Module :- Régulation et asservissement

Rappel :-

- Définition :- La Régulation est dit : réguler, Commander, guider, asservir, ...

- La Régulation automatique :- c'est la conduite d'un système en absence l'être humaine (sans l'action l'être humain)

- Objectif de la régulation d'un système :-
  - Simplifier
  - Minimiser le temps (augmentation de la productivité)
  - La précision (- Les circuits intégrés - Les pièces a Outil)

- Comment réguler un \$ (Système) ?:-

[*] Il faut Connaître : les variables (Les entrées et les sorties)

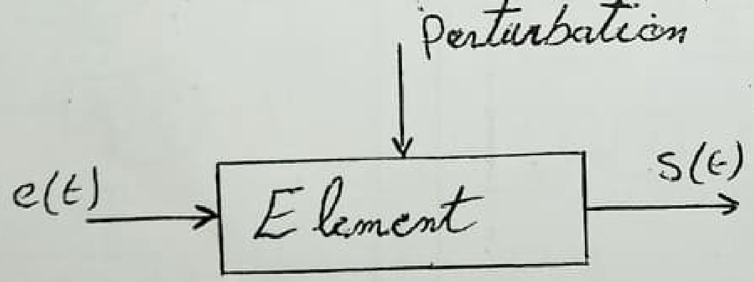
[*] " " " : les variables a réguler (ex : la vitesse) (ex : la pression)

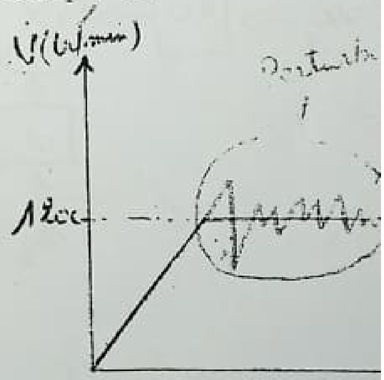
[*] Faire une relation (equation) entre les variables d'entrées et les variables de sorties $S(t) = f(e(t))$, la fonction de transfert

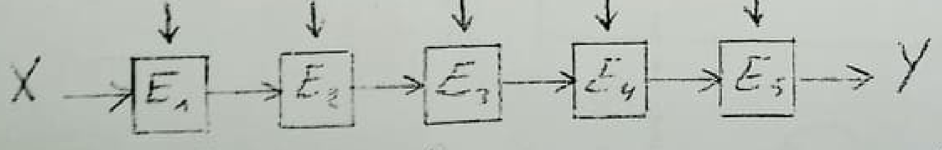
- Présentation graphique d'un \$ de réglage automatique :-

1) Diagramme fonctionnelle (Schéma fonctionnelle) :-

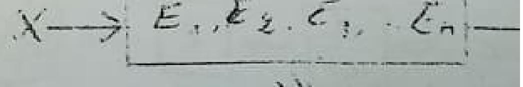
Ex :-



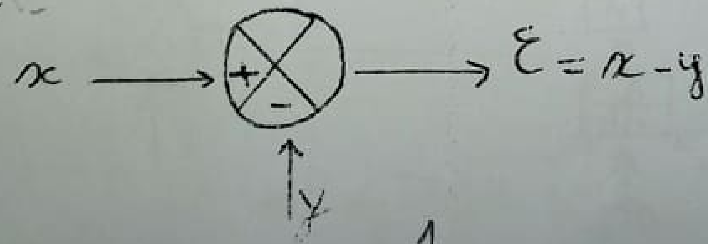




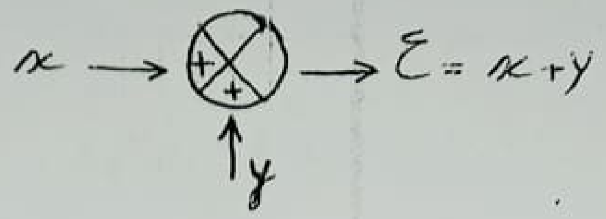
- Symbole fonctionnel :-



Comparateur :-

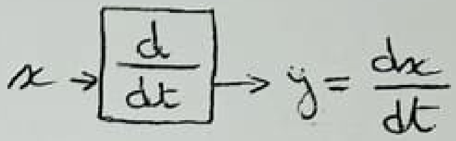


Sommateur :



Dérivateur :

$$x' = \frac{dx}{dt} = \dot{x}$$



2) Le graphe de fluence :

Exemple :

Representer le schema fonctionelle des relations suivantes

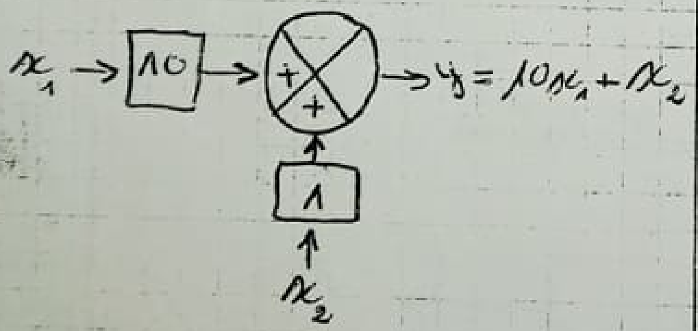
1) $y = 10x_1 + x_2$

2) $y = 5x_1 - 2x_2$

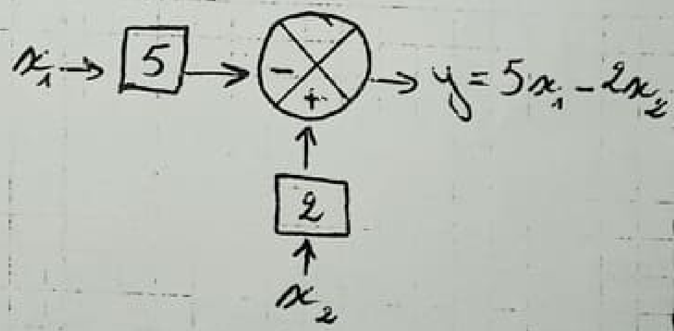
3) $y = 2x'_1 + x'_2$

– La solution :

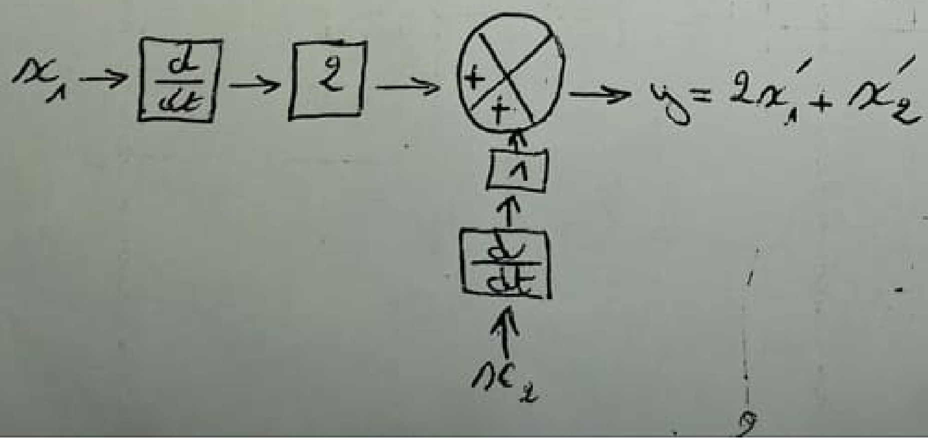
1) $y = 10x_1 + x_2$



2) $y = 5x_1 - 2x_2$



3) $y = 2x'_1 + x'_2$



Représenter le schéma fonctionnel des relations suivantes :-

(1) $y = 3x_1 - 2x_2 + x_3 - \frac{1}{2}x_4$

(2) $y = \frac{3}{2}\frac{dx_1}{dt} - \frac{dx_2}{dt} + x_3$

(3) $y = \sqrt{2}\int x_1\,dt - \frac{dx_2}{dt}$

(4) $y = \frac{1}{2}\frac{d^2x_1}{dt^2} + 2\frac{dx_2}{dt} - \int x_3$

(5) $y = \frac{1}{2}\int (x_1 + x_2)\,dt - 2\frac{d}{dt}(x_3 - x_4)$

(6) $y = \frac{1}{2}\int \left(x_1 - \frac{1}{2}x_2 + 3x_3\right) dt + \frac{d}{dt}(x_4 - x_5)$

