$$ العددية الدوال )حول \left(2\right) رقم تمارين سلسلة$$

$$ ( العكسية لدوال - الاستمرار-$$

$$ التالية الدوال تعريف مجموعة عين : \left(1\right)التمرين$$

$$ \left(1\right) f\left(x\right)=\sqrt{-x^{2}+5x-4} \left(2\right) f\left(x\right)=\frac{\sqrt{x-1}}{\sqrt{x^{2}-5x+4}} $$

$$\left(3\right)f\left(x\right)\sqrt{\frac{x-1}{x^{2}-5x+4}} \left(4\right) f\left(x\right)=\sqrt{E\left(x\right)-x} $$

$$\left(5\right) f\left(x\right)=ln\left(\frac{1-x}{1+x}\right) \left(6\right)^{\*} f\left(x\right)=ln\left[ln\left(lnx\right)\right] $$

$$ \left(7\right)^{\*}f\left(x\right)=\frac{1}{E\left(x\right)-x} \left(8\right) f\left(x\right)=ln\left(\sqrt{\frac{1-x}{1+x}}\right) $$

$$\left(9\right) f\left(x\right)= e^{\frac{1-x}{1+x}} $$

$$ ذالك غير او فردية او زوجية التالية الدوال كانت ان ادرس : \left(2\right)التمرين$$

$$ \left(1\right) f\left(x\right)=\frac{x}{1+\left|x\right|} \left(2\right) f\left(x\right)=\sqrt[3]{x^{5}+x} $$

$$\left(3\right)f\left(x\right)=ln\left(\frac{1-x}{1+x}\right) \left(4\right) f\left(x\right)=\left|x\right|\left(1-3x\right)\left(1+3x\right) $$

$$\left(5\right)^{\*} f\left(x\right)=\frac{sin^{2}\left(x\right)-cos3x}{x^{2}}$$

$$: \left(3\right)التمرين$$

$$ التالية النهايات احسب$$

$$\left(2\right)\lim\_{x\to +\infty }\left(\frac{x}{\sqrt{x+1}}-\frac{x}{\sqrt{x+2}}\right) \left(1\right)\lim\_{ x\to +0} \frac{\sqrt{1+x}-\sqrt{1-x}}{sinx}$$

$$\left(3\right)\lim\_{x\to +\infty }\sqrt{x^{2}+x-2}-\left(x-1 \right) \left(4\right)\lim\_{ x\to π}\frac{1+cosx}{x-π}$$

$$ \left(5\right)^{\*}\lim\_{ x\to \frac{π}{4}} \frac{sinx-cosx}{x-\frac{π}{4}} \left(6\right)^{\*}\lim\_{x\to +\infty }\sqrt{x^{2}+x-2}-\left(4x-1 \right) $$

$$حيث x\_{0}=1 عند f استمرارية ادرس \left(1\right) : \left(4\right)التمرين$$

$$f\left(x\right)=\left\{\begin{array}{c}\frac{\sqrt{2x+7}-3 }{\sqrt{2x+2}-2} , x\ne 1\\\frac{2}{3} , x=1\end{array}\right.$$

$$ 0 عند بالاستمرار التمديد تقبل التالية الدوال هل \left(2\right)$$

$$f\left(x\right)=\frac{\left(1-cosx\right)^{2}}{x^{4}} f\left(x\right)=\frac{sin2x}{\sqrt{x+1}-1}f\left(x\right)= 3x^{2}sin\left(\frac{1}{x}\right) $$

$$ :\left(5\right)التمرين$$

$$بين محصور حل كل حلول ثلاث تقبل x^{4}-5x+1=0 المعادلة ان بين$$

$$ متتاليين صحيحين عددين $$

$$ :\left(6\right)التمرين^{\*}$$

$$محصور حل كل حلول ثلاث تقبل x^{3}-5x^{2}+2x+1=0 المعادلة ان بين$$

$$ متتاليين صحيحين عددين بين $$

$$ -1, 0 , 1,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2} التالية للقيم Arccos , Arcsin احسب \left(1\right) : \left(7\right)التمرين$$

$$ -1, 0 , 1,\sqrt{3} التالية للقيم Arccotan , Arctan احسب \left(2\right) $$

$$ Arctan \left(tan\left(\frac{3π}{4}\right)\right),Arccos\left(cos\left(-\frac{π}{4}\right)\right), Arcsin\left(sin\frac{3π}{4}\right) احسب \left(3\right)^{\*}$$

$$ ان اثبت : \left(8\right)التمرين$$

$$\left(1\right) Arccos\left(x\right)+Arccos\left(-x\right)=π , x\in \left[-1,1\right] $$

$$ \left(2\right) Arcsin\left(x\right)+Arccos\left(x\right)=\frac{π}{2} , x\in \left[-1,1\right] $$

