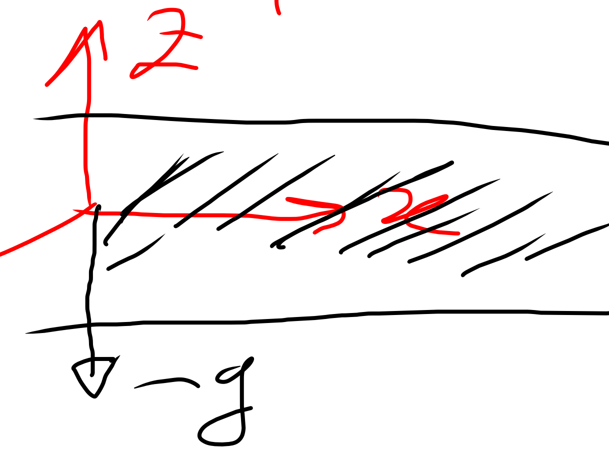
$$\tau_p = C_f \frac{1}{2} \bar{U}^2 \cdot \rho \Rightarrow C_f = \frac{\tau_p}{\frac{1}{2} \bar{U}^2 \cdot \rho}$$

11) Force de traînée (frottement) :

$$F_t = S \cdot \tau_p$$

Exo :
Le champ de vitesses d'un écoulement stationnaire et incompressible est donné par :
En considère la force volumique
$\vec{f}(f_x,f_y,f_z)=\vec{g}(g_x,g_y,g_z)=\vec{g}(0,0,-g)$
g:pesenteur:الجاذبية الارضية
Question : En utilisant les équations de Navier-stokes, trouver les valeurs les gradients de pression
$\partial P/\partial x$,
$\partial P/\partial y$ et
$\partial P/\partial z$

$$u = \delta(x^2 - y^2)$$
$$v = -\beta xy \equiv \vec{q}(u, v, w)$$
$$w = 0$$



$$\frac{D\vec{q}}{Dt} = \vec{f} - \frac{1}{\rho}\vec{\nabla} P + \nu \Delta \vec{q}$$

$$\frac{1}{\rho}\vec{\nabla} P = \vec{f} - \frac{D\vec{q}}{Dt} + \nu \Delta \vec{q}$$

$$\frac{\partial u}{\partial x} = 2\delta x$$
$$\frac{\partial u}{\partial y} = -2\delta y$$
...

$$\vec{\nabla} P = \rho \vec{f} - \rho \frac{D\vec{q}}{Dt} + \nu \Delta \vec{q}$$

$$\frac{\partial u}{\partial t} = 0 \qquad \frac{\partial u}{\partial x} = \cdots$$

$$x \Rightarrow \frac{\partial P}{\partial x} = \rho f_x - \rho \frac{Du}{Dt} + \nu \Delta u$$

$$y \Rightarrow \frac{\partial P}{\partial y} = \rho f_y - \rho \frac{Dv}{Dt} + \nu \Delta v$$

$$z \Rightarrow \frac{\partial P}{\partial z} = \rho f_z - \rho \frac{Dw}{Dt} + \nu \Delta w$$

$$\partial P / \partial x = f(x, y, z) = 2(x^2 - 2y) \cdots \cdots \cdots$$

TD 2 Ecoulement des fluides visqueux
Navier-Stokes Equation

$v = w = 0$

EXO

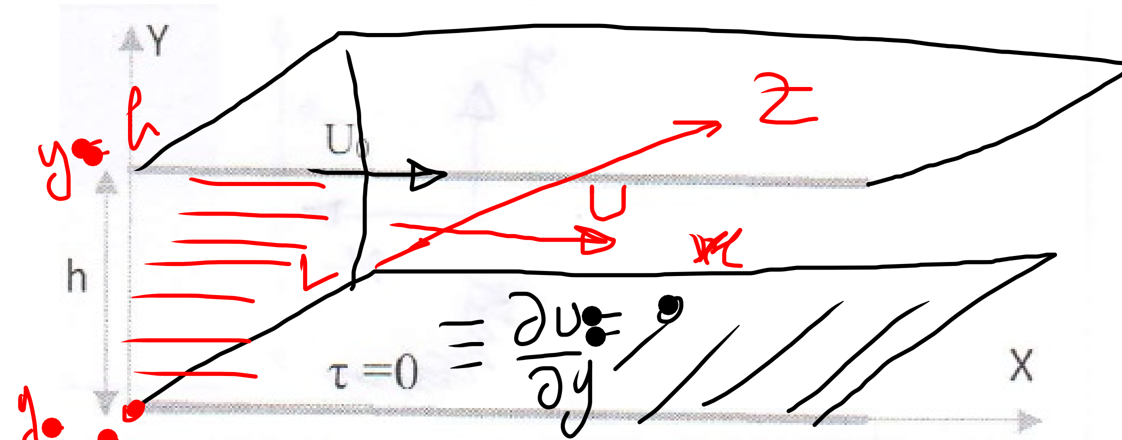
On considère un Ecoulement d'un fluide newtonien, incompressible, visqueux et permanant entre deux plaques horizontales de longueurs l. La plaque supérieur est animée d'une vitesse constante (Uo) . En plus, L'écoulement étant parallèle aux plaques de grandes largeur L dans le plan xoz (plan perpendiculaire à la feuille) et en négligeant les forces de pesanteurs:

$\partial / \partial z = 0$

1-déterminer l'expression du profil de vitesse. $\Rightarrow u$

2-Evaluer la vitesse débitante $\Rightarrow Q$, $\bar{U}$

3-Evaluer la vitesse maximale de l'écoulement $\Rightarrow U_{max}$



x E. continuity : $\rho = cte$

$$\operatorname{div} \vec{q} = 0 \Rightarrow \frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial w}{\partial z}} = 0$$

$$\frac{\partial u}{\partial x} = 0$$

$$\tau = \mu \cdot \frac{\partial u}{\partial y} = 0$$

$$\Rightarrow \frac{\partial u}{\partial y} = 0$$

N.S – selon l'axe $\underline{x}$

$$\cancel{\frac{\partial u}{\partial t}} + u\cancel{\frac{\partial u}{\partial x}} + \cancel{v}\frac{\partial u}{\partial y} + \cancel{w\frac{\partial u}{\partial z}} = \cancel{f_x} - \frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left[\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right]$$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \Rightarrow \int \frac{\partial^2 u}{\partial y^2} = \frac{1}{\rho \cdot \nu}\frac{\partial P}{\partial x} = A \Rightarrow$$

$$\Rightarrow \int \frac{\partial u}{\partial y} = \int Ay + C_1 \Rightarrow u(y) = \frac{Ay^2}{2} + C_1 y + C_2$$

C.L $\begin{cases} y = 0 \Rightarrow \tau = 0 \Rightarrow \tau = \mu\frac{\partial u}{\partial y} = 0 \Rightarrow \frac{\partial u}{\partial y} = 0 \\ y = h \Rightarrow u = u_0 \end{cases}$

$$u = \frac{A}{2}y^2 + C_1 y + C_2 \Rightarrow \frac{\partial u}{\partial y} = Ay + C_1$$

$$y = 0 \Rightarrow \frac{\partial u}{\partial y} = 0 = C_1 \Rightarrow C_1 = 0$$

$$y = h \Rightarrow u = u_0 = \frac{A}{2}h^2 + \cancelto{0}{C_1}\,h + C_2$$

$$C_2 = u_0 - \frac{A}{2}h^2$$

$$u = \frac{A}{2}y^2 + U_0 - \frac{A}{2}h^2$$

$$y = 0 \Rightarrow \frac{\partial u}{\partial y} = 0$$

$$y = h \Rightarrow U = U_0$$

1) $$U = \frac{A}{2}\left[y^2 - h^2\right] + U_0$$

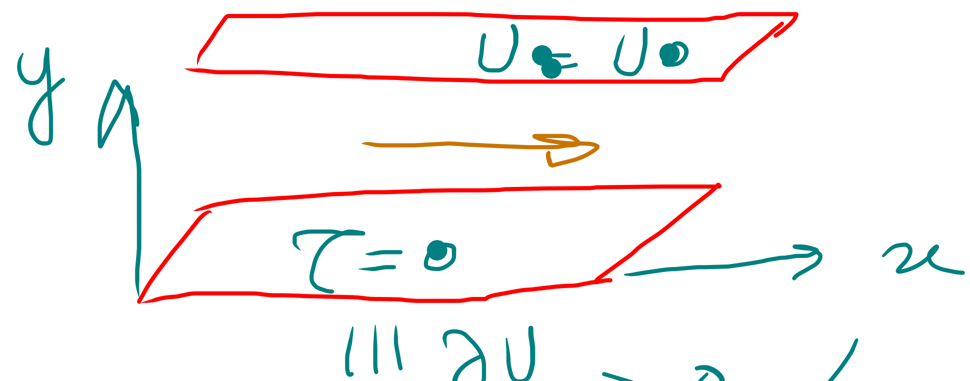
2) $\bar{U}$! $\Rightarrow$ $$Q_V = \bar{U} \cdot S = L\int_0^h u\,dy \Rightarrow \bar{U} = \frac{L}{S}\int_0^h u\,dy$$

$$\bar{U} = \frac{\cancel{L}}{\cancel{L} \cdot h}\int_0^h \left[\frac{A}{2}y^2 - \frac{A}{2}h^2 + U_0\right]dy = \frac{1}{h}\left[\frac{A}{6}y^3 - \frac{A}{2}h^2 y + U_0 y\right]_0^h$$

$$\bar{U} = \frac{1}{h}\left[\frac{A}{6}h^3 - \frac{A}{2}h^3 + U_0 h\right] = \frac{1}{h}\left[-\frac{1}{3}Ah^3 + U_0 h\right] = -\frac{1}{3}Ah^2 + U_0$$