(7) $y = \frac{5\,d^3x_1}{dt^3} - 2\frac{dx_2^2}{dt^2} + \frac{dx_3}{dt}$

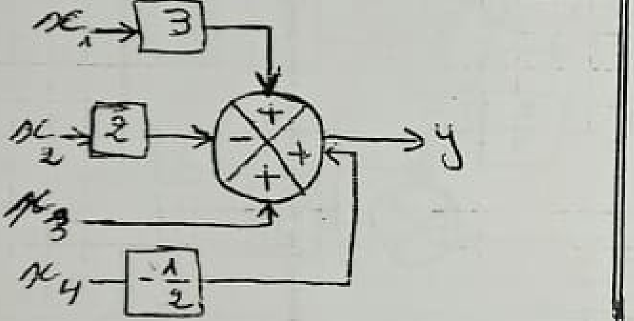
(8) $y = \frac{d}{dt}\left(\frac{2}{3}x_1 - x_2\right) - x_3$

(9) $y = \int x_1\,dt + \int \frac{1}{3}x_2\,dt + \frac{dx_3}{dt}$

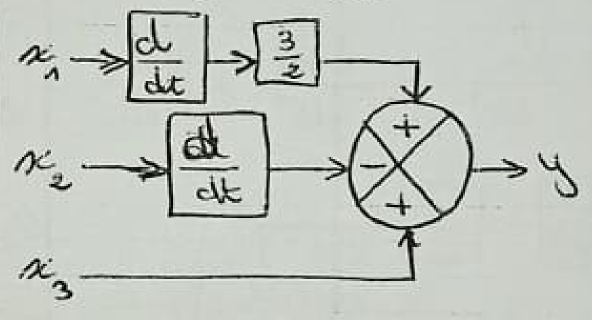
(10) $y = \frac{1}{\sqrt{2}}x_1' - \frac{3}{2}x_2'' + 4x_3'''$

* La solution :-

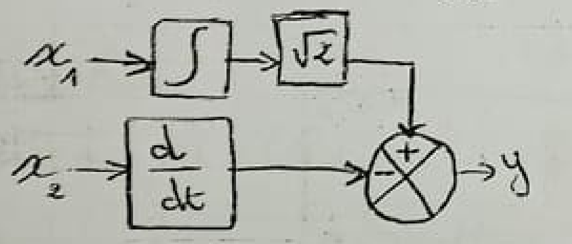
(1) $y = 3x_1 - 2x_2 + x_3 - \frac{1}{2}x_4$



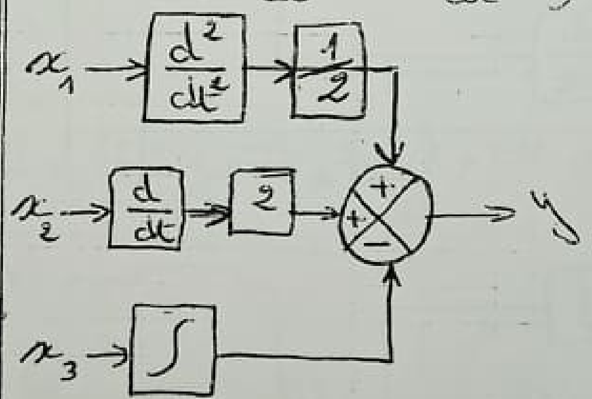
(2) $y = \frac{3}{2}\frac{dx_1}{dt} - \frac{dx_2}{dt} + x_3$