$$\left(3\right) Arctan\left(x\right)+Arctan\left(\frac{1}{x}\right)=\frac{π}{2} , x>0$$

$$\left(4\right) cos\left(Arcsin\left(x\right)\right)=\sqrt{1-x^{2}} , x\in \left[-1,1\right]$$

$$\left(5\right) sin\left(Arctan\left(x\right)\right)=\frac{x}{\sqrt{1+x^{2}}}$$

$$\left(6\right) sin\left(Arctan\left(x\right)\right)=\frac{\sqrt{1-x^{2}}}{x} , x\in \left[-1,1\right]-\left\{0\right\}$$

$$ كتايلي المعرفة f الدالة نعتبر : \left(9\right)التمرين$$

$$f\left(x\right)=Arccos\left(\frac{2x}{2-x}\right)$$

$$f الدالة تعريف مجموعة D\_{f} عين \left(1\right)$$

$$∀x\in D\_{f} f^{'}\left(x\right)=\frac{-4}{\left(2-x\right)\sqrt{4-4x-3x^{2}}} ان بين \left(2\right)$$

$$ البياني تمثيلها ارسم ثم f تغيرات جدول شكل \left(3\right)$$

$$ كتايلي المعرفة f الدالة نعتبر : \left(10\right)التمرين$$

$$f\left(x\right)=Arctan\left(\frac{1}{1-x}\right)$$

$$f الدالة تعريف مجموعة D\_{f} عين \left(1\right)$$

$$∀x\in D\_{f} f^{'}\left(x\right)=\frac{1}{\left(1-x\right)^{2}+1} ان بين \left(2\right)$$

$$ البياني تمثيلها ارسم ثم f تغيرات جدول شكل \left(3\right)$$

$$ كتايلي المعرفة f الدالة نعتبر : \left(11\right)التمرين^{\*}$$

$$f\left(x\right)=\frac{\sqrt{1-x}}{\sqrt{1+x}}Arccos\left(x\right)$$

$$f الدالة تعريف مجموعة D\_{f} عين \left(1\right)$$

$$ f استمرارية ادرس\left(2\right)$$

$$ f^{'}\left(x\right) عين \left(3\right)$$

$$ كتايلي المعرفة f الدالة نعتبر : \left(12\right)التمرين^{\*}$$

$$f\left(x\right)=Arctan\left(\frac{\sqrt{3}}{x-1}\right)+Arctan\left(x\frac{\sqrt{3}}{4-x}\right)$$

$$f الدالة تعريف مجموعة D\_{f} عين \left(1\right)$$

$$ا f\left(0\right), f\left(2\right), f\left(5\right) حسب\left(2\right)$$

$$ f ل بسيطة عبارة اوجد ثم f^{'}\left(x\right) عين \left(3\right)$$

$$ ان اثبت :13 التمرين $$

$$y=Argch\left(x\right) ⟺y=ln\left(x+\sqrt{x^{2}-1}\right) ∀x\in \left[1, +\infty \right[ \left(1\right)$$

$$y=Argsh\left(x\right) ⟺y=ln\left(x+\sqrt{x^{2}+1}\right) ∀x\in \left]-\infty , +\infty \right[ \left(2\right)$$

$$y=Argth\left(x\right) ⟺y=\frac{1}{2}ln\left(\frac{1+x}{1-v}\right) ∀x\in \left]-1, 1\right[ \left(3\right)$$

$$y=Argcoth\left(x\right) ⟺y=\frac{1}{2}ln\left(\frac{1+x}{v-1}\right) ∀x\in \left]-\infty , 1\right[ ∪\left]1, +\infty \right[ \left(4\right)$$

$$Argth\left(x\right)+Argth\left(y\right)=Argth\left(\frac{x+y}{1+xy}\right) ∀x,y\in \left]-1, 1\right[ \left(5\right)$$

$$Argch\left(\sqrt{1+x^{2}}\right)=Argsh\left(x\right) ∀x\in \left]-\infty ,+\infty \right[ \left(6\right)$$

$$Argsh\left(\sqrt{x^{2}-1}\right)=Argch\left(x\right) ∀x\in \left[1,+\infty \right[ \left(7\right)$$

$$ ∀θ\in \left]-\infty ,+\infty \right[ sh\left(3θ\right)=3shθ+4sh^{3}θ \left(8\right)$$

$$Argsh\left(3x+4x^{2}\right)=3Argshx ان استنتج$$

$$ Argsh\left[\sqrt{\frac{1+ch\left(x\right)}{2}}\right]-\frac{x}{2}=\left\{\begin{array}{c}-x, x\leq 0\\0, x>0\end{array}\right. \left(9\right)$$

