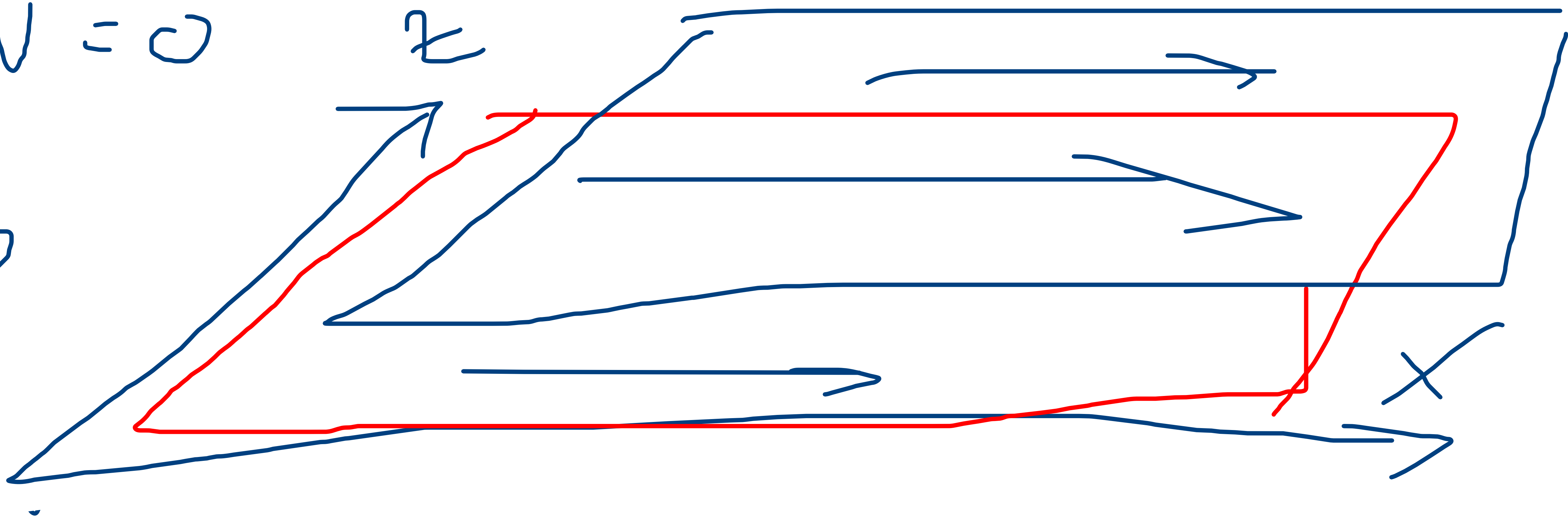
$v = w = 0$

$u \neq 0$



$$\frac{D\vec{q}}{Dt} = \vec{f} - \frac{1}{\rho}\,\vec{\mathrm{grad}}\,P + \nu\Delta\vec{q} \qquad \begin{matrix} \to x \\ \to y \\ z \end{matrix}$$

$$\vec{q}(u, v, w) \qquad \nabla, \quad \Delta = \nabla^2$$

$$\mathrm{div}\,\vec{q} = 0 \quad \text{Continuity} \quad \frac{D\rho}{\partial t} + \rho\,\mathrm{div}\,\vec{q} = 0$$

$$x \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = f_x - \frac{1}{\rho}\frac{\partial P}{\partial x} + \nu$$

$$\nu \Delta \vec{u}$$

$$\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$\underline{x}$

$$\cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial x}} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}} = \cancel{f_x} - \frac{1}{\rho}\frac{\partial p}{\partial x} +$$

$\frac{\partial}{\partial t} = 0$, $v = w = 0$

$f = 0$, $\begin{cases} y = d, & u = 0 \\ y = -d, & u = U_0 \end{cases}$

$$+ \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right) \quad 0$$

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial w}{\partial z}} = 0, \quad \text{Continuité}$$