(3) $y = \sqrt{2}\int x_1\,dt - \frac{dx_2}{dt}$

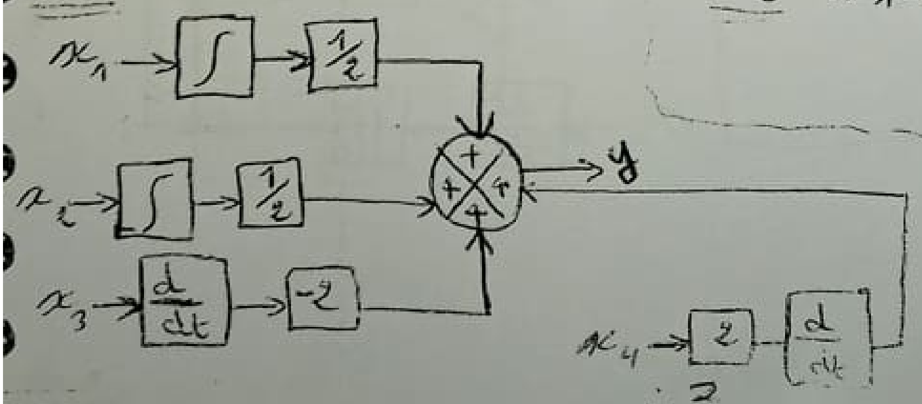


(4) $y = \frac{1}{2}\frac{d^2x_1}{dt^2} + 2\frac{dx_2}{dt} - \int x_3\,dt$

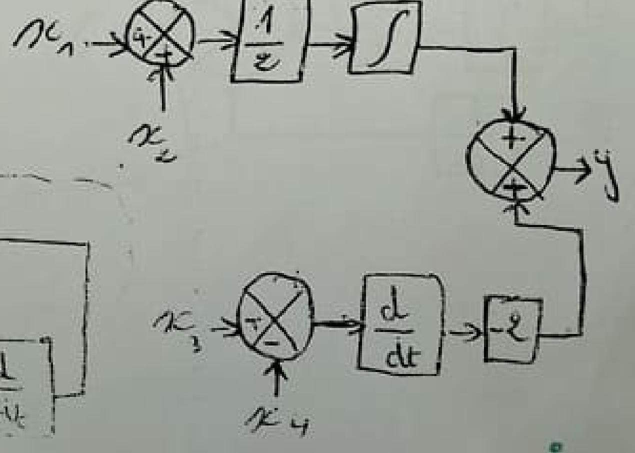


(5) $y = \frac{1}{2}\int (x_1 + x_2)\,dt - 2\frac{d}{dt}(x_3 - x_4)$

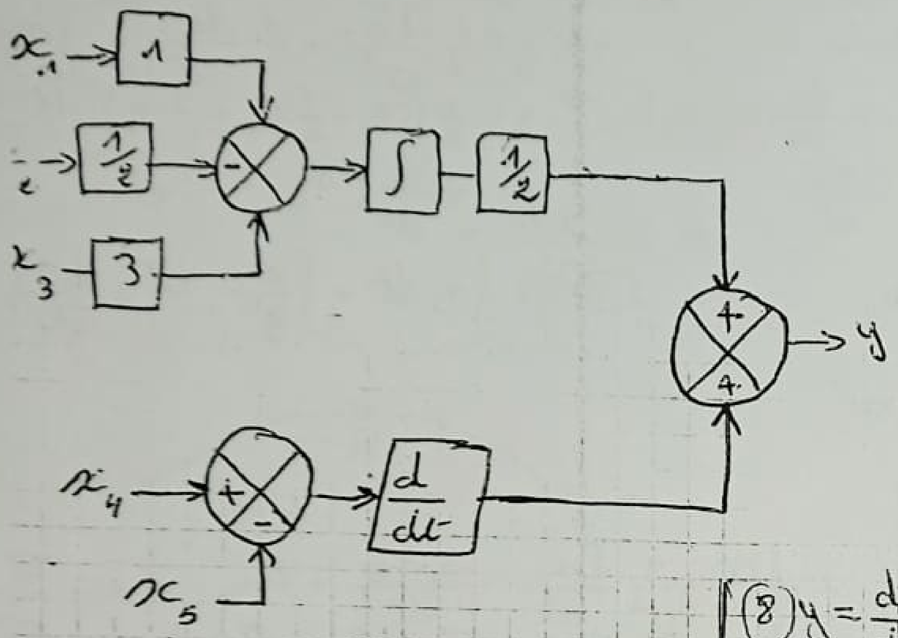
M_1



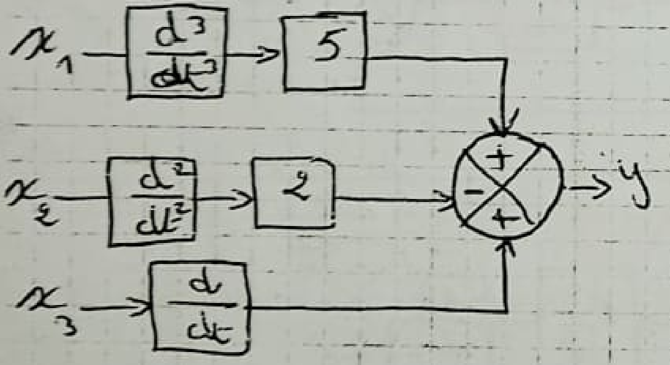
M_2



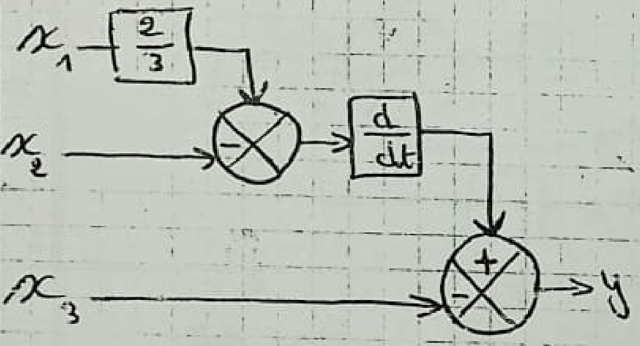
$y = \frac{1}{2}\int\left(x_1 - \frac{1}{2}x_2 + 3x_3\right)dt + \frac{d}{dt}\left(x_4 - x_5\right)$



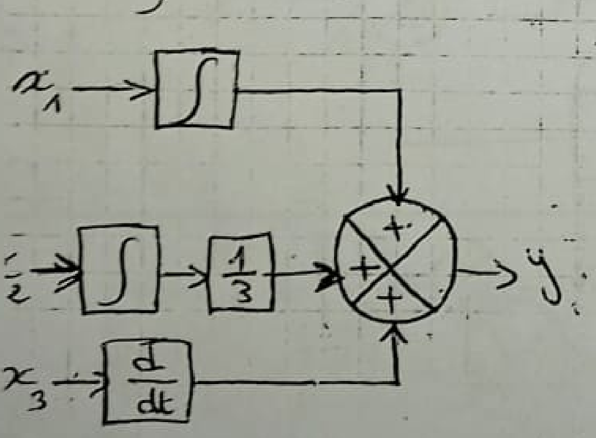
7) $y = 5\frac{d^3x_1}{dt^3} - 2\frac{d^2x_2}{dt^2} + \frac{dx_3}{dt}$



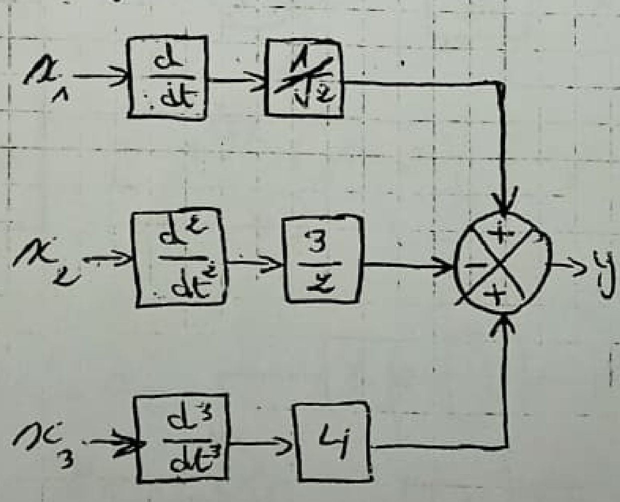
8) $y = \frac{d}{dt}\left(\frac{2}{3}x_1 - x_2\right) - x_3$



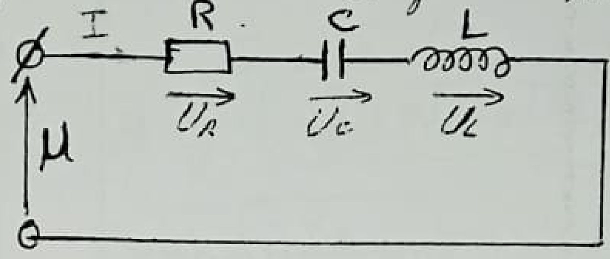
9) $y = \int x_1\,dt + \int \frac{1}{3}x_2\,dt + \frac{dx_3}{dt}$



10) $y =$



1) Ecrire la relation entre le signal d'entrée et le signal de sortie du circuit suivant :



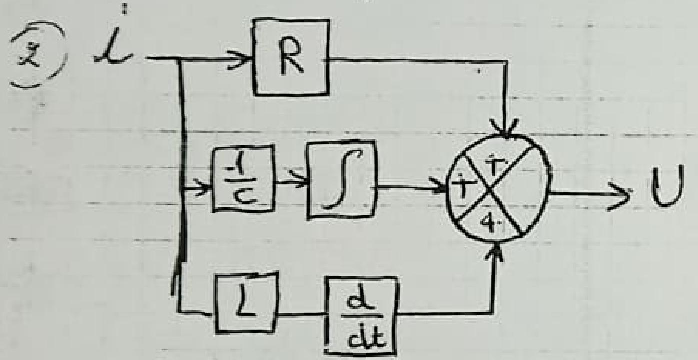
$\mathcal{U} = f(I)$?

2) Représenter le schéma fonctionnelle ?

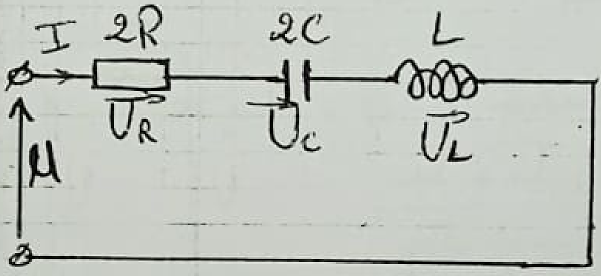
La solution :

1) On a $\mathcal{U} = U_R + U_C + U_L = R.I + \frac{1}{C}\int i\,dt + L\frac{di}{dt}$

2)



Exo3 :

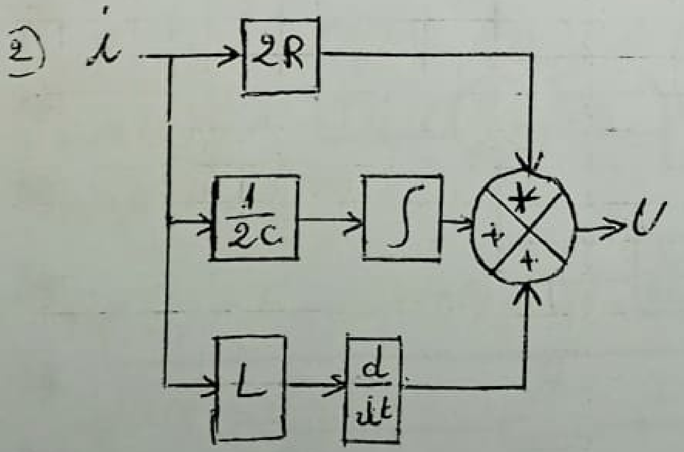


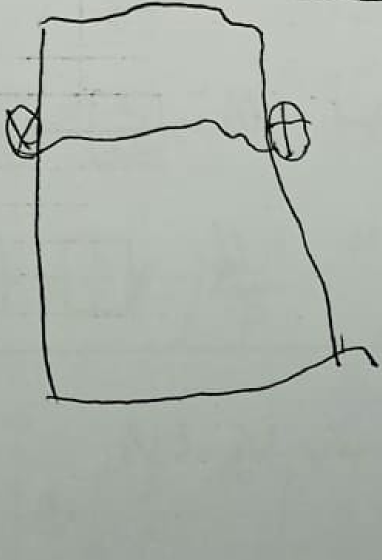
$\mathcal{U} = f(I)$

La solution

1) On a $\mathcal{U} = U_R + U_C + U_L = 2R.I + \frac{1\ c}{2C}\int i\,dt + L\frac{di}{dt}$