3) U_{max}? $\Rightarrow$ $y_0 \Rightarrow U_{max}$

$U_m = U(y_0)$



$$\frac{\partial U}{\partial y} = Ay = 0 \Rightarrow y = 0$$

$$\frac{\partial U}{\partial y} = 0 \, / \, y_0 = 0$$

$$U_{max} = U(y=0) = U_0 - \frac{A}{2} h^2$$

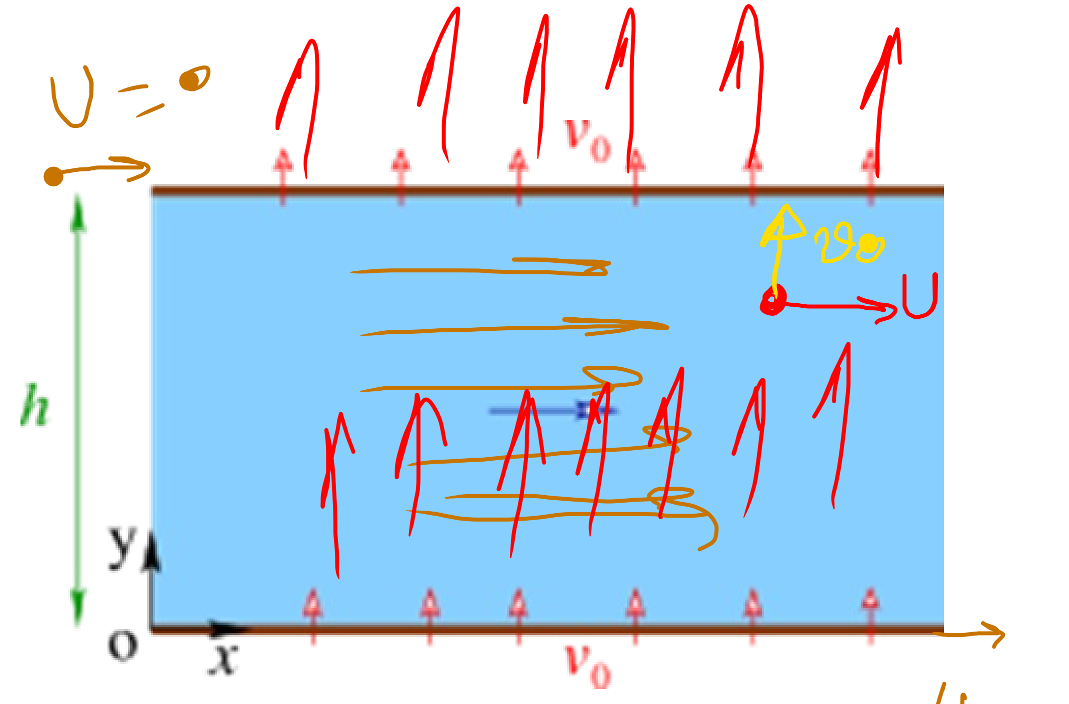
$$A = \frac{1}{\mu} \frac{\partial P}{\partial x}$$ — negative $(-)$

$$\frac{\partial P}{\partial x} = \frac{\overbrace{P(L) - P(0)}^{(-)}}{L}$$

$\frac{\partial}{\partial z} = 0$

EXO

On considère un écoulement stationnaire d'un fluide incompressible et visqueux entre deux plaques planes horizontales et parallèle aux plaques de grandes étendues dans le plan (xoz). Le même fluide est injecté à travers la plaque inférieure avec une vitesse uniforme vo perpendiculairement à la plaque. Le fluide est soutiré le long de la plaque supérieure avec la même vitesse uniforme vo perpendiculairement à cette dernière (voir figure). On suppose que la composante transversale vo de la vitesse est uniforme dans tout l'écoulement et que les forces de pesanteur est négligeables. Trouver l'expression du profil des vitesses de cet écoulement.



$$\begin{cases} U = U(y) \\ v = v_0 = cte \\ W = 0 \end{cases}$$

$\frac{\partial}{\partial x} = 0 \qquad \frac{\partial}{\partial t} = 0$

E. continuity $\Rightarrow$ f. in com $\Rightarrow$ $\dfrac{\partial u}{\partial x} + \cancel{\dfrac{\partial v}{\partial y}} + \cancel{\dfrac{\partial w}{\partial z}} = 0$

$$\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial}{\partial z} = 0 \quad (x \cdot z)$$

N.S $\Rightarrow$

$$v = v_0$$

$$\cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial x}} + v\frac{\partial u}{\partial y} + \cancel{w\frac{\partial u}{\partial z}} = \cancel{f_x} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$v_0\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\nu}{\nu}\frac{\partial^2 U}{\partial y^2} - \frac{v_0}{\nu}\frac{\partial U}{\partial y} = \frac{1}{\nu \cdot \rho}\frac{\partial P}{\partial x} \quad \left\{ \begin{array}{l} \dfrac{v_0}{\nu} = m \\ \dfrac{1}{\nu \cdot \rho} = \dfrac{1}{\mu} \end{array} \right.$$

$$\frac{\partial^2 U}{\partial y^2} - m\frac{\partial U}{\partial y} = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

EDP – 2ème ordre – non – homogène

$$U_{gen} = U_{hom} + U_{part}$$

$U_{homogène} \Rightarrow \dfrac{\partial^2 U}{\partial y^2} - m\dfrac{\partial U}{\partial y} = 0$

$$\circledast\ U = e^{ry}$$

$$r^2 e^{ry} - m r e^{ry} = 0 \Rightarrow$$

$$* \ \frac{\partial U}{\partial y} = r e^{ry}$$

$$\Rightarrow e^{ry}\left[r^2 - m r\right] = 0$$

$$r^2 - mr = 0 \Rightarrow \underbrace{r}_{=0}\underbrace{[r - m]}_{=0} = 0 \quad * \ \frac{\partial^2 U}{\partial y^2} = r^2 e^{ry}$$

$$\begin{cases} r = 0 \\ r = m \end{cases} \Rightarrow U_{hom} = C_1 e^{my} + C_2 e^{0 \cdot y}$$

$$* \; U_{hom} = C_1 e^{my} + C_2$$

$$* \; U_{part} = C_1(y) e^{my} + C_2(y)$$

La Méthode des variable constante

$$\begin{cases} C_1'(y) e^{my} + C_2'(y) = 0 \\ C_1'(y)(e^{my})' + C_2'(y)(1)' = \frac{1}{\nu}\frac{\partial P}{\partial x} \end{cases}$$

$$m \cdot C_1'(y) e^{my} = \frac{1}{\nu}\frac{\partial P}{\partial x}$$

$$\int C_1'(y) = \frac{1}{m\, e^{my} \cdot \nu} \frac{\partial P}{\partial x} = \int \left(\frac{1}{m \cdot \nu} \frac{\partial P}{\partial x}\right) e^{-my}$$

$$\int \frac{-m\, e^{-my}}{-m}\, dy$$

$$C_1(y) = \left[\frac{1}{m \cdot \nu} \frac{\partial P}{\partial x}\right] \cdot \frac{-1}{m} e^{-my}$$

$$\frac{-1}{m} e^{-my}$$

$$C_1(y) = -\frac{1}{m^2 \nu} \frac{\partial P}{\partial x} e^{-my}$$

$$C_2'(y) = C_1'(y) \cdot e^{my} = -\frac{1}{m^2 \nu} \frac{\partial P}{\partial x} \left(e^{-my+my}\right)$$

$$\int C_2'(y) = \int -\frac{1}{m^2 N}\frac{\partial P}{\partial x} \Rightarrow C_2(y) = -\frac{1}{m^2 N}\frac{\partial P}{\partial x} y$$

$$U_{pa} = -\frac{1}{m^2 N}\frac{\partial P}{\partial x} e^{-my} \cdot e^{my} - \frac{1}{m^2 N}\frac{\partial P}{\partial x} y$$

$$U_{pa} = -\frac{1}{m^2 N}\frac{\partial P}{\partial x} - \frac{1}{m^2 N}\frac{\partial P}{\partial x} y$$

$$U_{pa} = -\frac{1}{m^2 N}\frac{\partial P}{\partial x}\left[1 + y\right]$$

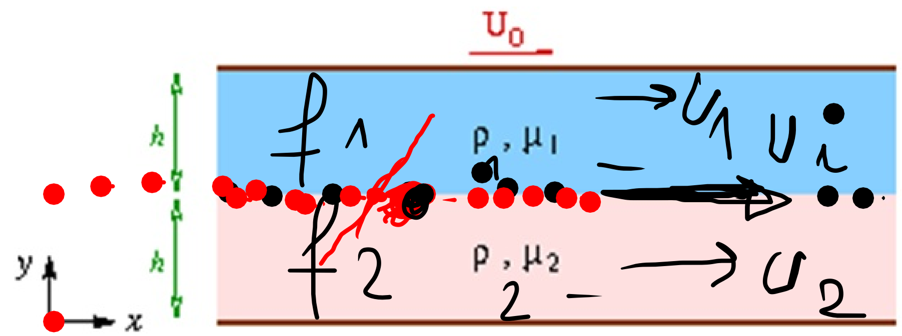
$$U_{gen} = U_{hom} + U_{par}$$

$$U_{gen} = C_1 e^{my} + C_2 - \frac{1}{\nu m^2} \frac{\partial P}{\partial x} \left[1 + y \right]$$

U/G

Exercice

On considère un écoulement stationnaire, incompressible et visqueux de deux fluides newtoniens et immiscibles entre deux plaques horizontales. La plaque supérieure est animée d'une vitesse constante Uo. L'écoulement s'effectuant avec un gradient de pression étant parallèle aux plaques de grandes étendues dans le plan (xoz). En négligeant les forces de pesanteur et en considérant la continuité des vitesses et des contraintes à l'interface. Déterminer la vitesse Ui à l'interface des deux fluides en fonction de N.B : Toutes les simplifications doivent être justifiées.



