UNIVERSITÉ ECHAHID HAMMA LAKHDAR EL-OUED

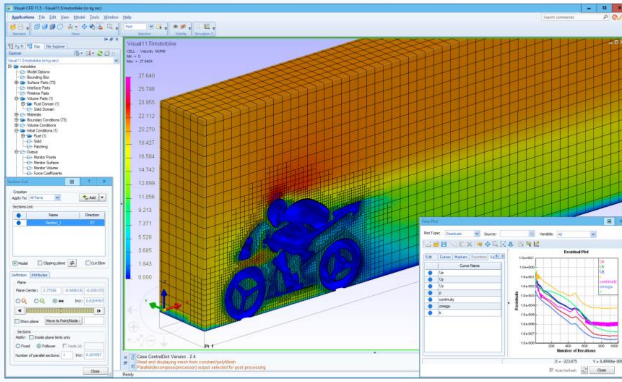
FACULTÉ DE TECHNOLOGIE

Cours de Mécanique des fluides Approfondie

Filière : Génie Mécanique
Spécialité : Energétique & Energies Renouvelables en Mécanique

Présentée par :

ATIA Abdelmalek



2020-2021

Semestre :1
Unité d'enseignement : UEF 1.1.1
Matière : Mécanique des fluides approfondie
VHS: 67 h30 (Cours: 3h00, TD: 1h30)
Crédits : 6
Coefficient : 3

Objectifs de l'enseignement :

Le but de la matière est de développer les connaissances de base de l'étudiant. La spécialité énergétique est étroitement liée à la phénoménologie des écoulements visqueux et turbulents observés dans les systèmes énergétiques, leur compréhension et analyse sont indispensables. L'imprégnation de l'étudiant des lois et modèles physiques et mathématiques de ces écoulements souvent complexes est un des fondamentaux de la spécialité dans l'acquisition d'un enseignement consistant nécessaire pour la recherche.

Connaissances préalables recommandées :

Base de Mécanique des fluides
Les mathématiques
Les méthodes numériques

Contenu de la matière :

Chapitre 1 : Dynamique des fluides et équations de transport : description du mouvement, tenseurs, dérivée particulaire, transport d'un volume infinitésimal, bilan de masse, de quantité de mouvement et d'énergie, fluides visqueux, équations de Navier-Stokes, éléments de rhéologie… **(4 semaines)**
Chapitre 2 : Fluide parfait et ses applications : écoulements potentiels, ondes d'interfaces **(2 semaines)**
Chapitre 3 : Dynamique des fluides réels : écoulement unidirectionnels, écoulement de Stokes, écoulement à faible vitesse, à faible nombre de Reynolds, lubrification hydrodynamique… **(3 semaines)**
Chapitre 4 : Couches limites : développement de la couche limite, solutions approchées, équation de Van Karman,… **(2 semaines)**
Chapitre 5 : Ecoulements turbulents : champ moyen et fluctuations, équations de Reynolds, modèle de Boussinesq, modèle de la longueur de mélange de Prandtl, échelles de turbulence, modèles de turbulence K-ε, K-ω, SST… **(4 semaines)**

Mode d'évaluation :

Contrôle Continu : 40%, Examen : 60%.

Références bibliographiques :

1- *Inge L. Ryhming, Dynamique des fluides, Presse Polytechniques et Universitaire Romandes.*
2- *P. Chassaing, Turbulence en mécanique des fluides, CEPADUES– Editions*
3- *R. Comolet, Mécanique expérimentale des fluides, Tome II, dynamique des fluides réels, turbomachines, Editions Masson, 1982.*
4- *T. C. Papanastasiou, G. C. Georgiou and A. N. Alexandrou, Viscous fluid flow, CRC Press LLC, 2000.*
5- *Adil Ridha, Cours de Dynamique des fluides réels, M1 Mathématiques et applications : spécialité Mécanique, Université de Caen, 2009.*
6- *R. W. Fox, A. T. Mc Donald and P. J. Pritchard, Introduction to fluid mechanics, sixth edition, Wiley and sons editor, 2003*
7- *Hermann Schlichting, Boundary layer theory, McGraw Hill book Company.*
8- *W.P. Graebel, Advenced fluid mechanics, Academic Press 2007.*
9- *H. Tennekes and J. L. Lumeley, A first course in turbulence, The MIT Press 1972*

Contenu de Cours

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Chapter 3
Boundary Layer
Turbulent Flow

Special Cases of Navier-Stocks Eqaution

Navier-Stocks Equation

$$\rho \frac{D\,u}{D\,t} = \rho\, f_x - \frac{\partial\, p}{\partial\, x} + \frac{\partial}{\partial\, x} \left[(\varepsilon + \mu)\; div\, \overrightarrow{q} \right] + \mu\, \Delta\, u$$

où $\Delta\, u \;\equiv\; \nabla^2 u$

Continuity Equation

$$\frac{D\,\rho}{D\,t} + \rho\, \overrightarrow{\nabla} . \overrightarrow{q} \;=\; 0$$

Incompressible Flow

et pour une viscosité dynamique μ constante :

$$\rho \frac{D\,u}{D\,t} = \rho\, f_x - \frac{\partial\, p}{\partial\, x} + \frac{\partial}{\partial\, x}\left[(\varepsilon + \mu)\; div\, \overrightarrow{q}\right] + \mu\, \Delta\, u \qquad (4.18)$$

où $\Delta\, u \;\equiv\; \nabla^2 u$

Pour le cas d'un fluide incompressible l'équation de Navier-Stokes suivant l'axe OX se réduit à :

$$\frac{D\,u}{D\,t} = f_x - \frac{1}{\rho}\frac{\partial\, p}{\partial\, x} + \nu\, \Delta\, u \qquad (4.19)$$

avec : $\nu = \frac{\mu}{\rho}$

Les équations d'Euler

Ecoulements à grand nombre de Reynolds
la viscosité du fluide est nulle

$$\vartheta \Delta u = 0$$

$$\begin{cases} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = f_x - \frac{1}{\rho} \frac{\partial P}{\partial x} \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = f_y + -\frac{1}{\rho} \frac{\partial P}{\partial y} \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = f_z - \frac{1}{\rho} \frac{\partial P}{\partial z} \end{cases}$$

écoulements de Stokes
"écoulements rampants"