2)

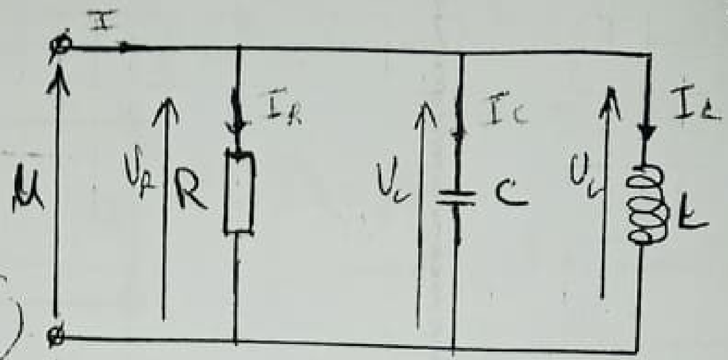




Exo 4 :
(meme questions de Exo 2)

Solution :

On a : $\boxed{I = I_R + I_C + I_L}$

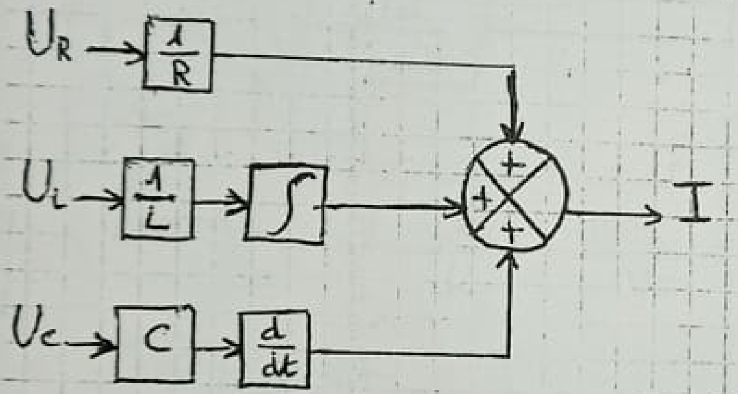


avec

$I_R = \dfrac{U_R}{R}$, $I_L = \dfrac{1}{L}\int U_L\,dt$

$I_C = C \cdot \dfrac{dU_C}{dt}$

Donc $I = \dfrac{U_R}{R} + \dfrac{1}{L}\int U_L\,dt + C \cdot \dfrac{dU_C}{dt}$

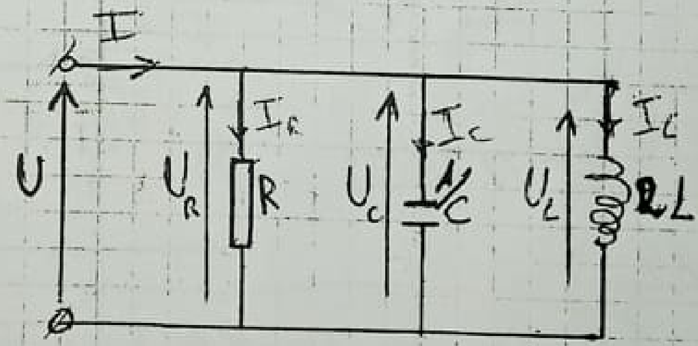


Exo 5 : (meme question).

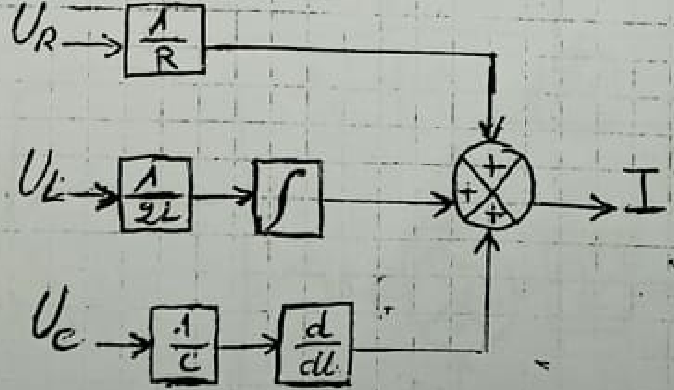
On a : $I = I_R + I_C + I_L$

$I_R = \dfrac{U_R}{R}$, $I_C = \dfrac{1}{C}\dfrac{dU_C}{dt}$

$I_L = \dfrac{1}{2L}\int U_L\,dt$



$U = U_R = U_C = U_L \Rightarrow$



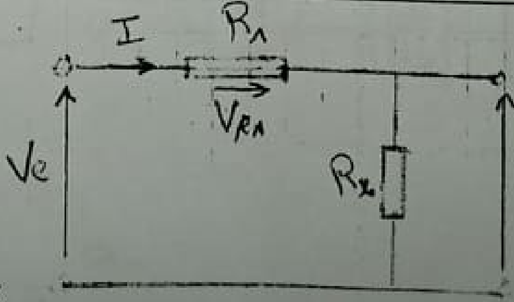
Exo 6 :

Ecrire la relation entre Ve et Vs

$Ve = f(Vs) = ?$

Ecrire la relation entre Vs et Ve

$Vs = f(Ve) = ?$



• Représenter le schéma fonctionnel ?

- La solution :

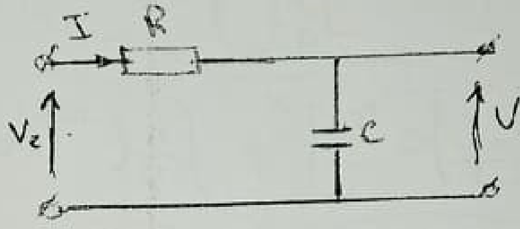
1. $V_e = (R_1 + R_2) I$ et $V_s = R_2 . I \Rightarrow I = \frac{V_s}{R_2}$ Donc

$V_e = (R_1 + R_2) . \frac{V_s}{R_2} \Leftrightarrow V_e = (R_1 + R_2) \frac{V_s}{R_2}$ ou $V_s = \frac{R_2}{R_1 + R_2} . V_e$

Donc : $V_s \rightarrow \boxed{\frac{R_1 + R_2}{R_2}} \rightarrow V_e$ $\quad V_e \rightarrow \boxed{\frac{R_2}{R_1 + R_2}} \rightarrow V_s$

Exo 7

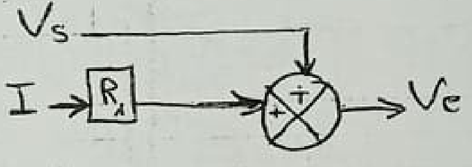
. Même question comme l'Exo 6 ?