$\mu_1,\ \mu_2,\ h,\ U_0$ et dp/dx.

$* \ \frac{\partial}{\partial x} = 0, \ (xoz) \Rightarrow \frac{\partial}{\partial z} = 0, \ f = 0$

N.S $\rightarrow$ fluide ①

N.S $\rightarrow$ fluide ②

$$* \ \cancel{\frac{\partial u_1}{\partial t}} + \cancel{u_1 \frac{\partial u_1}{\partial x}} + \cancel{v_1 \frac{\partial u_1}{\partial y}} + \cancel{w_1 \frac{\partial u_1}{\partial z}} = \cancel{f_{x_1}} - \frac{1}{\rho_1}\frac{\partial P}{\partial x} +$$

$$+ \nu_1 \left(\cancel{\frac{\partial^2 u_1}{\partial x^2}} + \frac{\partial^2 u_1}{\partial y^2} + \cancel{\frac{\partial^2 u_1}{\partial z^2}} \right)$$

$$* \ \frac{\partial u_1}{\partial x} + \cancel{\frac{\partial v_1}{\partial y}} + \cancel{\frac{\partial w_1}{\partial z}} = 0$$

$$\nu_1 \frac{\partial^2 u_1}{\partial x^2} - \frac{1}{\rho_1}\frac{\partial P}{\partial x} = 0 \Rightarrow \frac{\partial^2 u_1}{\partial x^2} = \frac{1}{\mu_1}\frac{\partial P}{\partial x}$$

$$* \; U_1 = \frac{A_1 y^2}{2} + C_1 y + C_2$$

$$* \; U_2 = \frac{A_2 y^2}{2} + C_3 y + C_4$$

$\boxed{y = h}$

$U_1 = U_2 = U_i$

$y = h \begin{cases} \tau \\ U \end{cases}$

$$\begin{cases} y = 0 \Rightarrow U_2 = 0 \quad (1) \\ y = 2h \Rightarrow U_1 = U_0 \quad (2) \end{cases}$$

$\tau_1 = \tau_2 \quad (3)$

$U_1 = U_2 = U_i \quad (4)$

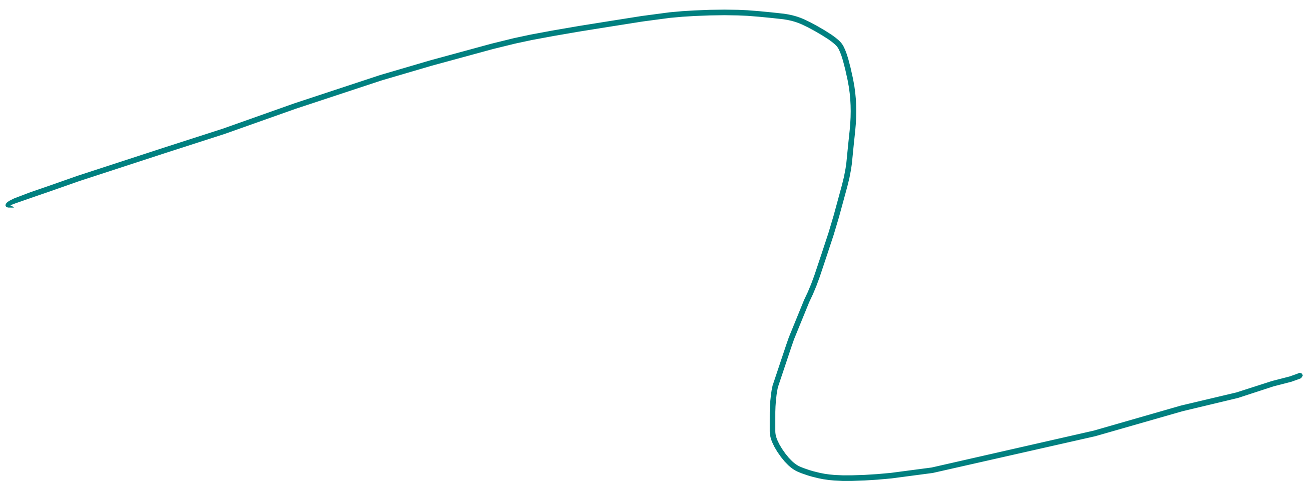
$$\tau_1 = \tau_2 \Rightarrow \mu_1 \frac{\partial U_1}{\partial y}\Big|_{y=h} = \mu_2 \frac{\partial U_2}{\partial y}\Big|_{y=h} \vdots$$

$$\left.\begin{aligned} \frac{\partial U_1}{\partial y} &= A_1 y + C_1 \\ \frac{\partial U_2}{\partial y} &= A_2 y + C_3 \end{aligned}\right\} \mu_1 [A_1 y + C_1] = \mu_2 [A_2 y + C_3]$$

$$U_1(y=h) = U_2(y=h)$$

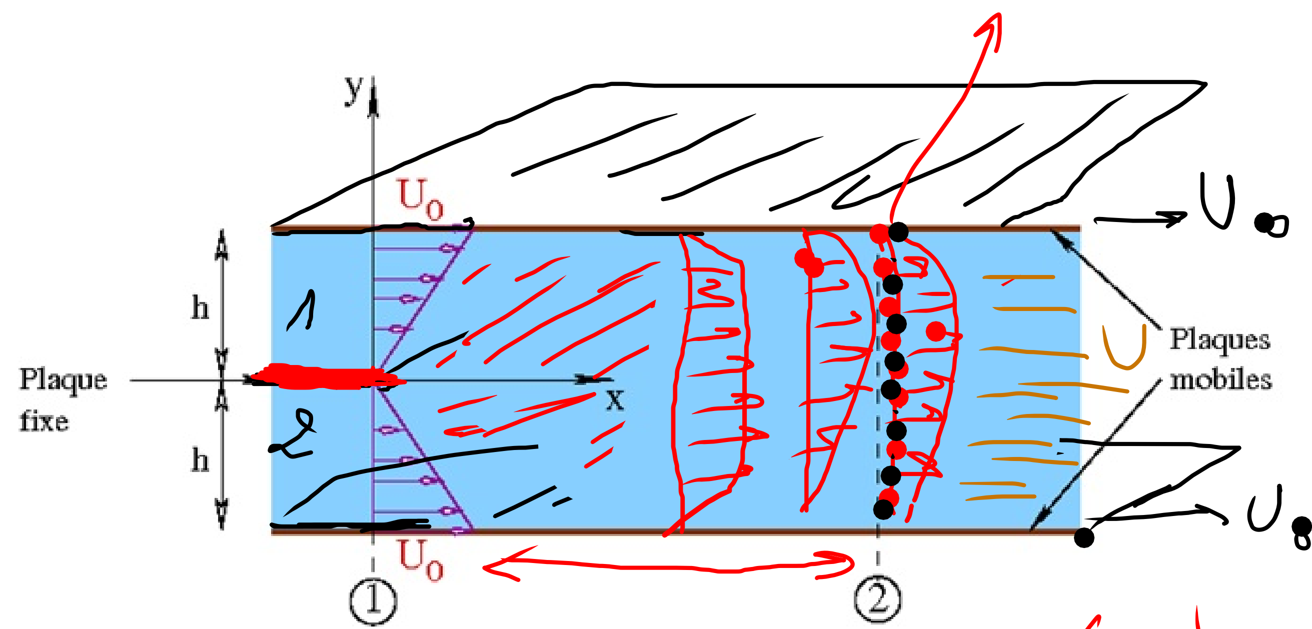
$$\frac{A_1}{2} h^2 + C_1 h + C_2 = \frac{A_2}{2} h^2 + C_3 h + C_4$$

C_1, C_2, C_3, et C_4



EXO
On considère un écoulement stationnaire d'un fluide newtonien, incompressible et visqueux (μ = Cte) entre deux plaques planes horizontales de grande largeur L dans le plan (xoz) et animées d'une vitesse uniforme U_0. Une plaque très fine est placée au milieu de ce canal et l'écoulement est parallèle aux plaques. Les deux parties du fluide séparées par la plaque fine se rejoignent à l'extrémité de cette dernière à la section (1). L'écoulement est dû à un gradient de pression que l'on déterminera à postériori. On demande de déterminer complètement le profil de vitesse supposé établi à partir de la section (2).