$= 0$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad // \ \nu = 0.9$$

$$\int \frac{\partial^2 u}{\partial y^2} = \int \boxed{\frac{1}{\nu}\frac{\partial P}{\partial x}} \, A$$

$$\int \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \int A$$

$$\int \partial\left(\frac{\partial u}{\partial y}\right) = \int A\,dy$$

$$\int \partial u = \int A y \, dy + \int c_1 \, dy$$

$$u = A\frac{y^2}{2} + c_1 y + c_2$$

$$U = \frac{A}{2}y^2 + c_1 y + C_2$$

$$\begin{cases} y = d, & U = 0 \\ y = -d, & U = U_0 \end{cases}$$

$$0 = \frac{A}{2}d^2 + \cancel{c_1 d} + C_2$$

$$+$$

$$U_0 = \frac{A}{2}d^2 - \cancel{c_1 d} + C_2$$

$$U_0 = A d^2 + 2C_2$$

$$C_2 = \frac{U_0}{2} - \frac{A d^2}{2}$$

$$c_1 = -\frac{A}{2}d - \frac{C_2}{d}$$

$$c_1 = -\cancel{\frac{A}{2}d} - \frac{U_0}{2d} + \cancel{\frac{A}{2}d}$$

$$C_1 = -\frac{U_0}{2d}$$

$$y = d, \quad u = 0$$

$$C_2 = \frac{U_0}{2} - \frac{A d^2}{2}$$

$$\boxed{u = \frac{A}{2}\left(y^2 - d^2\right) + \frac{U_0}{2}\left(1 - \frac{y}{d}\right)}$$

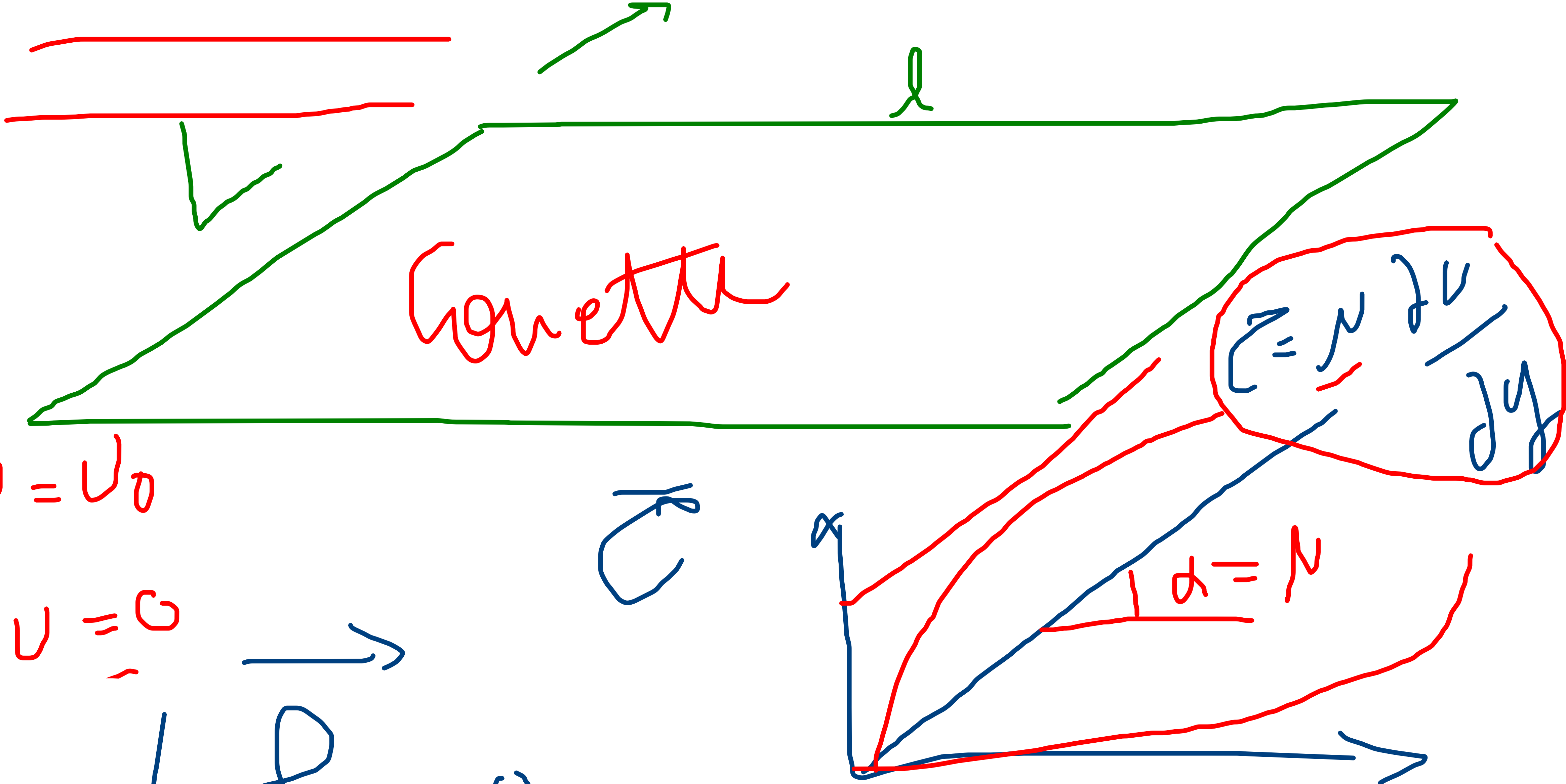
$$u = \frac{A y^2}{2} - \frac{U_0}{2d} y + \frac{U_0}{2} - \frac{A d^2}{2}$$