$$ التالية العبارات بسط :14 التمرين $$

$$Argth\left(\frac{\sqrt{chx-1}}{\sqrt{chx+1}}\right), Argsh\left(2x\sqrt{x^{2}+1}\right) , Argth\left(\frac{3x+x^{2}}{1+3x^{2}}\right)$$

$$ كتايلي المعرفة f الدالة نعتبر : \left(15\right)التمرين$$

$$f\left(x\right)=Argth\left(\frac{x+2}{x-2}\right)$$

$$ D\_{f} اطراف على النهايات احسب f الدالة تعريف مجموعة D\_{f} عين \left(1\right)$$

$$البياني تمثيلها ارسم ثم f تغيرات جدول شكل f^{'}\left(x\right) عين \left(2\right)$$

$$الحلول$$

$$ التالي التالية الدوال تعريف مجموعة عين : \left(1\right)التمرين$$

$$ \left(1\right) f\left(x\right)=\sqrt{-x^{2}+5x-4} \left(2\right) f\left(x\right)=\frac{\sqrt{x-1}}{\sqrt{x^{2}-5x+4}} $$

$$\left(3\right)f\left(x\right)\sqrt{\frac{x-1}{x^{2}-5x+4}} \left(4\right) f\left(x\right)=\sqrt{E\left(x\right)-x} $$

$$\left(5\right) f\left(x\right)=ln\left(\frac{1-x}{1+x}\right) \left(6\right)^{\*} f\left(x\right)=ln\left[ln\left(lnx\right)\right] $$

$$ \left(7\right)^{\*}f\left(x\right)=\frac{1}{E\left(x\right)-x} \left(8\right)^{\*} f\left(x\right)=ln\left(\sqrt{\frac{1-x}{1+x}}\right) $$

$$\left(9\right) f\left(x\right)= e^{\frac{1-x}{1+x}} $$

$$ \left(1\right) معرفة f\left(x\right)=\sqrt{-x^{2}+5x-4}⟺ -x^{2}+5x-4 \geq 0 $$

$$⟺x\in \left]-\infty ,-1\right]∪\left[4,+\infty \right[$$

$$D\_{f}=\left]-\infty ,-1\right]∪\left[4,+\infty \right[$$

$$\left(2\right) معرفة f\left(x\right)=\frac{\sqrt{x-1}}{\sqrt{x^{2}-5x+4}}⟺\left\{\begin{array}{c}x\in R :x-1\geq 0\\∧\\x^{2}-5x+4>0 \end{array}\right. $$

$$ ⟺\left\{\begin{array}{c}x\in R x\in \left]1,\infty \right[\\∧\\\left]-\infty ,1\right[∪\left]4,+\infty \right[\end{array}\right.$$

$$⟺ \left]4,+\infty \right[$$

$$D\_{f}=\left]4,+\infty \right[$$

$$\left(3\right) معرفة f\left(x\right)=\sqrt{\frac{x-1}{x^{2}-5x+4}}⟺\left\{\begin{array}{c}\frac{x-1}{x^{2}-5x+4}\geq 0\\∧\\x^{2}-5x+4\ne 0\end{array}\right.$$

$$ ⟺x\in \left]4,+\infty \right[$$

$$D\_{f}=\left]4,+\infty \right[$$

$$\left(4 \right) معرفة f\left(x\right)=\sqrt{E\left(x\right)-x}⟺E\left(x\right)-x\geq 0$$

$$⟺E\left(x\right)\geq x⟺x\leq E\left(x\right)$$

$$ ∀x\in R E\left(x\right)\geq x ان نعلم لكن$$

$$وعليه$$

$$معرفة f\left(x\right)=\sqrt{E\left(x\right)-x}⟺E\left(x\right)=x⟺x\in Z$$

$$D\_{f}=Z$$

$$\left(5\right) معرفة f\left(x\right)=ln\left(\frac{1-x}{1+x}\right)⟺\frac{1-x}{1+x}>0$$

$$⟺\left(1-x\right)\left(1+x\right)>0$$

$$⟺x\in \left]-1,1\right[$$

$$D\_{f}=\left]-1,1\right[$$

$$\left(9\right)معرفة f\left(x\right)= e^{\frac{1-x}{1+x}} ⟺1+x\ne 0 $$

$$⟺x\ne -1$$

$$⟺x\in R∖\left\{-1\right\}$$

$$D\_{f}= R∖\left\{-1\right\}$$

$$ ذالك غير او فردية او زوجية التالية الدوال كانت ان ادرس : \left(2\right)التمرين$$

$$ \left(1\right) f\left(x\right)=\frac{x}{1+\left|x\right|} \left(2\right) f\left(x\right)=\sqrt[3]{x^{5}+x} $$