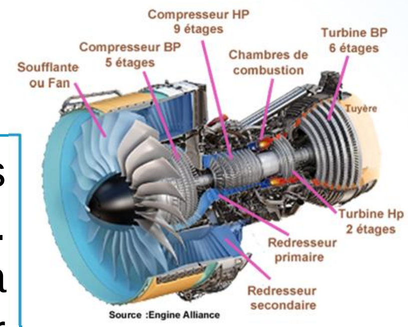
Ecoulements à faible nombre de Reynolds
les forces d'inertie sont négligables devant les forces visqueuses et volumiques

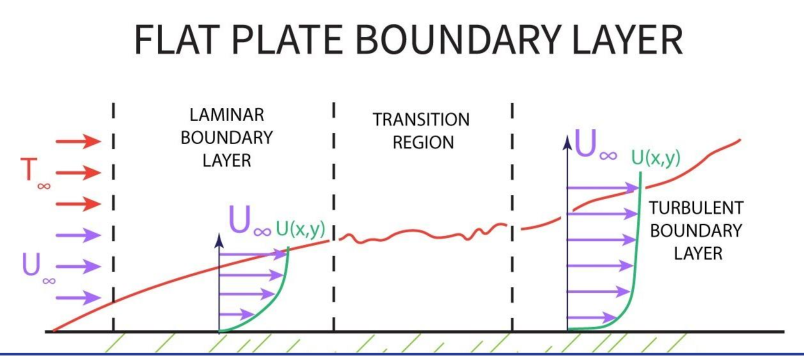
$$\frac{Du}{Dt} \ll \vartheta \Delta u + f$$

$$\begin{cases} \frac{1}{\rho}\frac{\partial P}{\partial x} = f_x + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \frac{1}{\rho}\frac{\partial P}{\partial y} = f_y + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ \frac{1}{\rho}\frac{\partial P}{\partial z} = f_z + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{cases}$$

Couche Limite

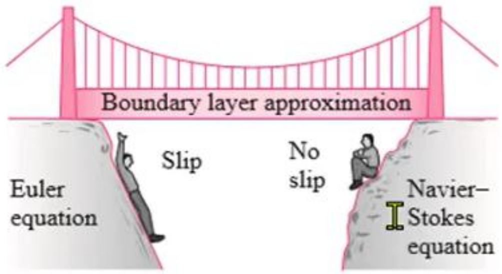
Avant 1860, l'intérêt technique de la mécanique des fluides était pratiquement limité à l'écoulement de l'eau. Le développement de l'industrie chimique pendant la dernière partie du XIXe siècle a porté l'attention sur d'autres liquides et sur les gaz. L'intérêt pour l'aérodynamique débuta avec les études de l'ingénieur en aéronautique allemand Otto Lilienthal à la fin du XIXe siècle ; on assista alors à des avances majeures après le succès du premier vol motorisé, effectué par les inventeurs américains Orville et Wilbur Wright en 1903.





La complexité des écoulements visqueux, en particulier des écoulements turbulents, a longtemps limité les progrès en dynamique des fluides. En 1904, l'ingénieur allemand Ludwig Prandtl indiqua que l'écoulement des fluides visqueux présente deux zones principales. Une, proche de la surface, est constituée d'une fine couche et concentre les effets de la viscosité. Son traitement par un modèle mathématique peut être simplifié compte tenu de sa faible épaisseur. En dehors de cette couche frontière, les effets de la viscosité peuvent être négligés et des équations mathématiques plus simples, adaptées à l'absence de frottement, peuvent alors s'appliquer. La théorie des couches limites a permis de développer les ailes d'avions modernes, la conception des turbines à gaz et des compresseurs,…etc.

Couche Limite – définition



(*b*)

FIGURE 10–75

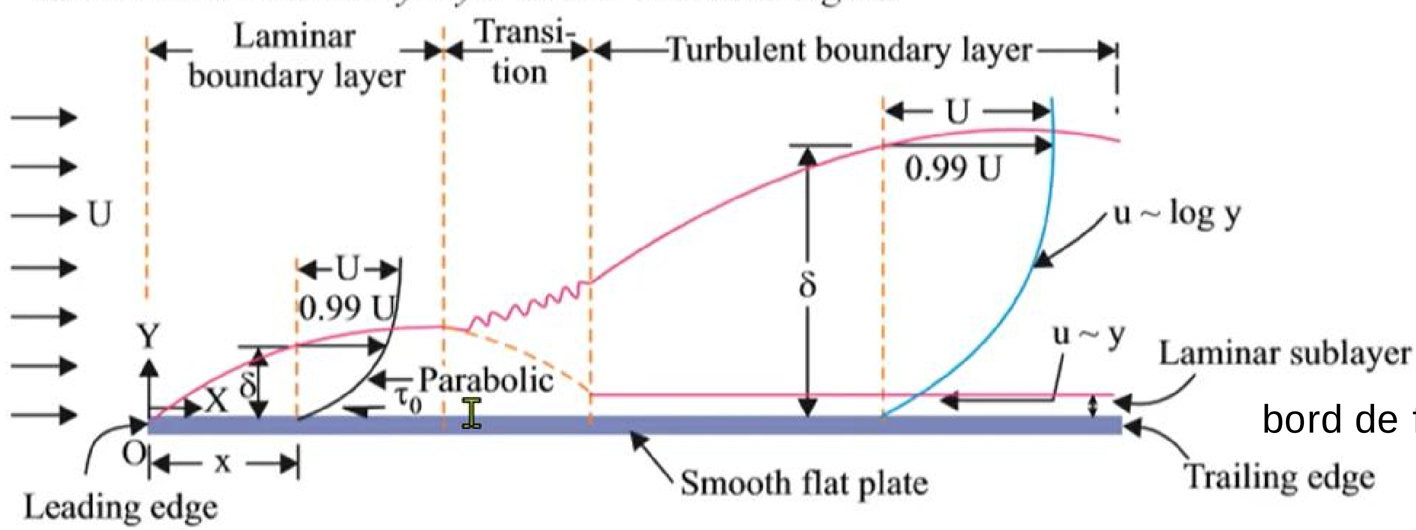
(*a*) A huge gap exists between the Euler equation (which allows slip at walls) and the Navier–Stokes equation (which supports the no-slip condition); (*b*) the boundary layer approximation bridges that gap.