Etablie



$\rightarrow U(y)$

$* \ \dfrac{\partial}{\partial t} = 0$

* $U(y) \rightarrow (2)$

$$\cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial x}} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}} = \cancel{f_x} - \frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

* Continuité $\Rightarrow \dfrac{\partial u}{\partial x} = 0$

$$\rightarrow 0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\nu}\underbrace{\frac{\partial P}{\partial x}}_{\neq 0} \rightarrow$$

$$\rightarrow U(y) = \frac{Ay^2}{2} + C_1 y + C_2 \quad \begin{cases} y = -h \Rightarrow U = U_0 \\ y = h \Rightarrow U = U_0 \end{cases}$$

$$\begin{cases} U(-h) = U_0 = \dfrac{Ah^2}{2} - C_1 h + C_2 \quad * \; (1) \\ \quad + \qquad\qquad\qquad + \\ U(h) = U_0 = \dfrac{Ah^2}{2} + C_1 h + C_2 \quad * \; (2) \end{cases}$$

$$2U_0 = Ah^2 + 2C_2 \Rightarrow C_2 = U_0 - \frac{1}{2}Ah^2$$

$$(2) \Rightarrow C_1 = \left[\cancel{U_0} - \frac{Ah^2}{2} - \cancel{U_0} + \cancel{\frac{1}{2}Ah^2}\right] \cdot \frac{1}{h}$$

$$C_1 = 0$$

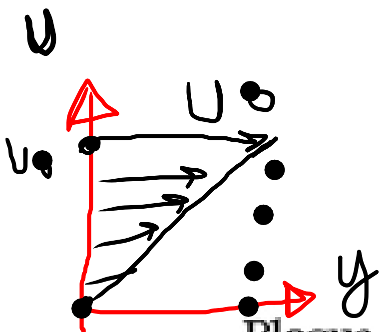
$$U_2(y) = \frac{Ay^2}{2} + U_0 - \frac{1}{2}Ah^2 \quad \begin{cases} U(-h) = U_0 \\ U(h) = U_0 \end{cases}$$

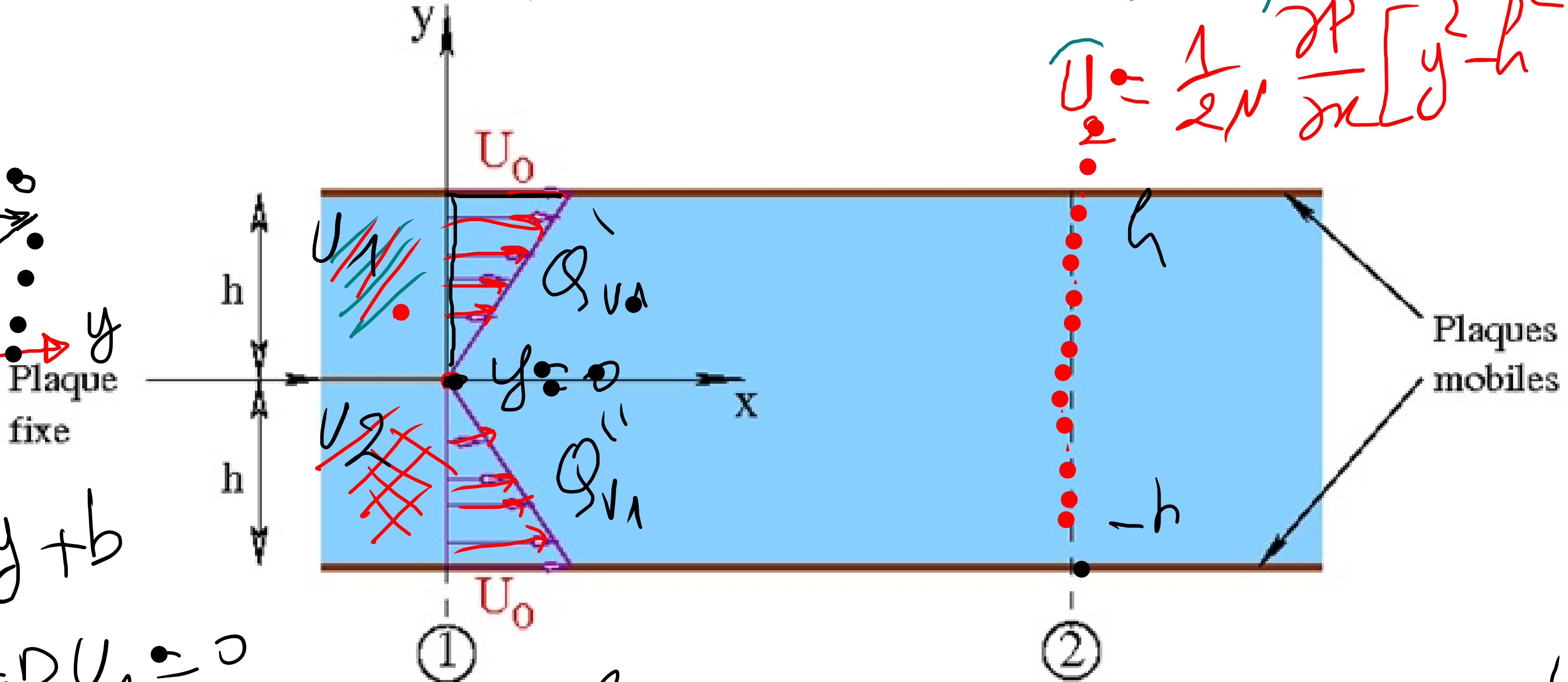
$$U_2(y) = \frac{A}{2}\left[y^2 - h^2\right] + U_0$$

$$U_2(y) = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left[y^2 - h^2\right] + U_0$$

$$Q_{v1} = Q_{v2}$$

$$U_2 = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left[y^2 - h^2\right] + U_0$$





$$\left\{\begin{array}{l} U_1 = ay + b \\ y = 0 \Rightarrow U_1 = 0 \\ y = h \Rightarrow U_1 = U_0 = ah \Rightarrow a = \frac{U_0}{h} \end{array}\right\} \Rightarrow U_1 = \frac{U_0}{h}\, y$$

$$* \; y = 0 \Rightarrow U_2 = 0,\; y = -h \Rightarrow U_2 = U_0 \Rightarrow U_2 = -\frac{U_0}{h}\, y$$

$$Q_{v_1} = Q'_{v_1} + Q''_{v_1} = L\int_0^h u_1\, dy + L\int_{-h}^0 u_2\, dy$$

$$Q_{v_1} = L\int_0^h \frac{U_0}{h} y\, dy + L\int_{-h}^0 -\frac{U_0}{h} y\, dy =$$

$$= L\left(\left[\frac{U_0}{2h} y^2\right]_0^h - \left[\frac{U_0}{2h} y^2\right]_{-h}^0\right) = \frac{L U_0}{2h}\left[2h^2\right]$$

$$(h^2) \qquad +(h^2)$$

$$= L U_0 h$$

$$Q_{v_2} = L\int_{-h}^{h} U\,dy = L\int_{-h}^{h}\left(\frac{A}{2}y^2 - \frac{A}{2}h^2 + U_0\right)dy$$

$$Q_{v_2} = L\left[\frac{Ay^3}{6} - \frac{A}{2}h^2 y + U_0 y\right]_{-h}^{h} =$$

$$Q_{v_2} = L\left(\left[\frac{Ah^3}{6} - \frac{A}{2}h^3 + U_0 h\right] - \left(-\frac{Ah^3}{6} + \frac{A}{2}h^3 - U_0 h\right)\right) =$$

$$= L\left(\frac{Ah^3}{3} - Ah^3 + 2U_0 h\right)$$

$$Q_{v_2} = L\left[A\left[\frac{h^3}{3} - h^3\right] + 2U_0 h\right] = L\left(-\frac{2}{3}Ah^3 + 2U_0 h\right)$$

$$* \; Q_{v_1} = Q_{v_2} \Rightarrow \cancel{L} U_0 \cancel{h} = \cancel{L}\left(-\frac{2}{3}A\cancel{h}^{3} + 2U_0 \cancel{h}\right)$$

$$U_0 = -\frac{2}{3}Ah^2 + 2U_0 \Rightarrow -\frac{2}{3}Ah^2 = -U_0$$

$$\Rightarrow A = \frac{3U_0}{2h^2} = \frac{1}{\mu}\frac{\partial P}{\partial x} \Rightarrow \boxed{\frac{\partial P}{\partial x} = \frac{3U_0\mu}{2h^2}}$$

$$U_2 = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left[y^2 - h^2\right] + U_0$$

$$U_2 = \frac{1}{2\cancel{\mu}}\left[\frac{3U_0\cancel{\mu}}{2h^2}\right]\left[y^2 - h^2\right] + U_0$$

$$U_2 = \frac{3U_0}{4h^2}\left[y^2 - h^2\right] + U_0$$

بالتوفيق

EXO : Ecoulemnt Potentielle

L'expression de la composante horizontale de vitesse est donnée par u=x(x+1)-y^2.