$$\left(3\right)f\left(x\right)=ln\left(\frac{1-x}{1+x}\right) \left(4\right) f\left(x\right)=\left|x\right|\left(1-3x\right)\left(1+3x\right) $$

$$\left(5\right)^{\*} f\left(x\right)=\frac{sin^{2}\left(x\right)-cos3x}{x^{2}}$$

$$ \left(1\right) ∀x\in R f\left(-x\right)=\frac{-x}{1+\left|-x\right|}=\frac{-x}{1+\left|x\right|}=- f\left(x\right)$$

$$ فرديةf ومنه$$

$$\left(2\right) ∀x\geq 0 f\left(-x\right)=\sqrt[3]{\left(-x\right)^{5}-x}=\sqrt[3]{-x^{5}-x}=-\sqrt[3]{x^{5}+x}$$

$$=-f\left(x\right)$$

$$f\left(-x\right)=\sqrt[3]{\left(-x\right)^{5}-x}=\sqrt[3]{-x^{5}-x}⟺f\left(-x\right)^{3}=-x^{5}-x$$

$$⟺\left[-f\left(-x\right)\right]^{3}=x^{5}+x$$

$$⟺-f\left(-x\right)=\sqrt[3]{x^{5}+x}=f\left(x\right)$$

$$ فرديةf ومنه$$

$$\left(3\right)∀x\in \left]-1,1\right[ f\left(-x\right)=ln\left(\frac{1+x}{1-x}\right)==-ln\left(\frac{1-x}{1+x}\right)=- f\left(x\right) $$

$ فرديةf ومنه$

$$\left(4\right)∀x\in R f\left(-x\right)=\left|-x\right|\left(1+3x\right)\left(1-3x\right)=\left|x\right|\left(1+3x\right)\left(1-3x\right)$$

$$= f\left(x\right)$$

$$زوجية f ومنه$$

$$: \left(3\right)التمرين$$

$$ التالية النهايات احسب$$

$$\left(2\right)\lim\_{x\to +\infty }\left(\frac{x}{\sqrt{x+1}}-\frac{x}{\sqrt{x+2}}\right) \left(1\right) \lim\_{ x\to +0} \frac{\sqrt{1+x}-\sqrt{1-x}}{sinx}$$

$$\left(3\right)\lim\_{x\to +\infty }\sqrt{x^{2}+x-2}-\left(x^{2}-1\right)$$

$$\left(4\right) \lim\_{ x\to \frac{π}{4}} \frac{sinx-cosx}{x-\frac{π}{4}} \left(5\right) \lim\_{ x\to π}\frac{1+cosx}{x-π}$$

$$\left(1\right)\lim\_{ x\to +0} \frac{\sqrt{1+x}-\sqrt{1-x}}{sinx}==\lim\_{ x\to +0} \frac{\left(\sqrt{1+x}-\sqrt{1-x}\left(\sqrt{1+x}+\sqrt{1-x}\right)\right)}{\left(\sqrt{1+x}+\sqrt{1-x}\right)sinx}=\lim\_{ x\to +0}\frac{2x}{\left(\sqrt{1+x}+\sqrt{1-x}\right)sinx}=\lim\_{ x\to +0}\frac{2}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}×\frac{1}{\frac{sinx}{x}}=1$$

$$\left(2\right)\lim\_{x\to +\infty }\left(\frac{x}{\sqrt{x+1}}-\frac{x}{\sqrt{x+2}}\right) =\lim\_{x\to +\infty }x\left(\frac{1}{\sqrt{x+1}}-\frac{1}{\sqrt{x+2}}\right)\left(\frac{1}{\sqrt{x+1}}+\frac{1}{\sqrt{x+2}}\right)=\lim\_{x\to +\infty }x\left(\frac{1}{x+1}-\frac{1}{x+2}\right)=\lim\_{x\to +\infty }\frac{x}{\left(x+1\right)\left(x+2\right)}=0$$

$$\left(3\right)\lim\_{x\to +\infty }\sqrt{x^{2}+x-2}-\left(x-1\right)=\lim\_{x\to +\infty }\frac{\left(\sqrt{x^{2}+x-2}-\left(x-1\right)\right)\left(\sqrt{x^{2}+x-2}+\left(x-1\right)\right)}{\sqrt{x^{2}+x-2}+\left(x^{2}-1\right)}$$

$$=\lim\_{x\to +\infty }\frac{x-1}{\sqrt{x^{2}+x-2}+\left(x-1\right)}=\lim\_{x\to +\infty }\frac{x\left(1-\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{1}{x}-\frac{2}{x^{2}}}+1-\frac{1}{x}\right)}=\frac{1}{2}$$

$$\left(4\right)\lim\_{ x\to π}\frac{1+cosx}{\left(x-π\right)^{2}}=\left\{\begin{array}{c}z=x-π\\\lim\_{ z\to 0}\frac{1+cos\left(z+π\right)}{z^{2}}\end{array}\right.$$