— The edge facing the direction of flow is called *leading edge*.

— The rear edge is called the *trailing edge*.

— Near the leading edge of a flat plate, the boundary layer is wholly *laminar*. For a laminar boundary layer, the velocity distribution is *parabolic*.

— The thickness of the boundary layer (δ) increases with distance from the leading edge x, as more and more fluid is slowed down by the viscous boundary, becomes unstable and breaks into *turbulent boundary layer* over a transition region.



Bord d'attaque

bord de fuite

The *characteristics* of a boundary layer may be summarised as follows:

(*i*) δ (thickness of boundary layer) increases as distance from leading edge x increases.

(*ii*) δ decreases as U increases.

(*iii*) δ increases as kinematic viscosity (ν) increases.

(*iv*) $\tau_0 \approx \mu \left(\frac{U}{\delta}\right)$; hence τ_0 decreases as x increases. However, when boundary layer becomes turbulent, it shows a sudden increase and then decreases with increasing x.

(*v*) When U increases in the downward direction, boundary layer growth is reduced.

(*vi*) When U decreases in the downward direction, flow near the boundary is further retarded, boundary layer growth is faster and is susceptible to separation.

(*vii*) The various characteristics of the boundary layer on flat plate (e.g variation of δ, τ_0 or force F) are governed by inertial and viscous forces; hence they are functions of either $\frac{Ux}{\nu}$ or $\frac{UL}{\nu}$.

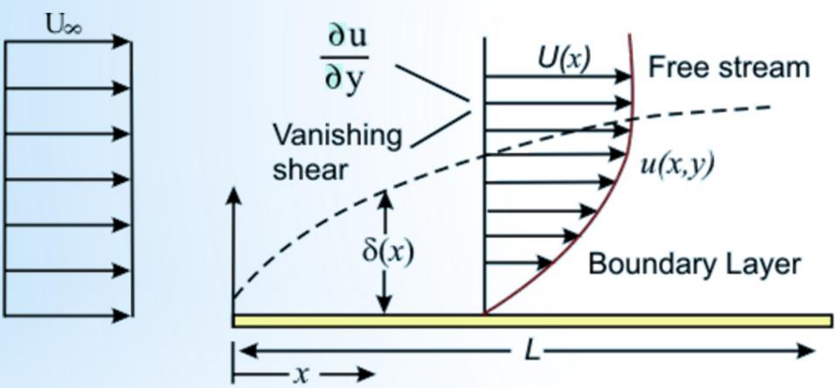
(*viii*) If $\frac{Ux}{\nu} < 5 \times 10^5$ ... boundary layer is *laminar* (velocity distribution is *parabolic*).

If $\frac{Ux}{\nu} > 5 \times 10^5$ ... boundary layer is *turbulent* on that portion (velocity distribution follows *Log law* or a *power law*).

Paramètres caractéristiques de la couche limite

Pour étudier et modéliser la couche limite on utilise, en plus de l'épaisseur δ de la couche limite, les épaisseurs δ1 de déplacement et δ2 de quantité de mouvement.

Epaisseur de déplacement
Elle correspond au déficit de débit lié à la présence de la couche limite



$$\delta_1 U_e = \int_0^\infty (U_e - u)\, dy$$

$$\delta_1 = \int_0^\delta \left(1 - \frac{u}{U_e}\right) dy$$

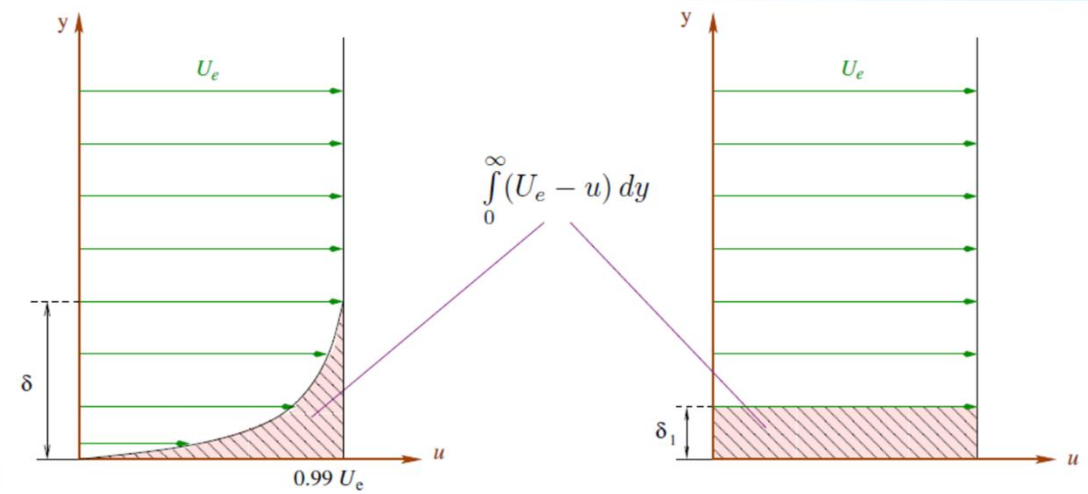


FIGURE 5.2: *Perte de débit liée à la présence de la couche limite.*

δ1 est aussi appelée : épaisseur de perte de débit.

Epaisseur de quantité de mouvement

On définit de la même façon l'épaisseur δ2 correspondant au déficit de quantité de mouvement.

$$\rho U_e^2 \delta_2 = \int_0^\infty \rho u \left(U_e - u\right) dy$$

$$\delta_2 = \int_0^\delta \frac{u}{U_e}\left(1 - \frac{u}{U_e}\right) dy$$

Facteur de forme

$$H = \frac{\delta_1}{\delta_2}$$

Ce paramètre caractérise la forme du profil de vitesse dans la couche limite. Il prend des valeurs différentes selon la nature laminaire ou turbulente de l'écoulement dans la couche limite. Pour une CLL sur une plaque plane, il passe pratiquement du double au simple : d'environ 2, 6 pour la CLL à 1, 3 pour la CLT.