$$A = \frac{1}{\mu} \frac{\partial p}{\partial x} = \frac{1}{\mu} \frac{\Delta p}{\ell}$$

$$\frac{\partial}{\partial t} = 0$$

$$g = dt$$

$$\begin{cases} y = -d, \; v = v_0 \\ y = d, \; v = 0 \end{cases}$$




$$\frac{\partial}{\partial z} = 0 \qquad \vec{f}_g = 0$$

$$\frac{\partial v}{\partial y}$$

1) $$\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{q} + \cancel{\operatorname{div}\left[\left(\frac{\varepsilon}{\rho} + \frac{\lambda}{\rho}\right)\operatorname{div}\vec{q}\,\right]}$$

2) $$\cancel{\frac{D\rho}{Dt}}^{\,0} + \rho \cdot \operatorname{div}\vec{q} = 0 \quad \Big| \quad \operatorname{div}\vec{q} = 0$$

x)

$$\cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial x}} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}} = \cancel{f_x} - \frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right) = 0$$

y)

$$\cancel{\frac{\partial v}{\partial t}} + \cancel{u\frac{\partial v}{\partial x}} + \cancel{v\frac{\partial v}{\partial y}} + \cancel{w\frac{\partial v}{\partial z}} = \cancel{f_y} - \frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\cancel{\frac{\partial^2 v}{\partial x^2}} + \cancel{\frac{\partial^2 v}{\partial y^2}} + \cancel{\frac{\partial^2 v}{\partial z^2}}\right)$$

$$\operatorname{div} \vec{q} = 0$$

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial w}{\partial t}} = 0$$

$$\left\{\begin{array}{l} \dfrac{\partial u}{\partial x} = 0 \\[2ex] 0 = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu \dfrac{\partial^2 u}{\partial y^2} \\[2ex] 0 = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} \end{array}\right.$$

$$p = cst$$
$$\neq y$$

3) $U(y) = ?$

$$\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\nu}\frac{\partial p}{\partial x} = A$$

$$\int \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \int A\,dy \Rightarrow \frac{\partial u}{\partial y} = Ay + C_1$$

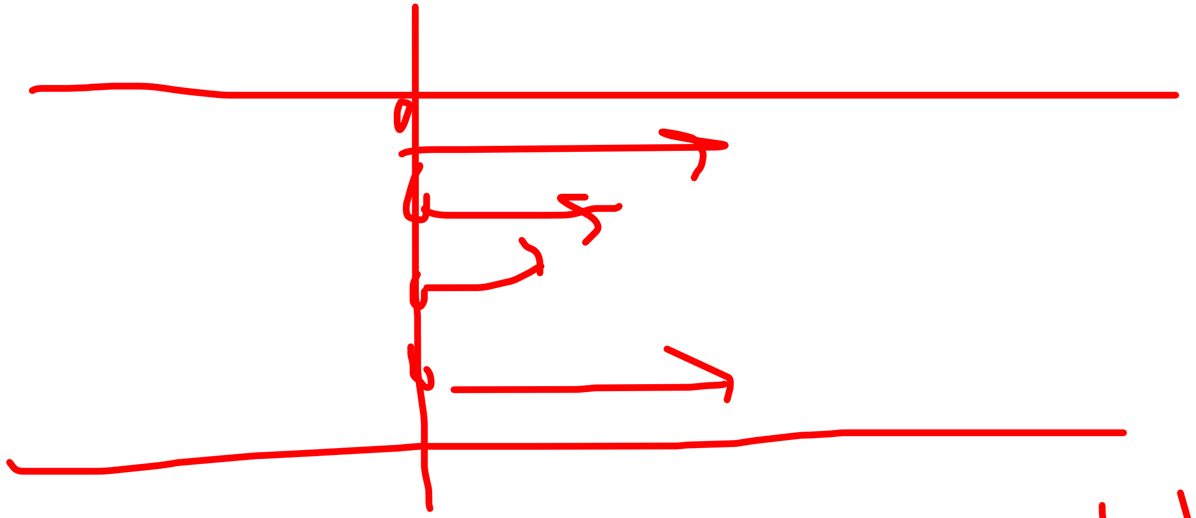
$$u = \frac{A}{2}y^2 + C_1 y + C_2$$

$$U = \frac{A}{2} y^2 + C_1 y + C_2 \quad \begin{cases} y = -d, \ U = U_0 \\ y = d, \ U = 0 \end{cases}$$