1. Trouvez la fonction potentielle sachant que l'écoulement est bidimensionnel, incompressible et permanent. Le point O(0,0) représente un point d' arrêt.
2. Evaluez le potentiel ∅ aux points O(0,0),B(2,3)et déduire Δ∅_(O-B) .
3. Retrouvez le résultat de la deuxième question en utilisant la méthode intégrale basée sur le produit scalaire de la vitesse et de l'élément différentiel de déplacement dr : ∫q⃗.(dr)⃗. _(O-B)

E. potentielle : ϕ

السريان

الجريان لا يقوم بحركة دورانية حول نفسه

$*$ Math :⇒ $\vec{Rot}\,\vec{q} = 0$

$\phi \Rightarrow \begin{cases} \vec{q} = \vec{\nabla}\phi \\ \vec{q}(U, V, W) = \end{cases}$

$U = \dfrac{\partial \phi}{\partial x}$

$V = \dfrac{\partial \phi}{\partial y}$

$W = \dfrac{\partial \phi}{\partial z}$

E. Continuité :

$\dfrac{D\rho}{Dt} + \rho\,\vec{\nabla}\cdot\vec{q} = 0$. incompressible ⇒

Si E. incompressible $\vec{\nabla}\cdot\vec{q} = 0$; div $\vec{q} = 0$

أي جريان من أجل أن يكون فيزيائيا ممكن يجب أن يحقق معادلة الاستمرارية

1) Φ ?, E.2D $\overset{y}{\uparrow}\!\rightarrow x$, incompressible, permanent, $O(0,0)$ pa

$\Phi \to U,\ v$

$\begin{cases} w = 0 \\ \frac{\partial}{\partial z} = 0 \end{cases}$

$\rho = cte$

$\operatorname{div}\vec{q} = 0$

$\begin{cases} \frac{\partial U}{\partial x} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}} = 0 \\ \frac{\partial U}{\partial x} + \frac{\partial v}{\partial y} = 0 \end{cases}$

$\frac{\partial}{\partial z} = 0$

$\begin{cases} U(O) = 0 \\ v(O) = 0 \end{cases}$

* $U = x(x+1) - y^2$
* $v = ?$ E.C

$$\frac{\partial v}{\partial y} = -\frac{\partial U}{\partial x} = -(2x+1) \Rightarrow$$

$$\int \partial v = \int -(2x+1)\,dy$$

$$v = -(2x+1)y + B(x)$$

$$v(x,y) = v(0,0) = 0 = -(2(0)+1)(0) + B(x)$$

$$\Rightarrow B(x) = 0$$

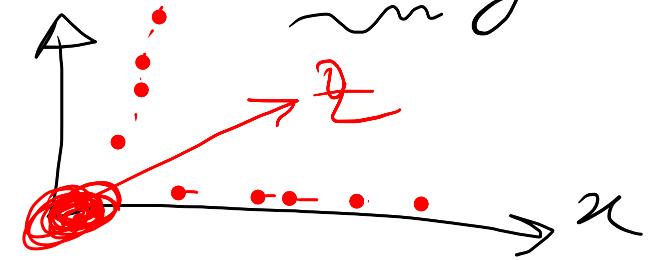
$$v(x,y) = -(2x+1)y$$

$$\Phi \Rightarrow \operatorname{Rot}\vec{q} = 0$$

$$\mathrm{Rot}\,\vec{q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

2D

$$2D \rightarrow \mathrm{Rot}\,\vec{q} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} = 0$$



$$* \quad \frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left[-(2x+1)y\right] = -\left[2y\right]$$

$$* \quad \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[x(x+1) - y^2\right] = -2y$$

$$\left.\right\} \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\mathrm{Rot}\,\vec{q} = 0 \Rightarrow \exists\, \phi$$

$$\vec{q} = \vec{\nabla}\cdot\phi \quad \begin{cases} u = \dfrac{\partial\phi}{\partial x} \\ v = \dfrac{\partial\phi}{\partial y} \end{cases} \Rightarrow \begin{cases} \int \partial\phi = \int u\, \partial x \\ \phi = \int u\, \partial x \end{cases}$$

$$\phi = \int (x^2 + x - y^2)\, dx = \left(\frac{x^3}{3} + \frac{x^2}{2} - y^2 x\right) + \lambda(y)$$

$$v = \frac{\partial\phi}{\partial y} = -(2x+1)y = -2yx + \lambda'(y)$$

$$\Rightarrow \lambda'(y) = \frac{\partial\lambda}{\partial y} = -2xy - y + 2yx = -y$$

$$\int \partial\lambda = \int -y\, dy \Rightarrow \lambda = -\frac{y^2}{2} + cte$$

$$\phi = \frac{x^3}{3} + \frac{x^2}{2} - y^2 x - \frac{y^2}{2} + cte$$

$\phi(0,0)$, $\phi(2,3)$

$$\phi = \frac{x^3}{3} + \frac{x^2}{2} - y^2 x - \frac{y^2}{2} + cte$$

$\times\ \phi(0,0) = cte$

$\times\ \phi(2,3) = \frac{8}{3} + \frac{4}{2} - 9 \cdot 2 - \frac{9}{2} + cte = \frac{16 + 12 - 108 - 27}{6} + cte$

$= -\frac{107}{6} + cte$

$\times\ \Delta\phi = \phi(B) - \phi(0)$

$\times\ \Delta\phi = -\frac{107}{6} + cte - cte = -\frac{107}{6}$

3) $\int_O^B \vec{q} \cdot d\vec{r}$, $\vec{q}(u, v, w)$, $d\vec{r}(dx, dy, dz)$ $\begin{cases} O(0,0) \\ B(2,3) \end{cases}$

$$\int_O^B (u\vec{i} + v\vec{j}) \cdot (dx\vec{i} + dy\vec{j}) = \int_O^B (u\,dx + v\,dy)$$

$$= \int_O^B u\,dx + \int_O^B v\,dy = \int_O^B (x^2 + x - y^2)\,dx + \int_O^B (2x+1)y\,dy$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} - y^2 x\right]_{(0,0)}^{(2,3)} + \left[-(2x+1)\frac{y^2}{2}\right]_{(0,0)}^{(2,3)} = \left[\frac{8}{3} + \frac{4}{2} - 9 \cdot 2\right] +$$

$$\left[-(\cancel{2}4+1)\frac{9}{2}\right] = -\frac{215}{6}.$$

EXO : Ecoulement En coordonne Cylindrique

On considère un écoulement laminaire, unidirectionnel et permanent d'un fluide visqueux dans une conduite cylindrique de rayon R et de longueur L. Sachant que la pression varie linéairement selon l'équation suivante :

1. Déterminer l'équation différentielle vérifiée par le profil de vitesse

2. Résoudre cette équation en tenant compte des conditions aux limites adéquates.

3. Exprimer la vitesse moyenne, le débit volumique de fluide ainsi que la contrainte visqueuse à la paroi et en déduire le coefficient de frottement .

$$\rho r\left[\cancel{\frac{\partial u_z}{\partial t}}^{0}+\cancel{u_z\frac{\partial u_z}{\partial z}}^{0}+\cancel{u_r\frac{\partial u_z}{\partial r}}^{0}\right]=-r\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}\left(\mu r\frac{\partial u_z}{\partial r}\right) \longrightarrow z$$

$U_z = W_z$

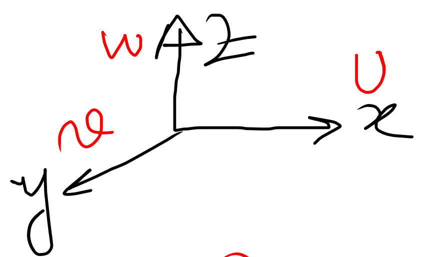
$$p(z)=a.z+b \quad \text{avec} \quad a \text{ et } b \text{ sont des constantes}$$

$\frac{\partial P}{\partial z} = a$

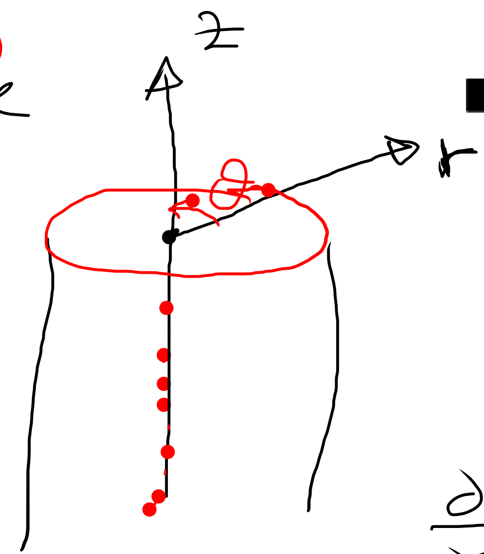
$u_z(r)$.

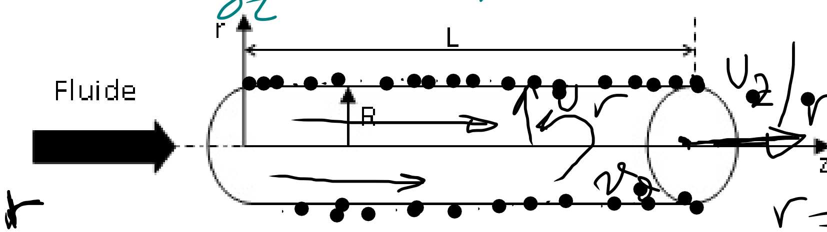
$$\boxed{\cancel{\frac{\partial\rho}{\partial t}}+\frac{1}{r}\cancel{\frac{\partial(\rho r u_r)}{\partial r}}+\frac{1}{r}\cancel{\frac{\partial(\rho v_\theta)}{\partial\theta}}+\frac{\partial(\rho w_z)}{\partial z}=0}$$

$\frac{\partial}{\partial z}(\rho w_z) = 0 \Rightarrow \frac{\partial w_z}{\partial z} = 0$



x	y	z
U	v	w
r	θ	z
U_r	v_θ	W_z





$U_z / r = R : 0$

$U_{z\,max}$ $r = 0$

$* W_z$; $U_r = v_\theta = 0$

$* \frac{\partial}{\partial z} = 0$

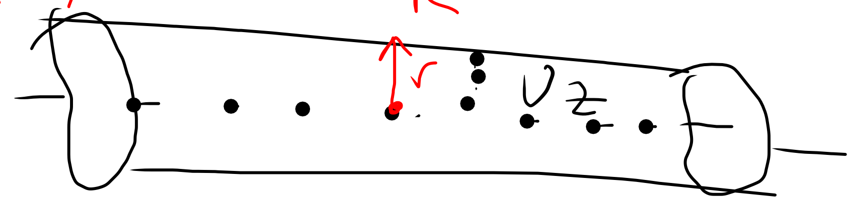
$$\frac{\partial}{\partial r}\left(\mu r\frac{\partial U_z}{\partial z}\right)-r\overbrace{\frac{\partial P}{\partial z}}^{a}=0$$