Epaisseur de la couche limite

$$\delta(x) = \frac{5\,x}{\sqrt{\Re_{ex}}}$$

$$\Re_{ex} = \frac{U_e\,x}{\nu}$$

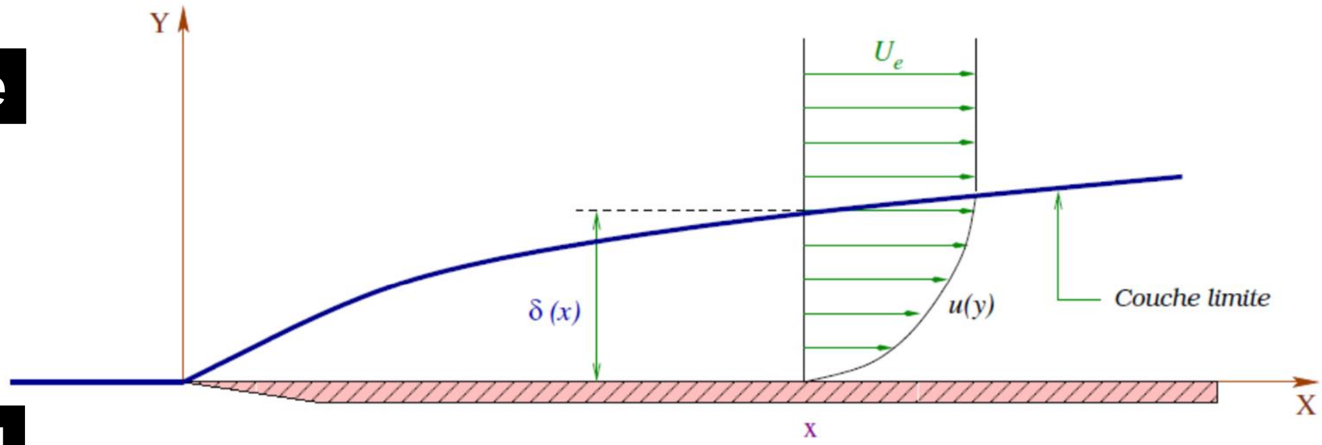


FIGURE 5.4: *Couche limite sur une plaque plane.*

$$\delta_1(x) = \frac{1,721\,x}{\sqrt{\Re_{ex}}}$$

$$\delta_2(x) = \frac{0,664\,x}{\sqrt{\Re_{ex}}}$$

Coefficient de frottement à la paroi : La contrainte de cisaillement locale à la surface de la plaque dépend seulement de x, elle est donnée par :

$$\tau_p = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu\, U_e \sqrt{\frac{U_e}{\nu\, x}}\, f''(0) = 0,332\, \mu\, U_e \sqrt{\frac{U_e}{\nu\, x}}$$

Le coefficient de frottement à la paroi est donnée par :

$$C_f = \frac{\tau_p}{\frac{1}{2}\rho U_e^2} = \frac{0,664}{\sqrt{\Re_{ex}}} = \frac{1,328}{\sqrt{\Re_{eL}}}$$

où ℜeL est le nombre de Reynolds basé sur la longueur de la plaque.

Equations régissant la couche limite laminaire

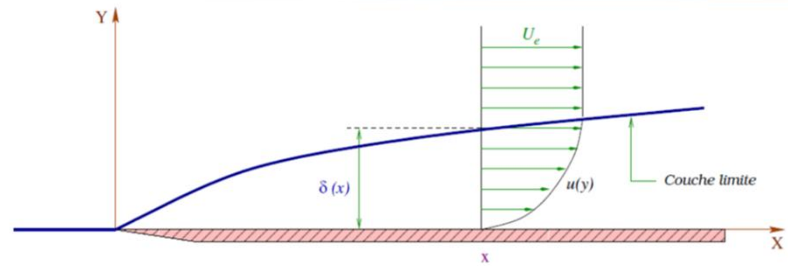


FIGURE 5.4: *Couche limite sur une plaque plane.*

$$\left\{ \begin{array}{rcl} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} & = & 0 \\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} & = & -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} & = & -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{array} \right. \quad (5.1)$$

Nous allons définir un certain nombre de références permettant d'adimensionnaliser les variables.

- U_e : vitesse de référence (par exemple la vitesse à l'extérieur de la couche limite suivant X) ;
- L : longueur caractéristique dans la direction X ;
- δ : longueur caractéristique dans la direction Y ($\delta/L << 1$)
- $U_e\,\delta/L$: vitesse de référence dans la direction Y ;
- ρU_e^2 : pression de référence ;
- L/U_e : temps de référence.

within the boundary layer at some value of x, the orders of magnitude are

$$u \sim U \qquad P - P_\infty \sim \rho U^2 \qquad \frac{\partial}{\partial x} \sim \frac{1}{L} \qquad \frac{\partial}{\partial y} \sim \frac{1}{\delta} \tag{10–62}$$

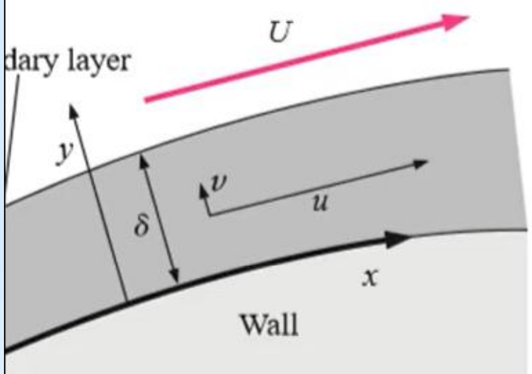
The order of magnitude of velocity component v is not specified in Eq. 10–62, but is instead obtained from the continuity equation. Applying the orders of magnitude in Eq. 10–62 to the incompressible continuity equation in two dimensions,

$$\underbrace{\frac{\partial u}{\partial x}}_{\sim U/L} + \underbrace{\frac{\partial v}{\partial y}}_{\sim v/\delta} = 0 \qquad \rightarrow \qquad \frac{U}{L} \sim \frac{v}{\delta}$$

Since the two terms have to balance each other, they must be of the same order of magnitude. Thus we obtain the order of magnitude of velocity component v,

$$v \sim \frac{U\delta}{L} \tag{10–63}$$

Since $\delta/L \ll 1$ in a boundary layer (the boundary layer is very thin), we conclude that $v \ll u$ in a boundary layer (Fig. 10–87). From Eqs. 10–62



E 10–87
y magnified view of the
lary layer along the surface of a
showing that velocity
onent v is much smaller than u.