$$\begin{cases} U_0 = \dfrac{A}{2} d^2 - \cancel{C_1 d} + C_2 \\ + \\ 0 = \dfrac{A}{2} d^2 + \cancel{C_1 d} + C_2 \end{cases}$$

$$U_0 = A d^2 + 2C_2 \Rightarrow C_2 = \frac{U_0}{2} - \frac{A}{2} d^2$$

$$\frac{\partial u}{\partial y} = 0 \quad , V_{max}$$



$$\frac{\partial u}{\partial y} = Ay - \frac{U_0}{2d} = 0 \quad \Big\} \quad y = \frac{U_0}{2Ad}$$

$$V_{max} = U\left(y = \frac{U_0}{2Ad}\right)$$

$$C_1 = -\frac{Ad}{2} - \frac{C_2}{d} = -\frac{A\cancel{d}}{2} - \frac{1}{d}\left(\frac{U_0}{2} - \frac{A\cancel{d^2}}{2}\right)$$

$$\boxed{\frac{\partial P}{\partial x} = \frac{\Delta P}{\ell}}$$

$$= -\frac{U_0}{2d}$$

$$\left\{\begin{array}{l} y = -d \quad U = U_0 \\ y = d \quad U = 0 \end{array}\right.$$

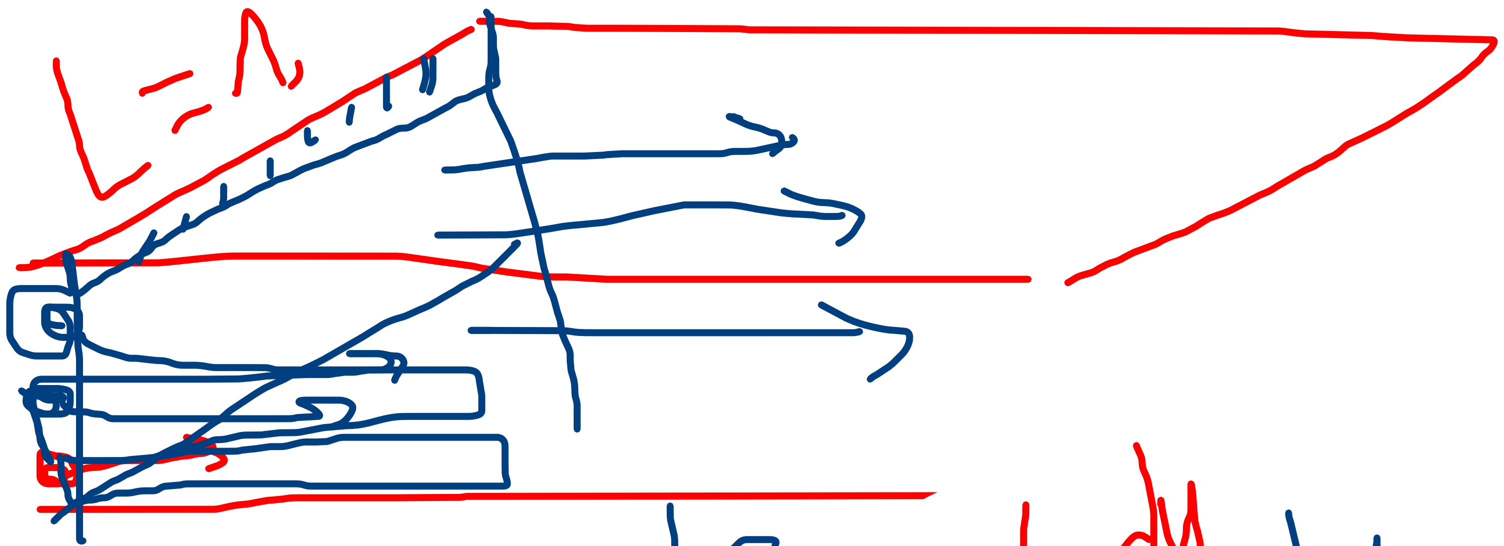
$$U = \frac{Ay^2}{2} - \frac{U_0}{2d}y + \frac{U_0}{2} - \frac{Ad^2}{2}$$

$$U = \frac{A}{2}\left(y^2 - d^2\right) + \frac{U_0}{2}\left(1 - \frac{y}{d}\right)$$