1) $\frac{\partial}{\partial r}\left(\mu r \frac{\partial U_z}{\partial z}\right) - ra = 0$

$r = 0 \Rightarrow U_z = U_{z\,max} \Rightarrow \frac{\partial U_z}{\partial r} = 0$

2) C.L:

$r = R \Rightarrow U_z = 0$



$\int \frac{\partial}{\partial r}\left(\mu r \frac{\partial U_z}{\partial z}\right) = \int ra\,dr \Rightarrow \mu r \frac{\partial U_z}{\partial z} = a\frac{r^2}{2} + C_1$

$\int \frac{\partial U_z}{\partial z} = \int \frac{a}{2\mu} r + \int \frac{C_1}{\mu r}$

$U_z = \frac{a}{4\mu} r^2 + \frac{C_1}{\mu} \ln r + C_2$

$\Rightarrow \left.\frac{\partial U_z}{\partial r}\right|_{r=0} = 0 \Rightarrow \left.\mu r \frac{\partial U_z}{\partial z}\right|_{r=0} = 0 = \frac{a(0)^2}{2} + C_1 \Rightarrow C_1 = 0$

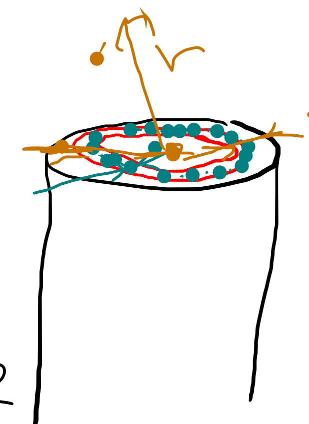
$r = R \Rightarrow U_z = 0 = \frac{a}{4N}(R^2) + C_2 \Rightarrow C_2 = -\frac{a}{4N}R^2$

$U_z = \frac{ar^2}{4N} - \frac{a}{4N}R^2 \Rightarrow U_z = \frac{a}{4N}\left[r^2 - R^2\right]$

$\to U_z(r=R) = 0$

$\to \left.\frac{\partial U_z}{\partial r}\right|_{r=0} = 0$

3) Q_V, $\bar{U}$, τ_r, C_f



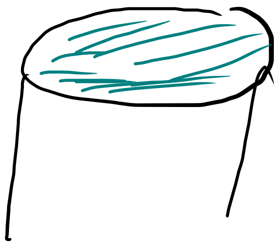
$ds = 2\pi r\,dr$

$Q_V = \int U_z\,ds = \int_0^R U_z\,2\pi r\,dr$

$Q_V = 2\pi \cdot \frac{a}{4N}\int_0^R r\cdot(r^2 - R^2)\,dr = \frac{\pi a}{2N}\int_0^R (r^3 - rR^2)\,dr = \frac{\pi a}{2N}\left[\frac{r^4}{4} - R^2\frac{r^2}{2}\right]_0^R$

$Q_V = \frac{\pi a}{2N}\left[\frac{R^4}{4} - \frac{R^4}{2}\right] = -\frac{\pi a}{4N}R^4$

$$\varphi_V = \bar{U}_z S = \int^R U_z \, dS \Rightarrow \bar{U}_z = \frac{\varphi_V}{S} = \frac{\varphi_V}{\pi R^2}$$



$$* \quad \bar{U}_z = -\frac{\cancel{\pi} a}{4N} R^{\cancel{4}\,2} \times \frac{1}{\cancel{\pi} R^2} = -\frac{a R^2}{4N}$$

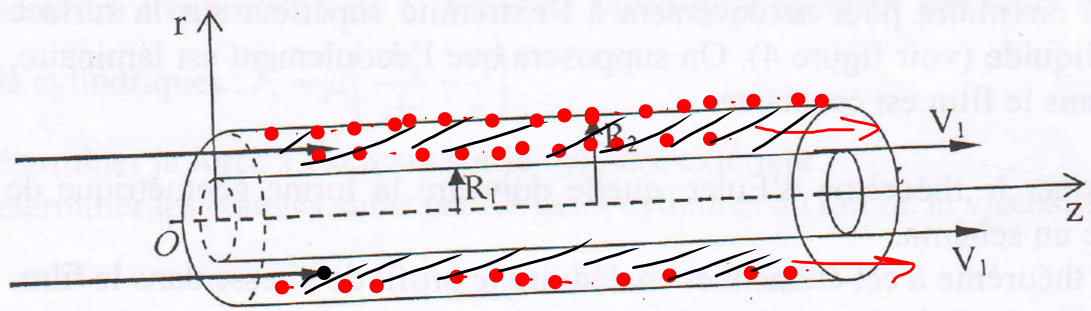
$$* \quad \tau_r \Big|_{r=R} = N \frac{\partial u_z}{\partial r} = N \left[\frac{a}{2N} R \right] = \frac{aR}{2}$$

$$* \quad C_f = \frac{\tau_{r=R}}{\frac{1}{2} \rho \bar{U}^2} = \frac{2 \times a R}{2 \rho \cdot \left(-\frac{a R^2}{4N} \right)^2} = \frac{16 N^2 \, 2 a R}{2 \rho a^2 R^4}$$

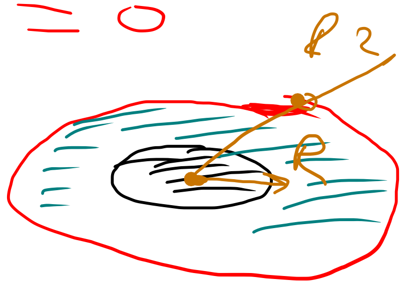
$$C_f = \frac{16 N^2}{\rho a R^3}$$

Exercice 2

Soit un fluide visqueux, incompressible, de viscosité dynamique μ confiné entre deux cylindres coaxiaux d'axe Oz et de rayon R_1 et R_2 le fluide initialement au repos, est mis en mouvement par le déplacement du cylindre intérieur selon Oz à la vitesse constante V_1, le cylindre extérieur restant immobile (fig. 2). On considère le régime laminaire permanent et sans différentiel de pression selon z.



* $\rho = cte$
* $U_z, \ v_\theta = U_r = 0$
* $\frac{\partial}{\partial z} = 0$
* $\frac{\partial P}{\partial z} = 0$



1) $\begin{cases} r = R_1 \Rightarrow U_z = V_1 \\ r = R_2 \Rightarrow U_z = 0 \end{cases}$

1- Exprimer les conditions aux limites associées à cet écoulement.
2- Déterminer l'expression du profile de vitesse.
3- Calculer le débit volumique dans une section droite.
4- Déduire l'expression de la vitesse moyenne.
5- Exprimer la force de frottement par une unité de langueur agissant sur chaque cylindre.

2) U_z ? $\Rightarrow$

$$\rho r\left[\cancel{\frac{\partial u_z}{\partial t}}+\cancel{u_z\frac{\partial u_z}{\partial z}}+\cancel{u_r\frac{\partial u_z}{\partial r}}\right]=\cancel{-r\frac{\partial p}{\partial z}}+\underline{\frac{\partial}{\partial r}\left(\mu r\frac{\partial u_z}{\partial r}\right)}$$

$$\int \frac{\partial}{\partial r}\left(\mu r \frac{\partial u_z}{\partial r}\right) = \int 0 \Rightarrow \mu \cancel{r} \frac{\partial u_z}{\partial r} = C_1 \Rightarrow \int \frac{\partial u_z}{\partial r} = \int \frac{C_1}{\mu r}$$

$$\Rightarrow U_z = \frac{C_1}{\mu}\ln r + C_2$$

$$U_z(R_1) = \underline{V_1} = \frac{C_1}{\mu}\underline{\ln R_1} + C_2$$

$$U_z(R_2) = 0 = \frac{C_1}{\mu}\ln R_2 + C_2$$

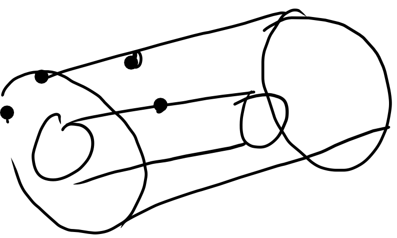
$$V_1 = \frac{C_1}{\mu}\left[\ln R_1 - \ln R_2\right]$$

$$V_1 = \frac{C_1}{\mu}\ln\frac{R_1}{R_2}$$

$$\Rightarrow C_1 = \frac{\mu V_1}{\ln\frac{R_1}{R_2}} = \frac{\mu V_1}{\ln\left(\frac{1}{\frac{R_2}{R_1}}\right)} = \frac{-\mu V_1}{\ln\frac{R_2}{R_1}}$$

$$C_2 = -\frac{C_1}{\mu}\ln R_2 = -\frac{\cancel{\mu} V_1}{\cancel{\mu}\cdot\ln\frac{R_1}{R_2}}\ln R_2 = -\frac{V_1\ln R_2}{\ln\frac{R_1}{R_2}}$$

$$U_2 = \frac{\cancel{\mu} V_1}{\ln\frac{R_1}{R_2}}\cdot\frac{1}{\cancel{\mu}}\ln r - V_1\frac{\ln R_2}{\ln\frac{R_1}{R_2}} \to U_2 = \frac{V_1}{\ln\frac{R_1}{R_2}}\left[\ln r - \ln R_2\right]$$