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{\delta} \qquad u^* = \frac{u}{U} \qquad v^* = \frac{vL}{U\delta} \qquad P^* = \frac{P - P_\infty}{\rho U^2}$$

Since we used appropriate scales, all these nondimensional variables are of order unity—i.e., they are *normalized* variables (Chap. 7).

We now consider the x- and y-components of the Navier–Stokes equation. We substitute these nondimensional variables into the y-momentum equation, giving

$$\underbrace{u}_{u^*U}\ \underbrace{\frac{\partial v}{\partial x}}_{\frac{\partial}{\partial x^*}\frac{v^*U\delta}{L^2}} + \underbrace{v}_{v^*\frac{U\delta}{L}}\ \underbrace{\frac{\partial v}{\partial y}}_{\frac{\partial}{\partial y^*}\frac{v^*U\delta}{L\delta}} = \underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial y}}_{\frac{1}{\rho}\frac{\partial}{\partial y^*}\frac{P^*\rho U^2}{\delta}} + \underbrace{\nu\frac{\partial^2 v}{\partial x^2}}_{\nu\frac{\partial^2}{\partial x^{*2}}\frac{v^*U\delta}{L^3}} + \underbrace{\nu\frac{\partial^2 v}{\partial y^2}}_{\nu\frac{\partial^2}{\partial y^{*2}}\frac{v^*U\delta}{L\delta^2}}$$

After some algebra and after multiplying each term by $L^2/(U^2\delta)$, we get

$$u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = -\left(\frac{L}{\delta}\right)^2\frac{\partial P^*}{\partial y^*} + \left(\frac{\nu}{UL}\right)\frac{\partial^2 v^*}{\partial x^{*2}} + \left(\frac{\nu}{UL}\right)\left(\frac{L}{\delta}\right)^2\frac{\partial^2 v^*}{\partial y^{*2}} \quad \textbf{(10–64)}$$

$$\underbrace{u}_{u^*U}\underbrace{\frac{\partial v}{\partial x}}_{\frac{\partial}{\partial x^*}\frac{v^*U\delta}{L^2}} + \underbrace{v}_{v^*\frac{U\delta}{L}}\underbrace{\frac{\partial v}{\partial y}}_{\frac{\partial}{\partial y^*}\frac{v^*U\delta}{L\delta}} = \underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial y}}_{\frac{1}{\rho}\frac{\partial}{\partial y^*}\frac{P^*\rho U^2}{\delta}} + \underbrace{\nu\frac{\partial^2 v}{\partial x^2}}_{\nu\frac{\partial^2}{\partial x^{*2}}\frac{v^*U\delta}{L^3}} + \underbrace{\nu\frac{\partial^2 v}{\partial y^2}}_{\nu\frac{\partial^2}{\partial y^{*2}}\frac{v^*U\delta}{L\delta^2}}$$

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Comparing terms in Eq. 10–64, the middle term on the right side is clearly orders of magnitude smaller than any other term since $\mathrm{Re}_L = UL/\nu \gg 1$. For the same reason, the last term on the right is much smaller than the first term on the right. Neglecting these two terms leaves the two terms on the left and the first term on the right. However, since $L \gg \delta$, the pressure gradient term is orders of magnitude greater than the advective terms on the left side of the equation. Thus, the only term left in Eq. 10–64 is the pressure term. Since no other term in the equation can balance that term, we have no choice but to set it equal to zero. Thus, the nondimensional y-momentum equation reduces to

$$\frac{\partial P^*}{\partial y^*} \cong 0$$

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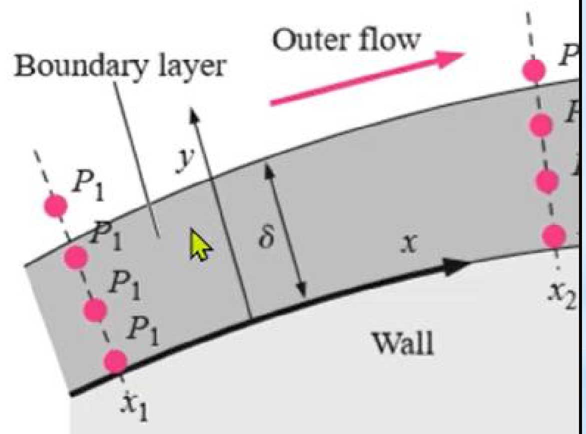
$$\frac{\partial P^*}{\partial y^*} \cong 0$$

or, in terms of the physical variables,

Normal pressure gradient through a boundary layer: $$\frac{\partial P}{\partial y} \cong 0 \qquad \textbf{(10–65)}$$

In words, although pressure may vary *along* the wall (in the x-direction), there is negligible change in pressure in the direction *normal* to the wall. This is illustrated in Fig. 10–88. At $x = x_1$, $P = P_1$ at all values of y across the boundary layer from the wall to the outer flow. At some other x-location, $x = x_2$, the pressure may have changed, but $P = P_2$ at all values of y across that portion of the boundary layer



FIGURE

Pressure may change a

boundary layer (x-direction), b

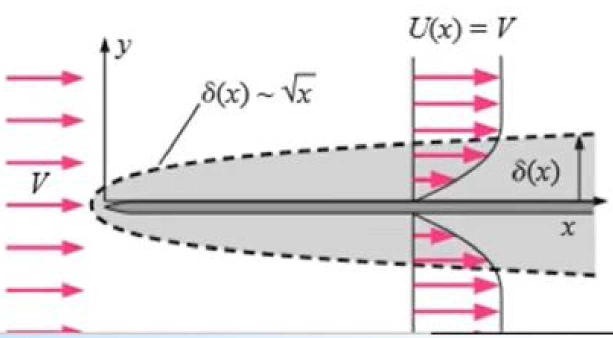
where P is the value of pressure calculated from our outer flow approximation (using either continuity plus Euler, or the potential flow equations plus Bernoulli). The x-component of the Navier–Stokes equation becomes

$$\underbrace{u}_{u^*U}\underbrace{\frac{\partial u}{\partial x}}_{\frac{\partial}{\partial x^*}\frac{u^*U}{L}} + \underbrace{v}_{v^*\frac{U\delta}{L}}\underbrace{\frac{\partial u}{\partial y}}_{\frac{\partial}{\partial y^*}\frac{u^*U}{\delta}} = \underbrace{-\frac{1}{\rho}\frac{dP}{dx}}_{\frac{1}{\rho}\frac{\partial}{\partial x^*}\frac{P^*\rho U^2}{L}} + \underbrace{\nu\frac{\partial^2 u}{\partial x^2}}_{\nu\frac{\partial^2}{\partial x^{*2}}\frac{u^*U}{L^2}} + \underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\nu\frac{\partial^2}{\partial y^{*2}}\frac{u^*U}{\delta^2}}$$