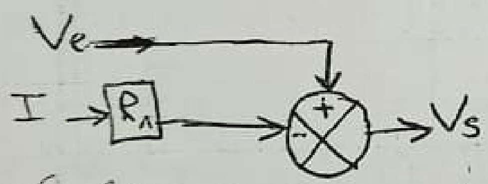


* Suite Exo 6 :

On a : $V_e = (R_1 + R_2) I = R_1 I + R_2 I$ et $V_s = I R_2$

$$\Rightarrow \begin{cases} V_e = I R_1 + V_s \\ V_s = V_e - I R_1 \end{cases} \Rightarrow$$





• La solution Exo 7 :

$V_e = RI + V_s$ et $V_s = V_e - RI$ ou $V_s = \frac{1}{C} \int i \, dt$

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* Transformation de Laplace :

Exo 1 $\quad u(t) = 1$, $u(t) = E_0$, $u(t) = \frac{1}{2}$

• $u(t) = 1 \Rightarrow u(P) = \int_0^{\infty} 1 . e^{-Pt} dt = 1 \left[-\frac{1}{P} e^{-Pt} \right]_0^{\infty} = \left[-\frac{1}{P} \left[e^{-P\infty} - e^{-P.0} \right] \right]$

$\Rightarrow u(P) = \frac{1}{P}$

• $u(t) = E_0 \Leftrightarrow u(P) = \int_0^{\infty} E_0 \, e^{-Pt} dt = E_0 \left[\frac{-1}{P} e^{-Pt} \right]_0^{\infty} = \frac{-E_0}{P} \left[e^{-P\infty} - e^{-P.0} \right]$

$\Rightarrow u(P) = \frac{E_0}{P}$

• $u(t) = \frac{1}{2} \Rightarrow u(P) = \int_0^{\infty} \frac{1}{2} . e^{-Pt} dt = \frac{1}{2} \left[-\frac{1}{P} e^{-Pt} \right]_0^{\infty} = -\frac{1}{2P} \left[e^{-P\infty} - e^{-P0} \right]$

$\Rightarrow u(P) = \frac{1}{2P}$

Ex 0 2 : $u(t) = t$, $u(t) = t^2$, $u(t) = \frac{1}{2}t + 3t^2$.

Ex 0 1 : $f(t) = t^n \Rightarrow F(P) = \mathcal{L}[f(t)] = \int_0^{\infty} f(t)e^{-Pt}\,dt$

$\Rightarrow F(P) = \int_0^{\infty} \underbrace{t^n}_{u} \underbrace{e^{-Pt}dt}_{dv}$ Donc $\begin{cases} U = t^n \\ du = n t^{n-1}\,dt \end{cases}$ et $\begin{cases} dv = e^{-Pt}\,dt \\ v = -\frac{1}{P}e^{-Pt} \end{cases}$

$$\boxed{\int_0^{+\infty} U\,dv = [u.v]_0^{+\infty} - \int_0^{+\infty} du.v}$$ où $\boxed{[U.v]_0^{+\infty} \longrightarrow 0}$ (Toujours)

$$F(P) = \left[t^n\left(-\frac{1}{P}\right)e^{-Pt}\right]_0^{\infty} + \int_0^{\infty} \frac{n}{P}t^{n-1}e^{-Pt}\,dt = \frac{n}{P}\int_0^{\infty} t^{n-1}e^{-Pt}\,dt$$

Donc $\boxed{F(P) = \mathcal{L}(t^n) = \frac{n!}{P^{n+1}}}$ $\boxed{F(P) = \mathcal{L}(t^n e^{-at}) = \frac{n!}{(P+a)^{n+1}}}$

Ex 2 : $f(t) = t$ $\Rightarrow F(P) = \mathcal{L}[f(t)] = \int_0^{\infty} f(t)e^{-Pt}\,dt = \int_0^{\infty} t e^{-Pt}\,dt$

où $\begin{cases} u = t \\ du = dt \end{cases}$ et $\begin{cases} dV = e^{-Pt}\,dt \\ V = -\frac{1}{P}e^{-Pt} \end{cases}$ Donc

$$\int_0^{\infty} U\,dV = [UV]_0^{\infty} - \int_0^{\infty} du.V \Rightarrow \int_0^{\infty} t e^{-Pt}\,dt = \left[t\left(-\frac{1}{P}\right)e^{-Pt}\right]_0^{\infty}$$

$$- \int_0^{\infty} 1\left(-\frac{1}{P}\right)e^{-Pt}\,dt = \frac{1}{P}\int_0^{\infty} e^{-Pt}\,dt = \frac{1}{P}\left[-\frac{1}{P}\left(e^{-Pt}\right)\right]_0^{\infty} = -\frac{1}{P}\left(\frac{1}{P}\right)(-1)$$

$$\Rightarrow F(P) = \mathcal{L}[f(t)] = \mathcal{L}[t] = \frac{1}{P^2}$$

Ex 3 : $f(t) = t^2$

$F(P) = \mathcal{L}[f(t)] = \int_0^{\infty} t^2 e^{-Pt}\,dt \Rightarrow \begin{cases} u = t^2 \\ du = 2t\,dt \end{cases}$ et $\begin{cases} dV = e^{-Pt}\,dt \\ V = -\frac{1}{P}e^{-Pt} \end{cases}$

$$\Rightarrow \int_0^{\infty} U.dV = \int_0^{\infty} t^2 e^{-Pt}\,dt = \left[t^2\left(-\frac{1}{P}\right)e^{-Pt}\right]_0^{\infty} - \int_0^{\infty} 2t\left(-\frac{1}{P}\right)e^{-Pt}\,dt$$

$$= \frac{2}{P}\left[\cancel{te^{-Pt}}\right]_0^{\infty} = \frac{2}{P} \times \left(\frac{1}{P^2}\right) = \frac{2}{P^3} \Rightarrow F(P) = \mathcal{L}(t^2) = \frac{2}{P^3}$$

Exc4 : $f(t) = \frac{1}{2}t + 3t^2$

$$F(p) = \mathcal{L}[f(t)] = \int_0^{\infty}\left(\frac{1}{2}t + 3t^2\right)e^{-pt}\,dt = \frac{1}{2}\int_0^{\infty} t\,e^{-pt}\,dt + 3\int_0^{\infty} t^2 e^{-pt}\,dt$$

$$= \frac{1}{2}\left[\frac{1}{p^2}\right] + 3\left[\frac{2}{p^3}\right] = \frac{p+12}{2p^3} \Rightarrow F(p) = \frac{p+12}{2p^3}$$

Exc5 : $f(t) = e^{-at}$ avec $a > 0$ ... $\mathbb{R}$

$$\Rightarrow F(p) = \mathcal{L}[f(t)] = \int_0^{\infty} e^{-at}\cdot e^{-pt}\,dt = \int_0^{\infty} e^{(-a-p)t}\,dt = \frac{-1}{a+p}\left[e^{-(a+p)t}\right]_0^{\infty}$$

$$= \frac{1}{a+p} \Rightarrow F(p) = \frac{1}{a+p}$$

Exc6 : $f(t) = \cos(wt)$

$\cos(wt) = \left(e^{jwt} + e^{-jwt}\right)/2$ Donc $\cos wt = \dfrac{e^{jwt} + e^{-jwt}}{2}$

$$F(p) = \mathcal{L}[f(t)] = \mathcal{L}[\cos wt] = \int_0^{\infty} \cos wt\, e^{-pt}\,dt = \int_0^{\infty}\left(\frac{e^{jwt} + e^{-jwt}}{2}\right)e^{-pt}\,dt$$

$$= \frac{1}{2}\int_0^{\infty}\left(e^{jwt} + e^{-jwt}\right)e^{-pt}\,dt = \frac{1}{2}\int_0^{\infty} e^{jwt}\cdot e^{-pt}\,dt + \frac{1}{2}\int_0^{\infty} e^{-jwt}\cdot e^{-pt}\,dt$$

$$= \frac{1}{2}\int_0^{\infty} e^{(jw-p)t}\,dt + \frac{1}{2}\int_0^{\infty} e^{-(jw+p)t}\,dt = \frac{1}{2}\int_0^{\infty} e^{-(-jw+p)t}\,dt + \frac{1}{2}\int_0^{\infty} e^{-(jw+p)t}\,dt$$

$$= \frac{-1}{2}\left(\frac{1}{p-jw}\right)\left[e^{-(-jw+p)t}\right]_0^{\infty} - \frac{1}{2}\left(\frac{1}{jw+p}\right)\left[e^{-(jw+p)t}\right]_0^{\infty}$$

$$= \frac{1}{2}\left[\frac{1}{p-jw} + \frac{1}{jw+p}\right] = \frac{1}{2}\left[\frac{jw+p+p-jw}{p^2-(jw)^2}\right] = \frac{1}{2}\left[\frac{2p}{p^2-((-1)w^2)}\right]$$