5) $\tau = \mu \dfrac{\partial u}{\partial y} = \mu \left[ A y - \dfrac{u_0}{2d} \right] \begin{cases} y = +d \\ y = -d \end{cases}$

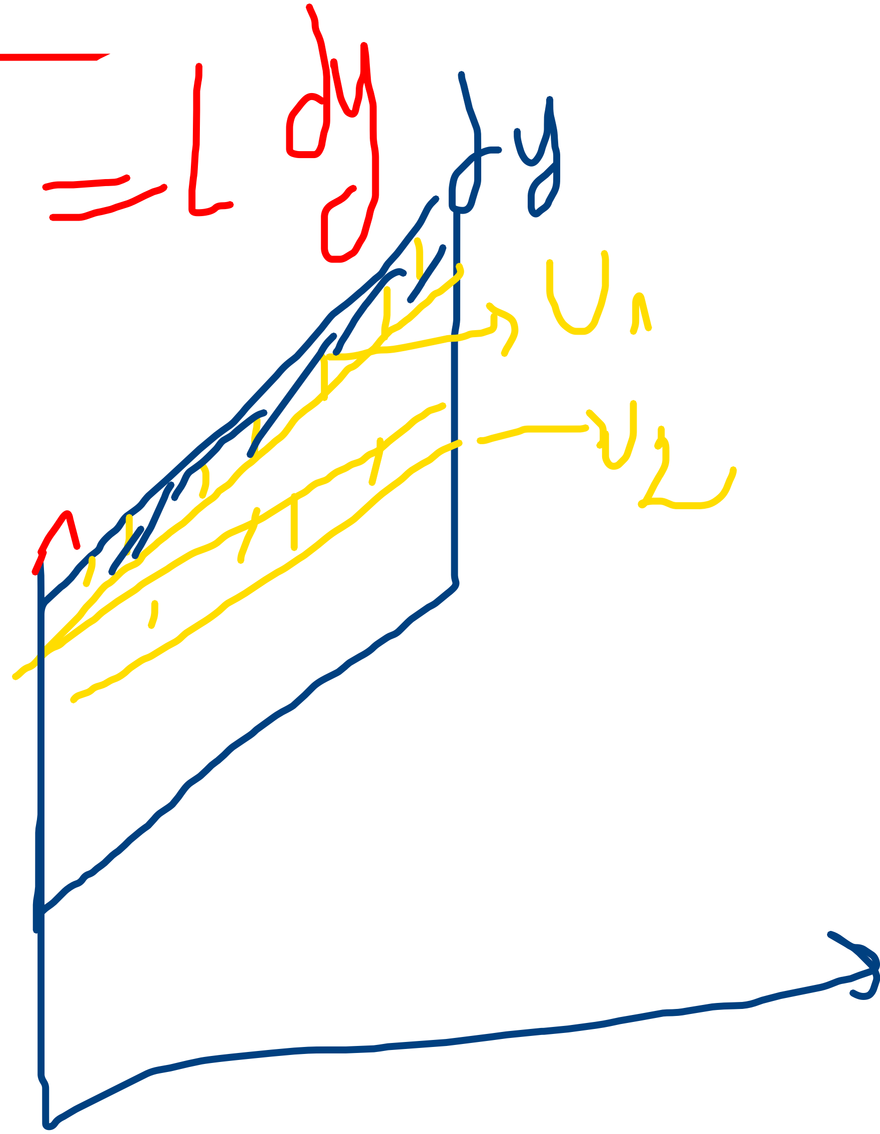
$\tau_{ps} = \tau(y-d)$

6) $Q_V$,




$$Q_V = \bar{U}\, S = \int u\, dS \quad , \quad dS = L\, dy$$

$$= \int_{-d}^{+d} u \cdot L\, dy$$



$$q = L \int_{-d}^{d} \left[ \frac{A}{2} y^2 - \frac{U_0}{2d} y + \frac{U_0}{2} - \frac{A}{2} d^2 \right] dy$$

$$q_v = L \left[ \frac{A}{6} y^3 - \frac{U_0}{4d} y^2 + \frac{U_0}{2} y - \frac{A}{2} d^2 y \right]_{-d}^{d}$$