$$V_1 \leftarrow R_1$$

$$0 \leftarrow R_2$$

$$U_2 = \frac{V_1}{\ln\frac{R_1}{R_2}}\left[\ln\frac{r}{R_2}\right]$$

$$Q_V = 2\pi\int_{R_1}^{R_2} U_2\, r\,dr = 2\pi A\int_{R_1}^{R_2} r\cdot\ln\frac{r}{R_2}\,dr = 2\pi A\int_{R_1}^{R_2} r\left[\ln r - \ln R_2\right]dr$$

$$Q_v = 2\pi \lambda \left[\int_{R_1}^{R_2} r \ln r \, dr - \int_{R_1}^{R_2} r \ln R_2 \, dr \right]$$

$$\ln R_2 \left[\frac{r^2}{2} \right]_{R_1}^{R_2} = \frac{1}{2} \ln R_2 \left[R_2^2 - R_1^2 \right]$$

intégrale par partie

$$\int U \, dV = UV - \int V \, dU$$

$$\begin{cases} U = \ln r \Rightarrow dU = \frac{1}{r} dr \\ dV = r\,dr \Rightarrow V = \frac{r^2}{2} \end{cases} \Rightarrow \int_{R_1}^{R_2} r \ln r \, dr = \left[\frac{r^2 \ln r}{2} - \int_{R_1}^{R_2} \frac{r^2}{2} \cdot \frac{1}{r} dr \right]$$

$$\int_{R_1}^{R_2} r \ln r \, dr = \left[\frac{r^2}{2} \ln r + \int_{R_1}^{R_2} \frac{r}{2} dr \right] = \left[\frac{r^2}{2} \ln r - \frac{r^2}{4} \right]_{R_1}^{R_2}$$

$$= \left[\frac{R_2^2 \ln R_2}{2} - \frac{R_2^2}{4} \right] - \left[\frac{R_1^2 \ln R_1}{2} - \frac{R_1^2}{4} \right] =$$

$$= \frac{1}{2} \left[R_2^2 \ln R_2 - R_1^2 \ln R_1 \right] - \frac{1}{4} \left[R_2^2 - R_1^2 \right]$$

$$Q_v = R_2^2 \ln R_2 - \frac{R_1^2}{2} \ln R_2 - \frac{R_1^2}{2} \ln R_1 - \frac{1}{4}\left[R_2^2 - R_1^2\right]$$

$$Q_v = R_2^2 \ln R_2 - \frac{R_1^2}{2} \ln R_2 \cdot R_1 - \frac{1}{4}\left[R_2^2 - R_1^2\right].$$

3) $\bar{U} = \dfrac{Q_v}{S} = \dfrac{Q_v}{\pi(R_2^2 - R_1^2)}$

$S = \pi R_2^2 - \pi R_1^2$

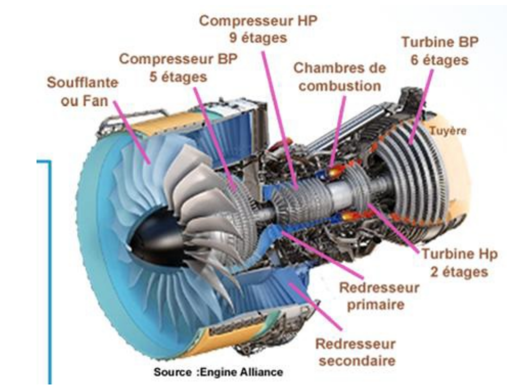
$S = \pi (R_2^2 - R_1^2)$

4) F_t ? $F_t = \tau_p \cdot S$

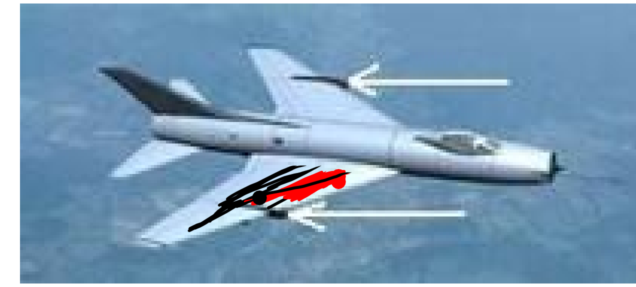
$S = 2\pi R \cdot l$

$F_t = \tau_p \cdot 2\pi R$; $\tau = \mu \dfrac{\partial U_z}{\partial r}$

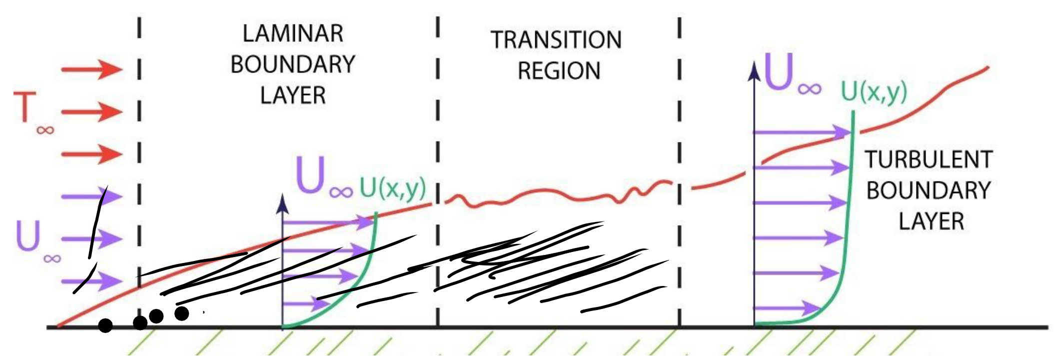
$F_{t_{R_1}} = \tau_{p(r=R_1)} \cdot 2\pi R_1$; $F_{t(R_2)} = \tau_{p(r=R_2)} \cdot 2\pi R_2$

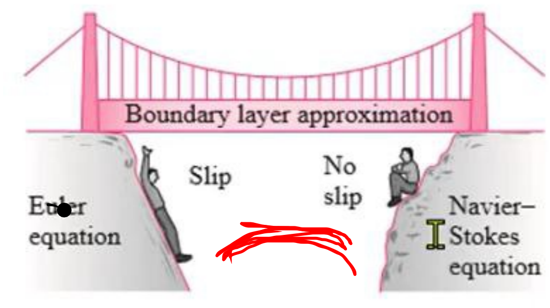


Couche limite



FLAT PLATE BOUNDARY LAYER

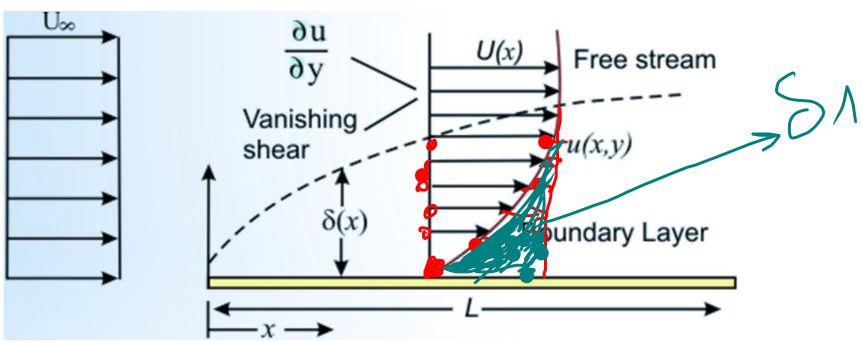


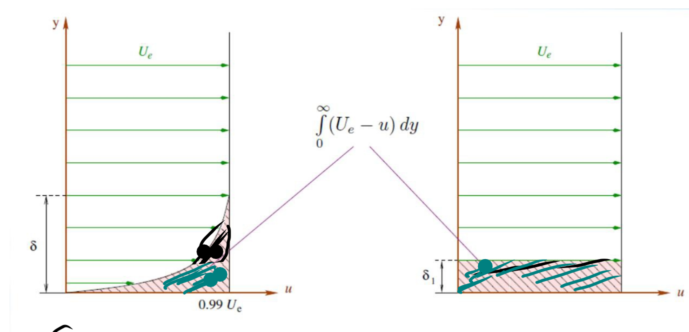


(b)

FIGURE 10–75

(a) A huge gap exists between the Euler equation (which allows slip at walls) and the Navier–Stokes equation (which supports the no-slip condition); (b) the boundary layer approximation bridges that gap.





δ FIGURE 5.2: *Perte de débit liée à la présence de la couche limite.*

Épaisseur de déplacement : $\delta = \int_0^{\delta} \left(1 - \frac{U}{U_\infty}\right) dy$;

δ_2 = E. qu. du Mouvement

$\delta_2 = \int_0^{\delta} \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dy$

$$\delta_2 = \int_0^{\delta} \frac{U}{Ue}\left(1 - \frac{U}{Ue}\right) dy \quad , \quad H = \frac{\delta_1}{\delta_2}$$

* U_∞, U_0, U, Ue

$$\begin{cases} H \approx 2{,}6 \longrightarrow C.L.L \\ H \approx 1{,}3 \longrightarrow C.LT \end{cases}$$

EXO **Couche limite**

De l'air s'écoule sur une plaque plane mince, on admet que la distribution de vitesse dans la couche limite laminaire obéit à la loi polynomiale suivante: $u/U=3/2\ (y/\delta)-1/2\ (y/\delta)^3$, U est la vitesse à l'extérieur de la couche limite.