$(j^2 = -1)$

$$= \frac{p}{p^2+w^2} \Rightarrow F(p) = \frac{p}{p^2+w^2}$$

Exc7 : $\sin\left(2t + \frac{\pi}{4}\right)$

1) $\sin(wt) = \dfrac{1}{2j}\left[e^{jwt} - e^{-jwt}\right]$

2) $\sin\left(2t + \frac{\pi}{4}\right) = \sin(2t)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right).\cos(2t)$

II- Transformées inverse de laplace ($\mathcal{L}^{-1}$) :

Exo 1. $F(P) = \dfrac{2}{P(P+1)(P-2)} \Rightarrow F(P) = 2\left[\dfrac{A}{P} + \dfrac{B}{P+1} + \dfrac{C}{P-2}\right]$

On a $P(P+1)(P-2) = 0 \Leftrightarrow$ il y a 3 solutions $(P=0, P=-1, P=2)$

Donc :

$$A = \lim_{P\to 0}\left[P \cdot \frac{1}{P(P+1)(P-2)}\right] = \left[\frac{1}{(0+1)(0-2)}\right] \Rightarrow \boxed{A = -\frac{1}{2}}$$

$$B = \lim_{P\to -1}\left[(P+1)\frac{1}{P(P+1)(P-2)}\right] = \left[\frac{1}{(-1)(-1-2)}\right] \Rightarrow \boxed{B = \frac{1}{3}}$$

$$C = \lim_{P\to 2} = \left[\cancel{(P-2)}\frac{1}{P(P+1)\cancel{(P-2)}}\right] = \frac{1}{2(2+1)} \Rightarrow \boxed{C = \frac{1}{6}}$$

Donc : $F(P) = 2\left[\dfrac{(-1/2)}{P} + \dfrac{(1/3)}{(P+1)} + \dfrac{(1/6)}{(P-2)}\right] = 2\left[(-1/2)\left(\dfrac{1}{P}\right) + \dfrac{1}{3}\left(\dfrac{1}{P+1}\right) + \dfrac{1}{6}\left(\dfrac{1}{P-2}\right)\right]$

$$\Rightarrow F(t) = 2\left[-\frac{1}{2}(1) + \frac{1}{3}e^{-t} + \frac{1}{6}e^{2t}\right]$$

$$\Rightarrow \boxed{F(t) = -1 + \frac{1}{3}\left[2e^{-t} + e^{2t}\right]} \quad \text{où } t \geq 0$$

Exo 2.

Determiner la Transformation de laplace de l'equation differentielle :

$$x(t) = \frac{d^2y(t)}{dt^2} + 3 \cdot \frac{dy(t)}{dt} + y(t)$$

On a : $\mathcal{L}\left[\dfrac{d^2y(t)}{dt^2}\right] = P^2 \cdot Y(P) - Py(0) - y'(0)$

et les conditions initiales $\{y(0) = 1 \ , \ y'(0) = 2\}$ Donc

$$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = P^2 Y(P) - P(1) - 2$$

$$\mathcal{L}\left[3\frac{dy(t)}{dt}\right] = 3\left[P \cdot Y(P) - y(0)\right] = 3\left[PY(P) - 1\right]$$

$$\mathcal{L}\left[y(t)\right] = Y(P)$$

Donc $\mathcal{L}[x(t)] = X(P) = P^2 Y(P) - P - 2 + 3P\,Y(P) - 3 + Y(P)$

$$\Rightarrow \boxed{X(P) = Y(P)\left[P^2 + 3P + 1\right] - P - 5}$$

Exc3

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} + 6y(t) = \frac{d^2x(t)}{dt^2} - x(t)$$

où $\frac{d^3y(t)}{dt^3} = P^3 Y(P) - P^2 y(0) - P y'(0) - y''(0)$ et les conditions initiales $\{ y(0) = 0,\ y'(0) = 0,\ y''(0) = 1$

La solution :

- $\frac{d^3y(t)}{dt^3} = P^3Y(P) - P^2y(0) - Py'(0) - y''(0) = P^3Y(P) - P^2 \cdot 0 - P \cdot 0 - 1 = P^3Y(P) - 1$
- $\frac{d^2y(t)}{dt^2} = P^2Y(P) - Py(0) - y'(0) = P^2Y(P) - P \cdot 0 - 0 = P^2Y(P)$
- $\frac{dy(t)}{dt} = PY(P) - y(0) = PY(P) - 0 = PY(P)$ / $y(t) = Y(P)$
- $\frac{d^2x(t)}{dt^2} = P^2X(P) - Px(0) - x'(0) = P^2X(P)$; $x(t) = X(P)$ Donc

$$P^3Y(P) - 1 + 3P^2Y(P) - PY(P) + 6Y(P) = P^2X(P) - X(P)$$

$$\Leftrightarrow \boxed{Y(P)\left[P^3 + 3P^2 + (-P + 6)\right] - 1 = X(P)\left[P^2 - 1\right]}$$

Exc4 : Déterminer de T.P pour :

$$2\frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} + x(t) = \frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t)$$

La solution

- $\frac{d^2x(t)}{dt^2} = P^2X(P) - Px(0) - x'(0) = P^2X(P)$ / $x(t) = X(P)$
- $\frac{dx(t)}{dt} = PX(P) - x(0) = PX(P)$
- $\frac{d^2y(t)}{dt^2} = P^2Y(P) - Py(0) - y'(0) = P^2Y(P)$ / $y(t) = Y(P)$
- $\frac{dy(t)}{dt} = PY(P) - y(0) = PY(P)$

Donc : $2P^2X(P) + PX(P) + X(P) = P^2Y(P) + 2PY(P) + 3Y(P)$

$$\Leftrightarrow X(P) = \left[2P^2 + P + 1\right] = Y(P)\left[P^2 + 2P + 3\right]$$

Exo 5 :

Déterminer la T.P de l'équation diff :

$\ddot{y}(t) + 4\dot{y}(t) + 20y(t) = 4$ où $\begin{cases} y(0) = -2 \\ \dot{y}(0) = 0 \end{cases}$

Solution :

$\mathcal{L}[\ddot{y}(t)] = P^2Y(P) - Py(0) - y'(0) = P^2Y(P) - P(-2) - 0 = P^2Y(P) + 2P$

$\mathcal{L}[\dot{y}(t)] = PY(P) - y(0) - y'(0) = PY(P) - (-2) - 0 = PY(P) + 2$

$\mathcal{L}[y(t)] = Y(P)$ et $4 \Rightarrow \mathcal{L}[4] = 4/P$

Donc $P^2Y(P) + 2P + 4(PY(P) + 2) + 20(Y(P)) = \frac{4}{P}$

$\Rightarrow P^2Y(P) + 2P + 4PY(P) + 8 + 20Y(P) = \frac{4}{P}$

$\Rightarrow Y(P)[P^2 + 4P + 20] + 2P + 8 = \frac{4}{P}$

$\Rightarrow Y(P)[P^2 + 4P + 20] = \frac{4}{P} - 2P - 8$

$\Leftrightarrow Y(P)[P^2 + 4P + 20] = \frac{4 - 2P^2 - 8P}{P} \Leftrightarrow Y(P) = \frac{-2P^2 - 8P + 4}{P[P^2 + 4P + 20]}$

Exo 6 :

$\frac{d^3y(t)}{dt^3} + 5\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} = 0$ où $\begin{cases} y(0) = 3 \\ \dot{y}(0) = -2 \\ \ddot{y}(0) = 7 \end{cases}$

Solution.