$$q = B$$

$$=$$

7) $\bar{U} = ?$

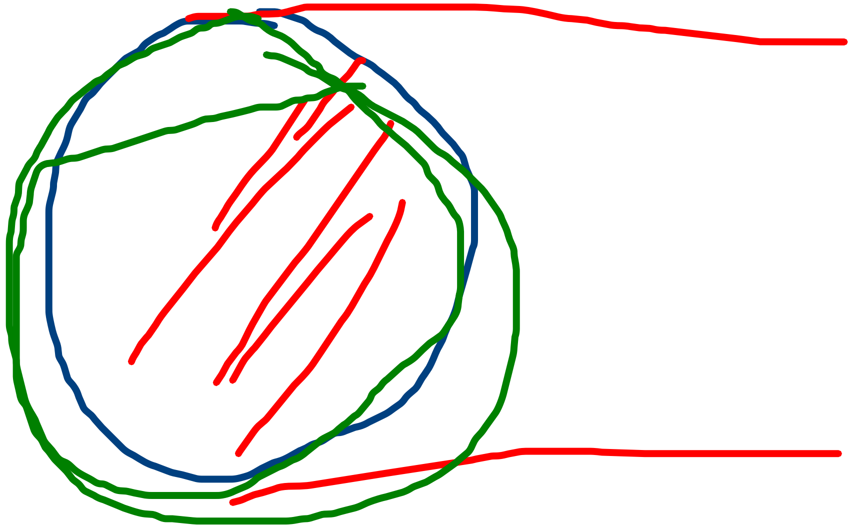
$$q_v = \bar{U} S = \int_{-d}^{d} u L \, dy = B$$

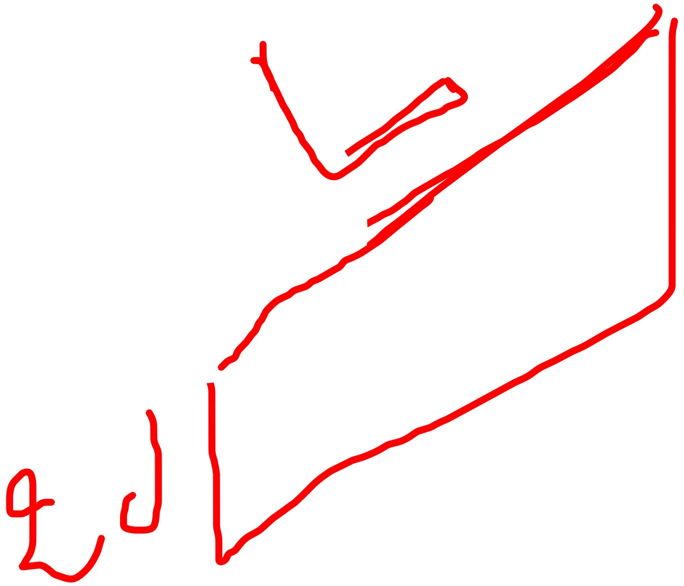
$$\boxed{\bar{U} = B / S}$$

$$\underline{\underline{\underline{S = L \cdot 2d}}}$$

a) $Re = \frac{\bar{u} D_H}{\nu}$ ; $D_H = \frac{4S}{P}$

$$D_H = \frac{\frac{\cancel{4}\,\cancel{\pi}\,\cancel{D}^2}{\cancel{4}}}{\cancel{\pi}\,D} = D$$






$$D_H = \frac{4S}{P}, \quad S = 2dL$$

$$P = 4d + 2L$$

$$D_H = \frac{4\cancel{2}dL}{\cancel{2}(2d+L)} = \frac{4dL}{2d+L}$$

Coefficient de pertes de charge

$$\Delta P = \lambda \frac{1}{2} \bar{U}^2 \cdot \rho \cdot \frac{\ell}{D_H} \Rightarrow \lambda = \frac{2 \Delta P \, D_H}{\rho \cdot \ell \, \bar{U}^2}$$