After some algebra, and after multiplying each term by L/U^2, we get

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{dP^*}{dx^*} + \left(\frac{\nu}{UL}\right)\frac{\partial^2 u^*}{\partial x^{*2}} + \left(\frac{\nu}{UL}\right)\left(\frac{L}{\delta}\right)^2\frac{\partial^2 u^*}{\partial y^{*2}} \qquad \textbf{(10–66)}$$

Comparing terms in Eq. 10–66, the middle term on the right side is clearly orders of magnitude smaller than the terms on the left side, since $\mathrm{Re}_L = UL/\nu \gg 1$. What about the last term on the right? If we neglect this term, we throw out all the viscous terms and are back to the Euler equation. Clearly this term must remain. Furthermore, since all the remaining terms in Eq. 10–66 are of order unity, the combination of parameters in parentheses in the last term on the right side of Eq. 10–66 must also be of order unity,



$$\left(\frac{\nu}{UL}\right)\left(\frac{L}{\delta}\right)^2 \sim 1$$

Again recognizing that $\mathrm{Re}_L = UL/\nu$, we see immediately that

$$\frac{\delta}{\ } \sim \frac{1}{\ } \qquad \textbf{(10–67)}$$

Note that the last term in Eq. 10–68 is not negligible in the boundary layer, since the y-derivative of velocity gradient $\partial u/\partial y$ is sufficiently large to offset the (typically small) value of kinematic viscosity ν. Finally, since we know from our y-momentum equation analysis that the pressure across the boundary layer is the same as that outside the boundary layer (Eq. 10–65), we apply the Bernoulli equation to the outer flow region. Differentiating with respect to x we get

$$\frac{P}{\rho} + \frac{1}{2}U^2 = \text{constant} \qquad \rightarrow \qquad \frac{1}{\rho}\frac{dP}{dx} = -U\frac{dU}{dx} \tag{10–69}$$

where we note that both P and U are functions of x only, as illustrated in Fig. 10–91. Substitution of Eq. 10–69 into Eq. 10–68 yields

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \tag{10–70}$$

and we have eliminated pressure from the boundary layer equations.

We summarize the set of equations of motion for a steady, incompressible, laminar boundary layer in the xy-plane without significant gravitational effects,

Boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10–71}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\ldots}$$

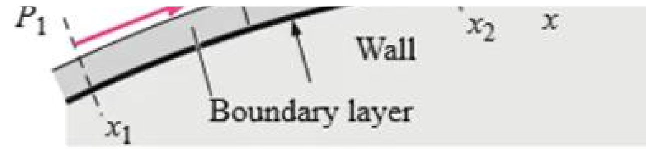


FIGURE 10–91

Outer flow speed parallel to the wall is $U(x)$ and is obtained from the outer flow pressure, $P(x)$. This speed appears in the x-component of the boundary layer momentum equation, Eq. 10–70.

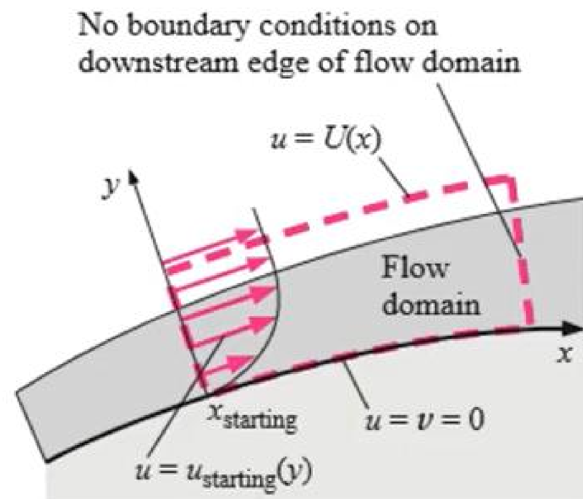
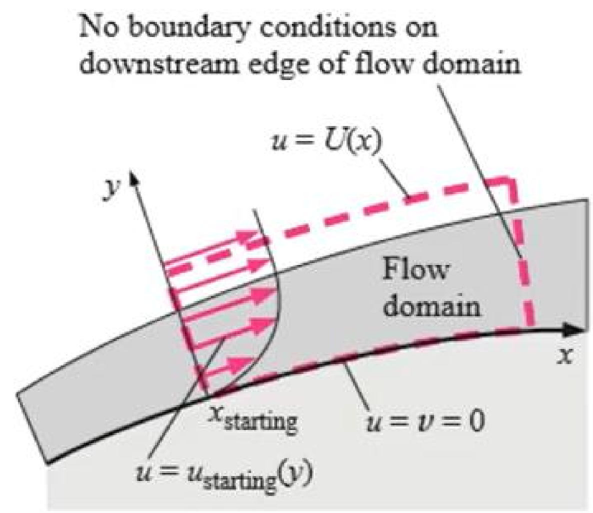


FIGURE 10–92

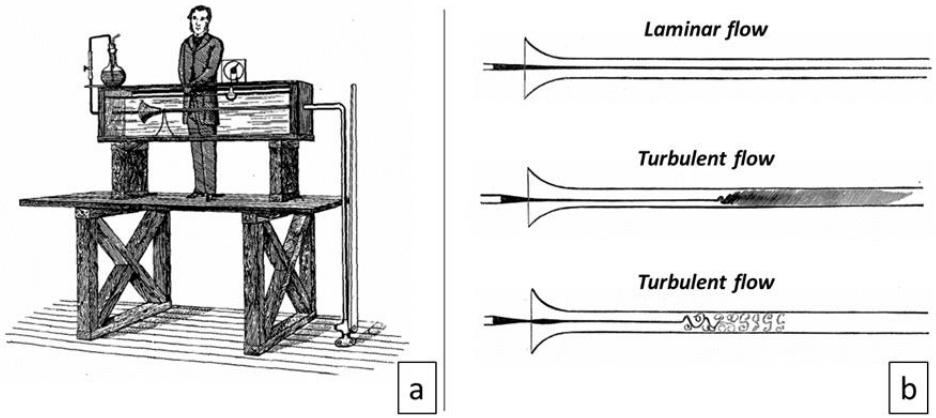
The boundary layer equation set is parabolic, so boundary conditions