$\mathcal{L}\left[\frac{d^3y(t)}{dt^3}\right] = P^3Y(P) - P^2y(0) - P\dot{y}(0) - \ddot{y}(0) = P^3Y(P) - 3P^2 + 2P - 7$

$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = P^2Y(P) - Py(0) - \dot{y}(0) = P^2Y(P) - 3P + 2$

$\mathcal{L}\left[\frac{dy(t)}{dt}\right] = PY(P) - y(0) = PY(P) - 3$ Donc

$P^3Y(P) - 3P^2 + 2P - 7 + 5P^2Y(P) - 15P + 10 + 6PY(P) - 18 = 0$

$\Rightarrow Y(P)[P^3 + 5P^2 + 6P] - 3P^2 - 13P - 15 = 0$

$\Rightarrow Y(P) = \frac{3P^2 + 13P + 5}{P^3 + 5P^2 + 6}$

$$\frac{d^2y(t)}{dt^2} + y(t) = \frac{3}{2}\sin(2t) \quad \text{où} \quad \begin{cases} y(0) = 1 \\ \dot{y}(0) = 2 \end{cases}$$

Solution

$$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = P^2Y(P) - Py(0) - \dot{y}(0) = P^2Y(P) - P - 2$$

$$\mathcal{L}[y(t)] = Y(P) \quad \text{et} \quad \mathcal{L}[\sin(2t)] = \frac{(2)}{P^2+(2)^2} = \frac{2}{P^2+4}$$

$$P^2Y(P) - P - 2 + Y(P) = \frac{3}{2}\left[\frac{2}{P^2+4}\right]$$

$$\Leftrightarrow Y(P)\left[P^2+1\right] - P - 2 = \frac{3}{P^2+4} \Leftrightarrow Y(P)\left[P^2+1\right] = \frac{3}{P^2+4} + P + 2$$

$$\Leftrightarrow Y(P)\left[P^2+1\right] = \frac{3 + P(P^2+4) + 2(P^2+4)}{(P^2+4)}$$

$$\Leftrightarrow Y(P) = \frac{P^3+2P^2+4P+11}{(P^2+1)(P^2+4)}$$

Ex 08.

même Exercice 07 mais les conditions initiales sont $\begin{cases} y(0) = -3 \\ \dot{y}(0) = -1 \end{cases}$

$$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right] = P^2Y(P) - Py(0) - \dot{y}(0) = P^2Y(P) + 3P + 1$$

$$\mathcal{L}[y(t)] = Y(P), \quad \mathcal{L}\left[\frac{3}{2}\sin(2t)\right] = \frac{3}{P^2+4} \quad \text{Donc}$$

$$P^2Y(P) + 3P + 1 + Y(P) = \frac{3}{P^2+4} \Leftrightarrow Y(P)\left[P^2+1\right] = -3P - 1 + \frac{3}{P^2+4}$$

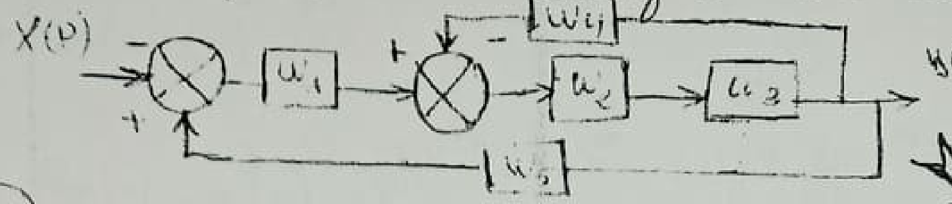
$$\text{où } -3P - 1 + \frac{3}{P^2+4} = \frac{-3P^3 - 12P - P^2 - 4 + 3}{P^2+4} = \frac{-3P^3 - 12P - P^2 - 1}{P^2+4}$$

$$\Rightarrow Y(P)\left(P^2+1\right) = \frac{-3P^3 - P^2 - 12P - 1}{P^2+4} \Leftrightarrow -3P^3 - P^2 - 12P - 1 = Y(P)(P^2+1)(P^2+4)$$

$$\Leftrightarrow Y(P) = \frac{-3P^3 - P^2 - 12P - 1}{(P^2+1)(P^2+4)}$$

Exo1 :

Simplifié, puis calculer la fonction de Transfert de bloc fonctionnel suivant :



$W_a = W_2 \cdot W_3$

$W_b = \frac{W_a}{1 \pm W_a \cdot W_4}$

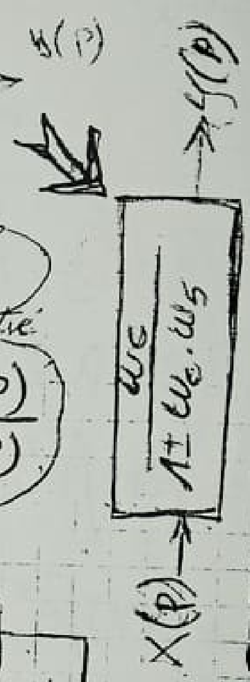
$W_c = W_b \cdot W_1$

$W_d = \frac{W_c}{1 \pm W_c \cdot W_5}$

Donc $F.T = \frac{W_c}{1 \pm W_c \cdot W_5} = \frac{S(P)}{E(P)} = \frac{y(P)}{X(P)}$

Sortie

Entrée



Exo2 :

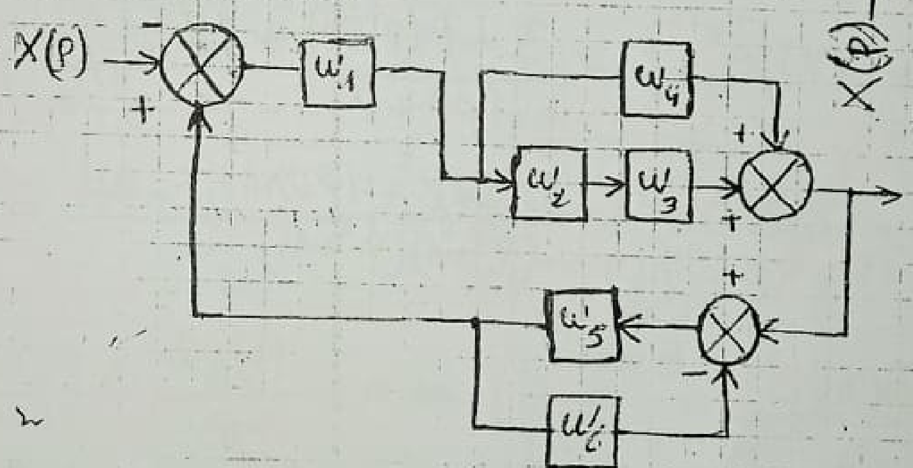
$W_a = W_2 \cdot W_3$

$W_b = W_a + W_4$

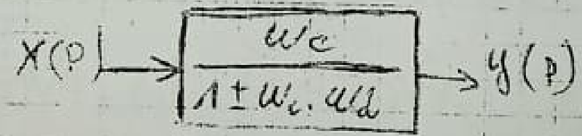
$W_c = W_1 \cdot W_b$

$W_d = \frac{W_5}{1 \pm W_5 \cdot W_6}$

$W_e = \frac{W_c}{1 \pm W_c \cdot W_d}$ Donc :



$F.T = \frac{W_c}{1 \pm W_c \cdot W_d} = \frac{y(P)}{X(P)}$



Exo3 :

1ère Solution :

Déplacement aval :