$$\Delta P = P_1 - P_2$$

## 10. Coefficient de frottement à la paroi

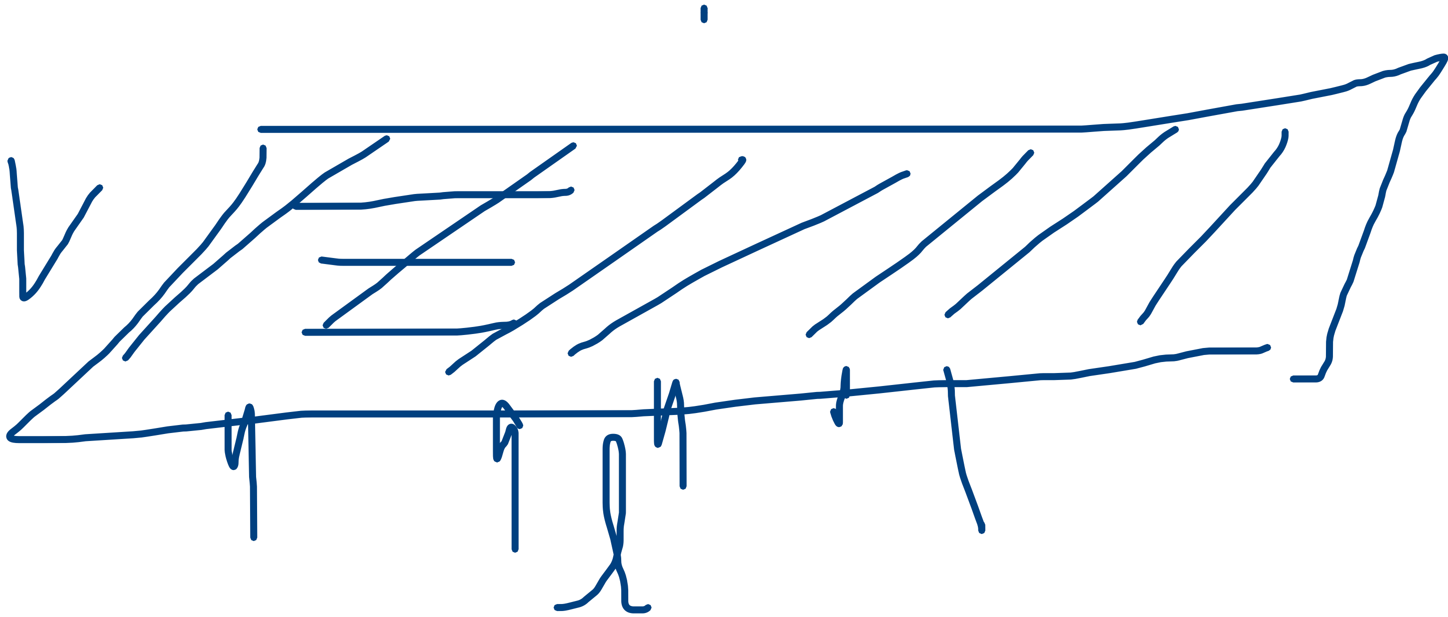
$$\tau = \mu \frac{\partial u}{\partial y} = C_f \frac{1}{2} \bar{U}^2 \rho$$

$$\underline{\underline{C_f}} = \frac{\tau}{\frac{1}{2} \bar{U}^2 \rho} \begin{cases} + & d \\ - & d \end{cases}$$

# 11. Force de traînée sur la plaque

$$F = \underline{\underline{\tau}} \cdot S$$

$$F = \mu \frac{\partial u}{\partial y} \cdot S$$



## Ex 14

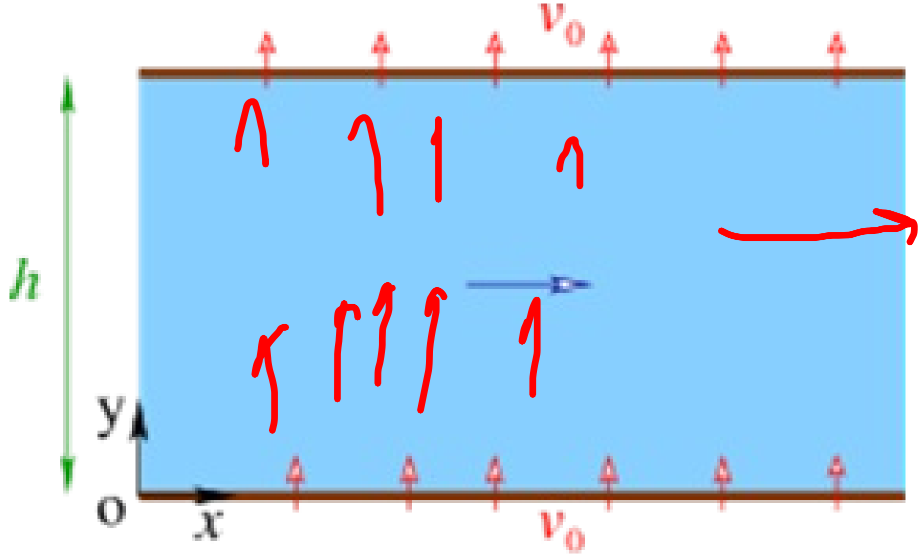
1) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$\phi \Rightarrow \vec{V} = \text{grad}\, \phi \quad \begin{cases} u = \frac{\partial \phi}{\partial x} \\ v = \frac{\partial \phi}{\partial y} \end{cases} \quad w = \frac{\partial \phi}{\partial z}$

$$\operatorname{div} \vec{q} = 0$$

$$\operatorname{div}(\overrightarrow{\operatorname{grad}} \phi) = 0 \Rightarrow \Delta\phi = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial t^2} = 0$$