$$=\lim\_{ z\to 0}\frac{1-cosz}{z^{2}}=\lim\_{ z\to 0}\frac{2sin^{2}z}{z^{2}}=2$$

$$ x\_{0}=1 عند f استمرارية ادرس \left(1\right) : \left(4\right)التمرين$$

$$f\left(x\right)=\left\{\begin{array}{c}\frac{\sqrt{2x+7}-3 }{\sqrt{2x+2}-2} , x\ne 1\\\frac{2}{3} , x=1\end{array}\right.$$

$$ 0 عند بالاستمرار التمديد تقبل التالية الدوال هل \left(2\right)^{\*}$$

$$f\left(x\right)=\frac{\left(1-cosx\right)^{2}}{x^{4}}, f\left(x\right)=\frac{sin2x}{\sqrt{x+1}-1}, f\left(x\right)= 3x^{2}sin\left(\frac{1}{x}\right) $$

$$ \left(1\right)\lim\_{ x\to 1}f\left(x\right)=\lim\_{ x\to 1}\frac{\sqrt{2x+7}-3 }{\sqrt{2x+2}-2}=\lim\_{ x\to 1}\frac{\left(\sqrt{2x+7}-3 \right)\left(\sqrt{2x+7}+3\right)\left(\sqrt{2x+2}+2\right)}{\left(\sqrt{2x+2}-2\right)\left(\sqrt{2x+2}+2\right)\left(\sqrt{2x+7}+3\right)}$$

$$=\lim\_{ x\to 1}\frac{\left(2x-2\right)\left(\sqrt{2x+2}+2\right)}{\left(2x-2\right)\left(\sqrt{2x+7}+3\right)}=\lim\_{ x\to 1}\frac{\left(\sqrt{2x+2}+2\right)}{\left(\sqrt{2x+7}+3\right)}=\frac{2}{3} =f\left(1\right)$$

$$ 1 عند مستمرة f ان ومنه$$

$$:\left(5\right)التمرين$$

$$بين محصور حل كل حلول ثلاث تقبل x^{5}-5x+1=0 المعادلة ان بين$$

$$ متتاليين صحيحين عددين$$

$$f^{'}\left(x\right)=5x^{4}-5=5\left(x^{4}-1\right)=5\left(x^{2}-1\right)\left(x^{2}+1\right)$$

$$ \left]-\infty ,-1\right[∪\left]1,+\infty \right[ المجال على تماما متزايده f$$

$$ \left]-1,1\right[ المجال على تماما ومتناقصة$$

$$ \left]-\infty ,-1\right[ المجال على \left( تماما متزايده \right) تماما رتيبة و مستمؤة f-$$

$$ المعادلة فان المتوسطة القيم نظرية حسب ومنه f\left(-2\right)×f\left(-1\right)<0 و$$

$$ \left]-\infty ,-1\right[ المجال على وحيدا حلا تقيل x^{5}-5x+1=0$$

$$ \left]1,+\infty \right[ المجال على \left( تماما متزايده \right) تماما رتيبة و مستمؤة f-$$

$$ المعادلة فان المتوسطة القيم نظرية حسب ومنه f\left(2\right)×f\left(1\right)<0 و$$

$$ \left]1,+\infty \right[ المجال على وحيدا حلا تقيل x^{5}-5x+1=0$$

$$ \left]-1,1\right[ المجال على \left( تماما متناقصة \right) تماما رتيبة و مستمؤة f-$$

$$ المعادلة فان المتوسطة القيم نظرية حسب ومنه f\left(-1\right)×f\left(1\right)<0 و$$

$$ \left]-1,1\right[ المجال على وحيدا حلا تقيل x^{5}-5x+1=0$$

$$-1, 0 , 1,\frac{\sqrt{2}}{2},\frac{\sqrt{3}}{2} التالية للقيم Arccos , Arcsin احسب \left(1\right) : \left(7\right)التمرين$$

$$ -1, 0 , 1,\sqrt{3} التالية للقيم Arccotan , Arctan احسب \left(2\right) $$

$$Arcsin\left(-1\right)=y⟺y\in \left[-\frac{π}{2},\frac{π}{2}\right] , siny=-1⟺y=-\frac{π}{2} $$

$$Arcsin\left(0\right)=y⟺y\in \left[-\frac{π}{2},\frac{π}{2}\right] , siny=0⟺y=0 $$

$$Arcsin\left(1\right)=y⟺y\in \left[-\frac{π}{2},\frac{π}{2}\right] , siny=1⟺y= \frac{π}{2} $$