Conditions aux limites

Mathematically, the full Navier–Stokes equation is **elliptic** in space, which means that boundary conditions are required over the entire boundary of the flow domain. Physically, flow information is passed in all directions, both upstream and downstream. On the other hand, the x-momentum boundary layer equation (the second equation of Eq. 10–71) is **parabolic.** This means that we need to specify boundary conditions on only three sides of the (two-dimensional) flow domain. Physically, flow information is not passed in the direction opposite to the flow (from downstream). This fact greatly reduces the level of difficulty in solving the boundary layer equations. Specifically, we don't need to specify boundary conditions *downstream*, only upstream and on the top and bottom of the flow domain (Fig. 10–92). For a typical boundary layer problem along a wall, we specify the no-slip condition at the wall ($u = v = 0$ at $y = 0$), the outer flow condition at the edge of the boundary layer and beyond [$u = U(x)$ as $y \to \infty$], and a starting profile at some upstream location [$u = u_{starting}(y)$ at $x = x_{starting}$, where $x_{starting}$ may or may not be zero]. With these boundary conditions, we simply march downstream in the x-direction, solving the boundary layer equations as we go. This is particularly attractive for numerical boundary layer computations, because once we know the profile at one x location



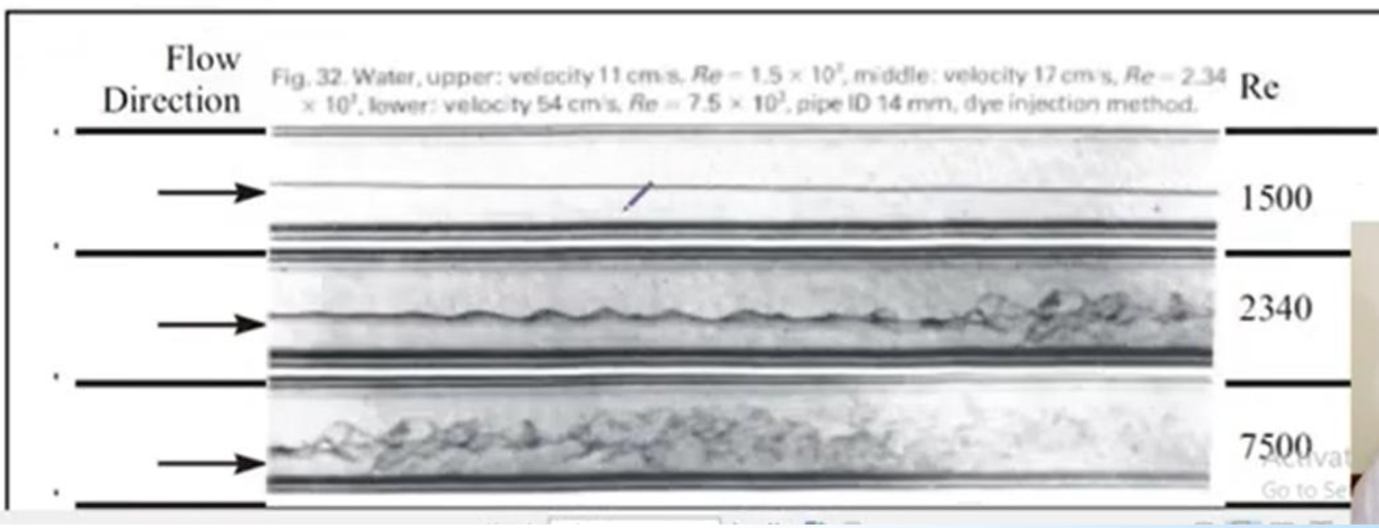
Turbulent Flow

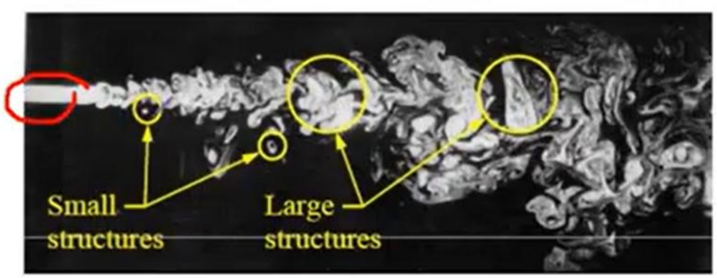


$$\underbrace{\frac{\rho \bar{U} D}{\mu}}_{Re} = 2300 \pm 200$$

These trajectories are straight parallel lines for simple pipe flows.

Consider Reynolds experiment (1882) - inject a thin stream of dye into a fully developed flow in a pipe;





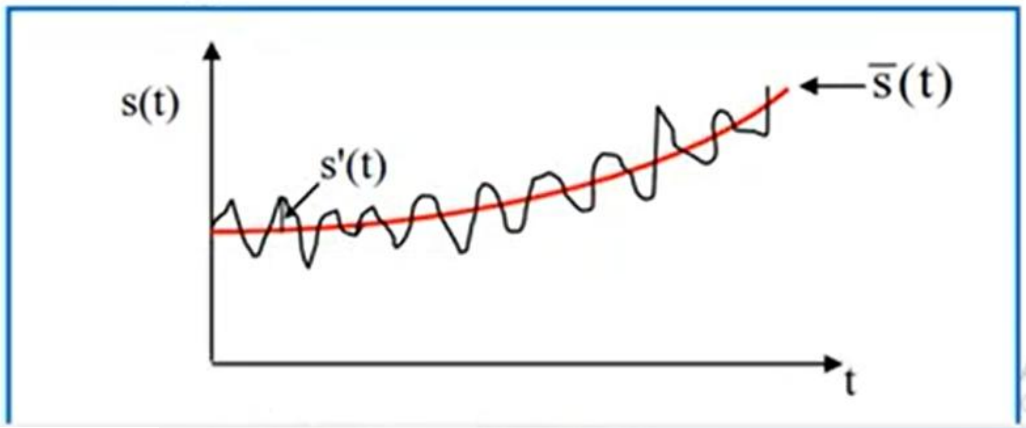
Time averaging technique (Time smoothing):