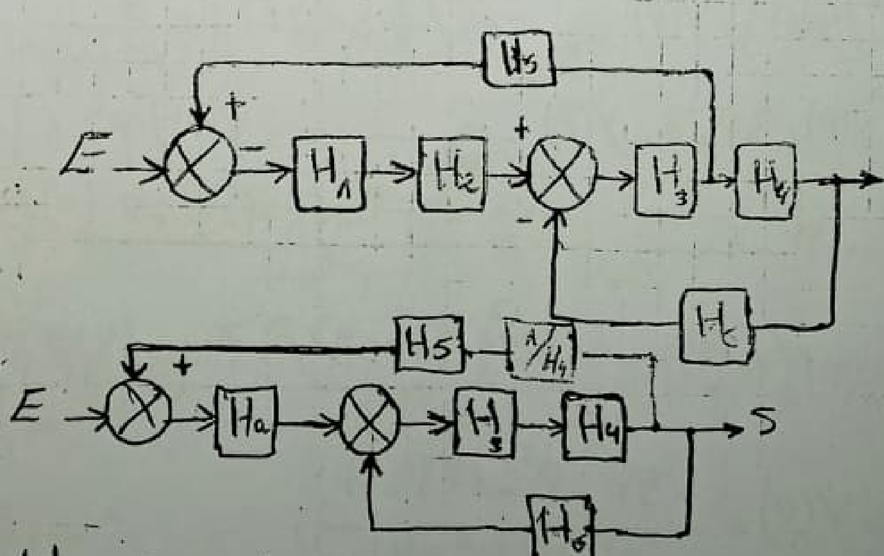
$H_a = H_1 \cdot H_2$

$H_b = H_3 \cdot H_4$

$H_c = H_5 \cdot \left(\frac{1}{H_4}\right)$

$H_d = \frac{H_b}{1 \pm H_b \cdot H_6}$

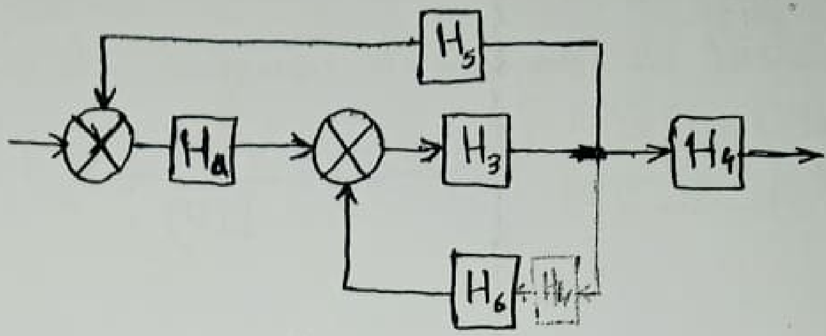
$H_e = H_a \cdot H_d$



$H_f = \frac{H_e}{1 \pm H_e \cdot H_c}$

2ème Solution :-

Déplacement amont :-



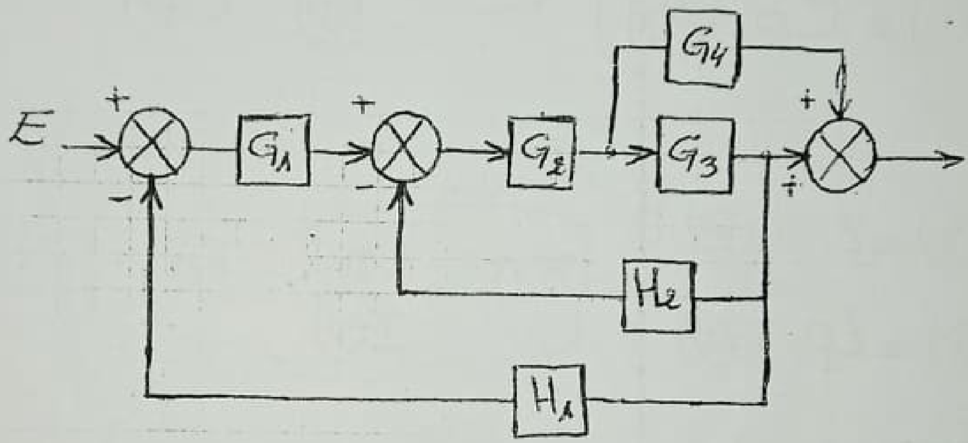
$H_a = H_1 . H_2$

$H_b = H_6 . H_4$

$H_c = \dfrac{H_3}{1 \pm H_3 . H_b}$

$H_d = H_a . H_c$, $H_e = \dfrac{H_d}{1 \pm H_d . H_5}$ $H_f = H_e . H_4$

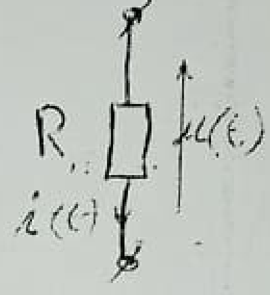
Ex 04 :-



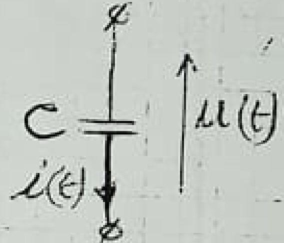
Rappel :-

Calcul de fonction de Transfert de circuit electrique

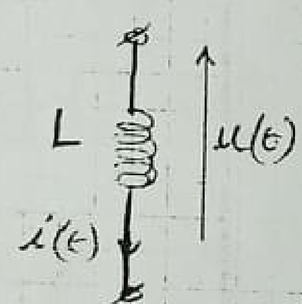
$$\left.\begin{array}{l} u(t) = R\, i(t) \\ V(P) = R.\, I(P) \end{array}\right\} \Rightarrow Z(P) = \frac{V(P)}{I(P)} = R$$



$$\left.\begin{array}{l} i(t) = C.\, \dfrac{du(t)}{dt} \\ I(P) = CP.\, V(P) \end{array}\right\} \Rightarrow Z(P) = \frac{V(P)}{I(P)} = \frac{1}{CP}$$



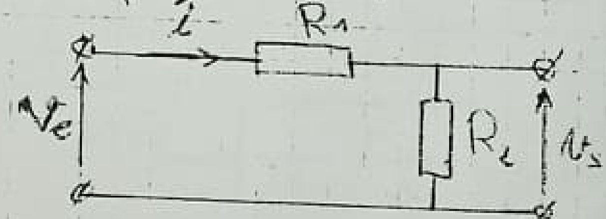
$$\left.\begin{array}{l} u(t) = L.\, \dfrac{di(t)}{dt} \\ V(P) = LP.\, I(P) \end{array}\right\} \Rightarrow Z(P) = \frac{V(P)}{I(P)} = LP$$



Exercice 01 :

Calculer la F.T de circuit electrique suivant :

$$F.T = W(P) = \frac{V_s}{V_e}$$

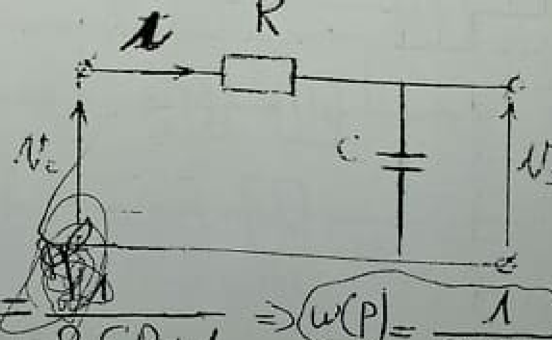


La solution :-

$$\left.\begin{array}{l} V_e = (R_1 + R_2)\, i \Rightarrow V(P)_e = (R_2 + R_1)\, I(P) \\ V_s = R_2.\, i \Rightarrow V(P)_s = R_2.\, I(P) \end{array}\right\} \Rightarrow W(P) = \frac{V_s}{V_e} = \frac{R_2\, \cancel{I(P)}}{(R_1 + R_2)\, \cancel{I(P)}}$$

$$\Rightarrow W(P) = \frac{R_2}{R_1 + R_2}$$

Exercice 02 :-



$$V_e = i.R + i\,\frac{1}{CP} \Rightarrow V_e = i\left(R + \frac{1}{CP}\right)$$

$$V_e = \left(\frac{CP.R + 1}{CP}\right) i$$

$$V_s = i\,\frac{1}{CP} \quad \text{Donc} \quad \frac{V_s}{V_e} = \frac{i\,\dfrac{1}{CP}}{i\,\dfrac{(CP.R + 1)}{CP}} = \frac{1}{R.CP + 1} \Rightarrow W(P) = \frac{1}{RCP + 1}$$