$$Arcsin\left(\frac{\sqrt{2}}{2}\right)=y⟺y\in \left[-\frac{π}{2},\frac{π}{2}\right] , siny=\frac{\sqrt{2}}{2}⟺y= \frac{π}{4} $$

$$Arcsin\left(\frac{\sqrt{3}}{2}\right)=y⟺y\in \left[-\frac{π}{2},\frac{π}{2}\right] , siny=\frac{\sqrt{3}}{2}⟺y= \frac{π}{6}$$

$$Arccos\left(-1\right)=y⟺y\in \left[0,π\right] , cosy=-1⟺y=π $$

$$Arccos\left(0\right)=y⟺y\in \left[0,π\right] ,cosy=0⟺y= \frac{π}{2} $$

$$Arccos\left(1\right)=y⟺y\in \left[0,π\right] , cosy=1⟺y= 0 $$

$$Arcsin\left(\frac{\sqrt{2}}{2}\right)=y⟺y\in \left[0,π\right] , cosy=\frac{\sqrt{2}}{2}⟺y= \frac{π}{4} $$

$$Arcsin\left(\frac{\sqrt{3}}{2}\right)=y⟺y\in \left[0,π\right] , cosy=\frac{\sqrt{3}}{2}⟺y= \frac{π}{3}$$

$$Arctan\left(-1\right)=y⟺y\in \left]-\frac{π}{2},\frac{π}{2}\right[ , tany=-1⟺y=- \frac{π}{4}$$

$$Arctan\left(0\right)=y⟺y\in \left]-\frac{π}{2},\frac{π}{2}\right[ , tany=0⟺y= 0$$

$$Arctan\left(1\right)=y⟺y\in \left]-\frac{π}{2},\frac{π}{2}\right[ , tany=1⟺y= \frac{π}{4}$$

$$Arctan\left(\sqrt{3}\right)=y⟺y\in \left]-\frac{π}{2},\frac{π}{2}\right[ , tany=\sqrt{3}⟺y= \frac{π}{6}$$

$$ان اثبت : \left(8\right)التمرين$$

$$\left(1\right) Arccos\left(x\right)+Arccos\left(-x\right)=π , x\in \left[-1,1\right] $$

$$ \left(2\right) Arcsin\left(x\right)+Arccos\left(x\right)=\frac{π}{2} , x\in \left[-1,1\right] $$

$$\left(3\right) Arctan\left(x\right)+Arctan\left(\frac{1}{x}\right)=\frac{π}{2} , x>0$$

$$\left(4\right) cos\left(Arcsin\left(x\right)\right)=\sqrt{1-x^{2}} , x\in \left[-1,1\right]$$

$$\left(5\right) sin\left(Arctan\left(x\right)\right)=\frac{x}{\sqrt{1+x^{2}}}, x\in \left[-1,1\right]$$

$$\left(6\right) tan\left(Arccos\left(x\right)\right)=\frac{\sqrt{1-x^{2}}}{x} , x\in \left[-1,1\right]-\left\{0\right\} $$

$$الحل$$

$$ان نعلم$$

$$\left(Arccos\left(x\right)\right)^{'}=\frac{-1}{\sqrt{1-x^{2}}} , x\in \left]-1,1\right[$$

$$\left(Arcsin\left(x\right)\right)^{'}=\frac{1}{\sqrt{1-x^{2}}} , x\in \left]-1,1\right[$$

$$\left(Arctan\left(x\right)\right)^{'}=\frac{1}{1+x^{2}} , x\in \left]-\infty ,+\infty \right[$$

$$\left(1\right) Arccos\left(x\right)+Arccos\left(-x\right)=π , x\in \left[-1,1\right]$$

$$f\left(x\right)= Arccos\left(x\right)+Arccos\left(-x\right) $$

$$ f^{'}\left(x\right)= \frac{-1}{\sqrt{1-x^{2}}}+\frac{1}{\sqrt{1-x^{2}}}=0 $$

$$ وعليه ثابتة f ان ومنه$$

$$∀x\in \left[-1,1\right] f\left(x\right)=c=f\left(0\right)$$

$$ Arccos\left(x\right)+Arccos\left(-x\right)=2 Arccos\left(x\right)=2\left(\frac{π}{2}\right)=π$$

$$∀x\in \left[-1,1\right]Arccos\left(x\right)+Arccos\left(-x\right)=π ومنه$$

$$\left(2\right) Arcsin\left(x\right)+Arccos\left(x\right)=\frac{π}{2} , x\in \left[-1,1\right] $$

$$f\left(x\right)= Arcsin\left(x\right)+Arccos\left(x\right) $$

$$ f^{'}\left(x\right)= \frac{1}{\sqrt{1-x^{2}}}-\frac{1}{\sqrt{1-x^{2}}}=0 $$

$$ وعليه ثابتة f ان ومنه$$

$$∀x\in \left[-1,1\right] f\left(x\right)=c=f\left(0\right)$$

$$ Arcsin\left(x\right)+Arccos\left(x\right)= Arcsin\left(0\right)+ Arccos\left(0\right)$$