First, we need to define what we mean by a **time-averaged** quantity. Suppose we have some property like velocity or pressure which fluctuated with time:

$$s = s(t) \quad \text{where s any property}$$

we can average over some time interval;

$$\bar{s}(t) = \frac{1}{\Delta t}\int_0^{\Delta t} s(t)dt \qquad \therefore s(t) = \bar{s}(t) + s'(t)$$



$$s'(t) = s(t) - \bar{s}(t) \qquad \underbrace{\bar{s'}(t)}_{\text{avarge of fluctuation}} = \frac{1}{\Delta t}\int_0^{\Delta t} s'(t)dt = 0$$

$u = \bar{u} + u'$

u = instantaneous velocity component in x-dir.

$$v = \bar{v} + v' \quad \text{and} \quad w = \bar{w} + w'$$

$$\overline{u+v} = \bar{u} + \bar{v} \qquad \because \overline{u'} = \overline{v'} = 0 \quad \text{and} \quad \overline{\bar{u}u'} = 0$$

$$\overline{uv} = \overline{(\bar{u}+u')(\bar{v}+v')} = \overline{\bar{u}\,\bar{v}} + \overline{\bar{u}v'}^{=0} + \overline{\bar{v}u'}^{=0} + \overline{u'v'}$$

$$\therefore \overline{uv} = \bar{u}\,\bar{v} + \overline{u'v'}$$

$$\overline{u^2} = \bar{u}^2 + \overline{u'^2}$$

$$\left(\overline{\frac{\partial u}{\partial x}}\right) = \frac{\partial \bar{u}}{\partial x} \quad , \quad \left(\overline{\frac{\partial u}{\partial t}}\right) = \frac{\partial \bar{u}}{\partial t}$$

$$u = \overline{u} + u' \quad avec \quad \begin{cases} \overline{u} = \frac{1}{T} \int_{t_0}^{t_0+T} u \, dt \\ \overline{u'} = 0 \end{cases}$$

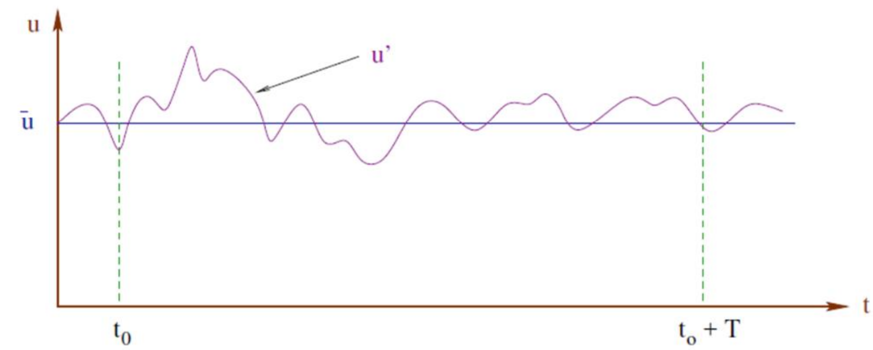


FIGURE 9.1: ***Fluctuations de la vitesse dans un écoulement turbulent.***

Les moyennes quadratiques étant non nulles :

$$\overline{u'^2} = \frac{1}{T} \int_{t_0}^{t_0+T} u'^2 \, dt > 0$$

On définit ainsi l'intensité de turbulence I par le rapport :

$$I = \frac{1}{\overline{u}} \sqrt{\frac{1}{3} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)}$$

(i) Mass Conservation Equation;

we will start with the equation of continuity for an incompressible flow;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (1) \qquad \text{where} \quad u=\bar{u}+u', v=\bar{v}+v', \text{and } w=\bar{w}+w'$$

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial u'}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial v'}{\partial y}+\frac{\partial \bar{w}}{\partial z}+\frac{\partial w'}{\partial z}=0$$

Integrating this eq. term by term over time, we have'

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0 \quad (2) \Rightarrow \nabla.\bar{\mathbf{V}}=0 \quad \text{or div } \bar{\mathbf{V}}=0$$

(ii) momentum eq. in x- dir

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u \qquad (3)$$

Multiply continuity eq. (1) by u and add to the momentum eq. (3), we have;

$$u\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0$$

$$\frac{\partial u}{\partial t}+2u\frac{\partial u}{\partial x}+\left(u\frac{\partial u}{\partial y}+v\frac{\partial u}{\partial y}\right)+\left(u\frac{\partial w}{\partial z}+w\frac{\partial u}{\partial z}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u$$

$$\frac{\partial u}{\partial t}+\frac{\partial(uu)}{\partial x}+\frac{\partial(uv)}{\partial y}+\frac{\partial(uw)}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u$$

$$\frac{\partial \bar{u}}{\partial t} + \text{div}\left(\bar{u}\,\bar{\mathbf{V}}\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \upsilon\,\text{div}\left(\text{grad}\left(\bar{u}\right)\right) + \frac{1}{\rho}\text{div}\left(-\rho\overline{u'\mathbf{V}'}\right) \qquad x-\text{dir.}$$

$$\frac{\partial \bar{v}}{\partial t} + \text{div}\left(\bar{v}\,\bar{\mathbf{V}}\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} + \upsilon\,\text{div}\left(\text{grad}\left(\bar{v}\right)\right) + \frac{1}{\rho}\text{div}\left(-\rho\overline{v'\mathbf{V}'}\right) \qquad y-\text{dir.}$$

$$\frac{\partial \bar{w}}{\partial t} + \text{div}\left(\bar{w}\,\bar{\mathbf{V}}\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z} + \upsilon\,\text{div}\left(\text{grad}\left(\bar{w}\right)\right) + \frac{1}{\rho}\underbrace{\text{div}\left(-\rho\overline{w'\mathbf{V}'}\right)}_{\textit{Reynolds stresses}} \qquad z-\text{dir.}$$

The above equations is called **Reynolds-averaged Navier-Stokes equations**

The **RANS** in ***tensor*** form:

References

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thank you
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teşekkür ederim
Баярлалаа
спасибо
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