1. Exprimez l'épaisseur de déplacement et de quantité de mouvement en fonction de l'épaisseur de la couche limite δ et déduire le facteur de forme H.
2. Sachant que les dimensions de la plaque sont (3cm de largeur(l),4cm de longueur (L)) et que la vitesse moyenne de l'écoulement est de l'ordre de(15m/s), montrez que la couche limite reste laminaire sur toute la surface de la plaque. $\nu_{air}=1.5.10^{-5}\ (m^2/s)$
3. Donnez l'expression de l'épaisseur de la couche limite puis l'évaluez au niveau des extrémités (x=0,x=L).

On donne : $\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \rho U^2 \frac{d\delta_2}{dx}$

4. Tracez approximativement l'allure de cette couche dans l'intervalle 0≤x≤L et commentez le résultat.

$U = 15\ m/s,\quad \nu = 1{,}5\ 10^{-5}\ \frac{m^2}{s}$

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}(y/\delta)^3$$

1) $$\delta_1 = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left(1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right) dy = \left[y - \frac{3}{4}\frac{y^2}{\delta} + \frac{1}{8}\frac{y^4}{\delta^3}\right]_0^\delta$$

$$\delta_1 = \delta\left[1 - \frac{3}{4} + \frac{1}{8}\right] = \left(\frac{8-6+1}{8}\right)\delta = \frac{3}{8}\delta = 0{,}375\,\delta$$

$$* \quad \delta_2 = \int_0^{\delta} \left(\frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty} \right) \right) dy = \int_0^{\delta} \left(\frac{u}{U_\infty} - \left(\frac{u}{U_\infty} \right)^2 \right) dy =$$

$$= \int_0^{\delta} \frac{u}{U_\infty}\, dy - \int_0^{\delta} \left(\frac{u}{U_\infty} \right)^2 dy = \int_0^{\delta} \left(\overbrace{\frac{3}{2} \frac{y}{\delta}}^{a} - \overbrace{\frac{1}{2} \left(\frac{y}{\delta} \right)^3}^{b} \right) dy -$$

$$- \int_0^{\delta} \left(\frac{9}{4} \left(\frac{y}{\delta} \right)^2 + \frac{1}{4} \left(\frac{y}{\delta} \right)^6 - \frac{3}{2} \left(\frac{y}{\delta} \right)^4 \right) dy$$

$$\delta_2 = \left[\frac{3}{4} \frac{y^2}{\delta} - \frac{1}{8} \frac{y^4}{\delta^3} \right]_0^{\delta} - \left[\frac{9}{12} \frac{y^3}{\delta^2} + \frac{1}{28} \frac{y^7}{\delta^6} - \frac{3}{10} \frac{y^5}{\delta^4} \right]_0^{\delta}$$

$$\delta_2 = \delta \left[\frac{3}{4} - \frac{1}{8} - \frac{9}{12} - \frac{1}{28} + \frac{3}{10} \right] = 0{,}14\, \delta$$

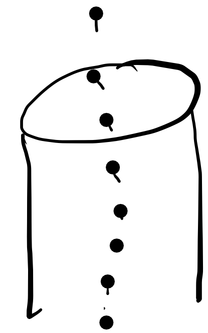
$H = \frac{\delta_1}{\delta_2} = \frac{0,3758}{0,148} \approx 2,67$, laminaire.

2) $Re = \frac{L \cdot \bar{U}}{\nu} = \frac{4\,10^{-2} \cdot 1,5}{1,5\,10^{-5}} = 40\,10^{3}$

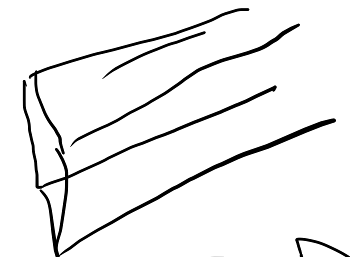
$= 4\,10^{4} < 5\,10^{5}$

$Re = \frac{x \cdot \bar{U}}{\nu}$

C.L. Reste laminaire sur toute la surface de la plaque.



$Re = \frac{D \cdot \bar{U}}{\nu}$



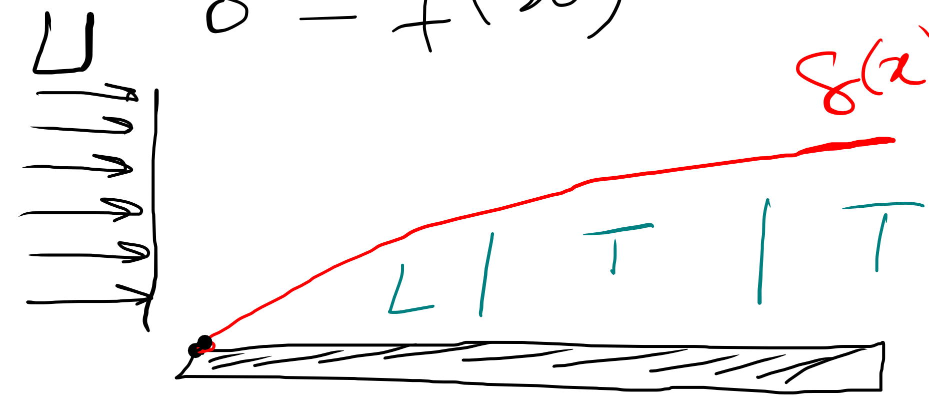
$Re = \frac{\bar{U} \cdot D_H}{\nu}$

$D_H = \frac{4S}{P}$

3) δ ? , $\mu \dfrac{\partial u}{\partial y}\Big|_{y=0} = \rho U^2 \dfrac{d\delta_2}{dx}$

$$\begin{cases} \dfrac{u}{U} = \dfrac{3}{2}\left(\dfrac{y}{\delta}\right) - \dfrac{1}{2}\left[\dfrac{y}{\delta}\right]^3 \\ \delta_2 = 0{,}14\,\delta \end{cases}$$

$\delta = f(x)$



$x = 0,\ \delta = 0\ \ldots\ldots$

$Re < 5\,10^5$

$$\frac{\partial u}{\partial y}\Big|_{y=0} = U\left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right] \quad y = 0$$

$$*\ \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{3U}{2\delta}$$

$$*\ \frac{\partial \delta_2}{dx} = \frac{\partial}{\partial x}\left[0{,}14\,\delta\right] = 0{,}14\,\frac{\partial \delta}{dx}$$

$$\frac{\mu 3U}{2\delta} = \rho \cdot U^2 \cdot 0{,}14\,\frac{\partial \delta}{\partial x} \Rightarrow \delta \frac{\partial \delta}{\partial x} = \frac{\mu}{\rho}\,\frac{U}{U^2} \cdot \frac{3}{0{,}28} = \frac{3}{0{,}28}\,\frac{\nu}{U}$$

$$\int \delta \, d\delta = \int \frac{3}{0{,}28} \frac{\nu}{U} \, dx$$

$$\frac{\delta^2}{2} = \frac{3}{0{,}28} \frac{\nu}{U} x + C_1$$

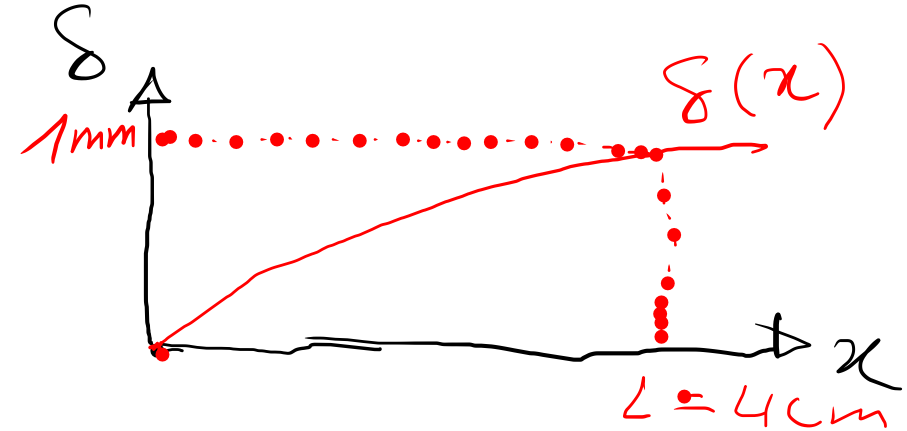
$$\delta = \sqrt{\frac{6}{0{,}28} \frac{\nu}{U} x + C_1} \qquad \begin{cases} x = 0 \, , \; \delta = 0 \\ 0 = \sqrt{C_1} \Rightarrow C_1 = 0 \end{cases}$$

$$\delta = \sqrt{\frac{6}{0{,}28} \frac{\nu}{U} x} \, , \qquad L = 4\,cm$$

$$\delta(L) = \sqrt{\frac{6}{0{,}128} \frac{\nu}{U} 4 \cdot 10^{-2}} = \sqrt{\frac{6}{0{,}128} \frac{1{,}5 \cdot 10^{-5}}{15} 4 \cdot 10^{-2}} \cong 0{,}92\,mm$$

4) $\delta = \sqrt{\frac{6}{0{,}28}\,\frac{\nu}{U}\,x} = \sqrt{\frac{6}{0{,}28}\cdot\frac{1{,}5\cdot 10^{-5}}{15}}\cdot\sqrt{x} \simeq 1{,}467\sqrt{x}$

$$\begin{cases} x = 0\,, & \delta = 0 \\ x = L\,; & \delta \simeq 1\,\text{mm} \end{cases}$$



* On observe que l'épaisseur de la couche limite évolue asymptotiquement.

بالتوفيق