$$ =\frac{π}{2}+0=\frac{π}{2}$$

$$∀x\in \left[-1,1\right] Arcsin\left(x\right)+Arccos\left(x\right)=\frac{π}{2} ومنه$$

$$\left(3\right) Arctan\left(x\right)+Arctan\left(\frac{1}{x}\right)=-\frac{π}{2} , x<0$$

$$f\left(x\right)= Arctan\left(x\right)+Arctan\left(\frac{1}{x}\right) $$

$$ f^{'}\left(x\right)= \frac{1}{1+x^{2}}-\left(\frac{1}{x}\right)^{2}\frac{1}{1+\left(\frac{1}{x}\right)^{2}}=\frac{1}{1+x^{2}}-\frac{1}{1+x^{2}}=0 $$

$$ وعليه ثابتة f ان ومنه$$

$$∀ x\in \left]-\infty ,0\right[ f\left(x\right)=c=f\left(-1\right)$$

$$ Arctan\left(x\right)+Arctan\left(\frac{1}{x}\right)= Arctan\left(-1\right)+Arctan\left(-1\right)$$

$$ =-\frac{π}{4}-\frac{π}{4}=-\frac{π}{2}$$

$$∀x\in \left]-\infty ,0\right[ Arctan\left(x\right)+Arctan\left(\frac{1}{x}\right)=-\frac{π}{2} ومنه$$

$$\left(4\right) cos\left(Arcsin\left(x\right)\right)=\sqrt{1-x^{2}} , x\in \left[-1,1\right]$$

$$Arcsin\left(x\right)=t , t\in \left[-\frac{π}{2}, \frac{π}{2}\right] نضع$$

$$x=sint=sin\left(Arcsin\left(x\right)\right) اذ$$

$$cos^{2}\left(Arcsin\left(x\right)\right)+sin^{2}\left(Arcsin\left(x\right)\right)=1 ان نعلم$$

$$cos^{2}\left(Arcsin\left(x\right)\right)=1-sin^{2}\left(Arcsin\left(x\right)\right)=1-x^{2}$$

$$cos\left(Arcsin\left(x\right)\right)=\sqrt{1-x^{2}}, x\in \left[-1,1\right] ومنه$$

$$\left(5\right) sin\left(Arctan\left(x\right)\right)=\frac{x}{\sqrt{1+x^{2}}}, x\in \left]-\infty ,+\infty \right[$$

$$Arctan\left(x\right)=t , t\in \left]-\frac{π}{2}, \frac{π}{2}\right[ نضع$$

$$ sin\left(Arctan\left(x\right)\right)=sint و x=tant ن اذ$$

$$ cost>0 فان t\in \left]-\frac{π}{2}, \frac{π}{2}\right[ كان اذا انه نعلم$$

$$\frac{1}{cos^{2}t}=1+tan^{2}t⟹cos^{2}t=\frac{1}{1+tan^{2}t}$$

$$⟹ cost=\sqrt{\frac{1}{1+tan^{2}t}}$$

$$⟹sint=tant×cost=tant\sqrt{\frac{1}{1+tan^{2}t}}=\frac{x}{\sqrt{1+x^{2}}}$$

$$⟹sin\left(Arctan\left(x\right)\right)=sint=\frac{x}{\sqrt{1+x^{2}}}, x\in \left]-\infty ,+\infty \right[ $$

$$\left(6\right) tan\left(Arccos\left(x\right)\right)=\frac{\sqrt{1-x^{2}}}{x} , x\in \left[-1,1\right]∖\left\{0\right\} $$

$$ x\in \left[-1,1\right]∖\left\{0\right\} Arccos\left(x\right)=t⟹ x=cost , t\in \left[0,π\right]∖\left\{\frac{π}{2}\right\} $$

$$ t\in \left[0,π\right]∖\left\{\frac{π}{2}\right\} sint >0و tan\left(Arccos\left(x\right)\right)=tant ن اذ$$

$$ tan\left(Arccos\left(x\right)\right)=tant=\frac{sint}{cost}=\frac{\sqrt{1-cos^{2}t}}{cost}= \frac{\sqrt{1-x^{2}}}{x} $$

$$كتايلي المعرفة f الدالة نعتبر : \left(9\right)التمرين$$

$$f\left(x\right)=Arccos\left(\frac{2x}{2-x}\right)$$

$$f الدالة تعريف مجموعة D\_{f} عين \left(1\right)$$

$$∀x\in D\_{f} f^{'}\left(x\right)=\frac{-4}{\left(2-x\right)\sqrt{4-4x-3x^{2}}} ان بين \left(2\right)$$

$$ البياني تمثيلها ارسم ثم f تغيرات جدول شكل \left(3\right)$$

$$الحل$$

$$ 2-x\ne 0 ∧-1 \leq \frac{2x}{2-x}\leq 1 ⟺ معرفة f$$

$$2-x\ne 0 ∧\frac{2x}{2-x}+1\geq 0 ∧ \frac{2x}{2-x}-1 \leq 0⟺$$

$$2-x\ne 0 ∧\frac{2+x}{2-x}\geq 0 ∧ \frac{3x-2}{2-x} \leq 0⟺$$

$$2-x\ne 0 ∧x\in \left[-2,2\right] ∧ x\in \left]-\infty ,\frac{2}{3}\right]∪\left[2,+\infty \right[ ⟺$$

$$ x\in \left[-2,\frac{2}{3}\right]⟺$$

$$D\_{f}=\left[-2,\frac{2}{3}\right]$$

$$∀x\in D\_{f} f^{'}\left(x\right)=\frac{-\left(\frac{2x}{2-x}\right)^{'}}{\sqrt{1-\left(\frac{2x}{2-x}\right)^{'2}}}=\frac{\frac{-4}{\left(2-x\right)^{2}}}{\sqrt{\frac{4-4x-3x^{2}}{\left(2-x\right)^{2}}}}$$

$$=\frac{-4}{\left(2-x\right)\sqrt{4-4x-3x^{2}}}<0 j$$

$$كتايلي المعرفة f الدالة نعتبر : \left(10\right)التمرين$$

$$f\left(x\right)=Arctan\left(\frac{1}{1-x}\right)$$

$$f الدالة تعريف مجموعة D\_{f} عين \left(1\right)$$

$$∀x\in D\_{f} f^{'}\left(x\right)=\frac{1}{\left(1-x\right)^{2}+1} ان بين \left(2\right)$$

$$ البياني تمثيلها ارسم ثم f تغيرات جدول شكل \left(3\right)$$

$$الحل$$

$$ 1-x\ne 0 ⟺ معرفة f$$

$$x\ne 1⟺ $$

$$D\_{f}=\left]-\infty ,1\right[∪\left]1,+\infty \right[$$

$$\lim\_{x\to -\infty }f\left(x\right)=\lim\_{x\to -\infty }Arctan\left(\frac{1}{1-x}\right)$$

$$ مركبة دالة نهاية باستخدام$$

$$\lim\_{x\to -\infty }\left(\frac{1}{1-x}\right)=0, \lim\_{x\to 0}Arctan\left(x\right)=0$$

$$\lim\_{s→1}Arctan\left(\frac{1}{1-x}\right)=0$$

$$\lim\_{x→1}f\left(x\right)=\lim\_{x→1}Arctan\left(\frac{1}{1-x}\right)$$

$$ مركبة دالة نهاية باستخدام$$

$$\lim\_{s→1}\left(\frac{1}{1-x}\right)=+\infty , \lim\_{x\to +\infty }Arctan\left(x\right)=\frac{π}{2}$$

$$\lim\_{x→1}Arctan\left(\frac{1}{1-x}\right)=\frac{π}{2}$$

$$\lim\_{x→1}f\left(x\right)=\lim\_{x→1}Arctan\left(\frac{1}{1-x}\right)$$

$$ مركبة دالة نهاية باستخدام$$

$$\lim\_{x→1}\left(\frac{1}{1-x}\right)=-\infty , \lim\_{x\to -\infty }Arctan\left(x\right)=-\frac{π}{2}$$

$$\lim\_{x→1}Arctan\left(\frac{1}{1-x}\right)=-\frac{π}{2}$$

$$\lim\_{x\to +\infty }f\left(x\right)=\lim\_{x\to +\infty }Arctan\left(\frac{1}{1-x}\right)$$

$$ مركبة دالة نهاية باستخدام$$

$$\lim\_{x\to +\infty }\left(\frac{1}{1-x}\right)=0, \lim\_{x\to 0}Arctan\left(x\right)=0$$

$$\lim\_{x\to +\infty }Arctan\left(\frac{1}{1-x}\right)=0$$

$$∀x\in D\_{f} f^{'}\left(x\right)=\frac{\left(\frac{1}{1-x}\right)^{'}}{1+\left(\frac{1}{1-x}\right)^{2}}=\frac{\frac{1}{\left(1-x\right)^{2}}}{\frac{\left(x-1\right)^{2}+1}{\left(1-x\right)^{2}}}$$

$$=\frac{1}{\left(x-1\right)^{2}+1}>0$$

$$ D\_{f} على تماما متزايدة f ومنه$$