$U_0$

$v = v_0 \qquad \partial / \partial t = 0$

$u(y) = ?$

$\dfrac{\partial}{\partial z} = 0$

$y = h \Rightarrow u = U_0$

$y = 0 \Rightarrow u = 0$

$$* \ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$$

$$\cancel{\frac{\partial u}{\partial t}} + \cancel{u \frac{\partial u}{\partial x}} + v \frac{\partial u}{\partial y} + \cancel{w \frac{\partial u}{\partial z}} = \cancel{f_x} - \cancel{\frac{1}{\rho} \frac{\partial p}{\partial x}} + \nu \left( \cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}} \right)$$

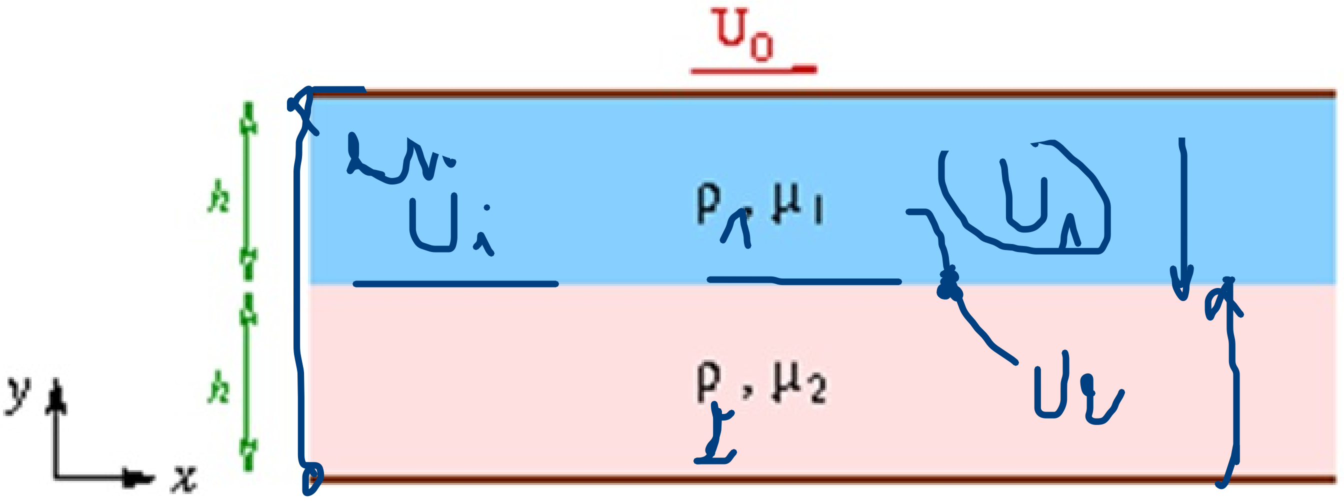
$$v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \Rightarrow \nu\frac{\partial^2 u}{\partial y^2}-v_0\frac{\partial u}{\partial y}=0$$

$$X^2 \qquad X$$

$$\nu X^2 - v_0 X = 0 \quad , \quad \begin{cases} \Delta = b^2-4ac = v_0^2 \\ \lambda_1 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{v_0}{\nu} \\ \lambda_2 = 0 \end{cases}$$

$$u = C_1 e^{\lambda_1 y} + C_2 e^{\lambda_2 y}$$

$$u(y) = \underline{\underline{C_1}}\, e^{\frac{v_0}{\nu} y} + \underline{\underline{C_2}}$$




$$\begin{cases} N.V \rightarrow f_1 \\ N.V \longrightarrow f_2 \end{cases}$$

$$U_1 = \frac{A_1}{2} y^2 + C_1 y + C_2$$

$$U_2 = \frac{A_2}{2} y^2 + C_3 y + C_4$$

$$A_1 = \frac{1}{\mu_1}\left(\frac{\partial P_1}{\partial x}\right)$$

$$A_2 = \frac{1}{\mu_2}\left(\frac{\partial P_2}{\partial x}\right)$$

$$U_i \rightarrow U_1(h) = U_2(h)$$

$$
\begin{cases}
y = 0, \quad u_1 = 0 \rightarrow (1) \\
y = 2h, \quad u_1 = U_0 \rightarrow (2) \\
y = h \quad \begin{cases} u_1 = u_2 \rightarrow (3) \\ \tau_1 = \tau_2 \Rightarrow \mu_1 \dfrac{\partial u_1}{\partial y} = \mu_2 \dfrac{\partial u_2}{\partial y} \rightarrow (4) \end{cases}
\end{cases}
$$

$C_1$

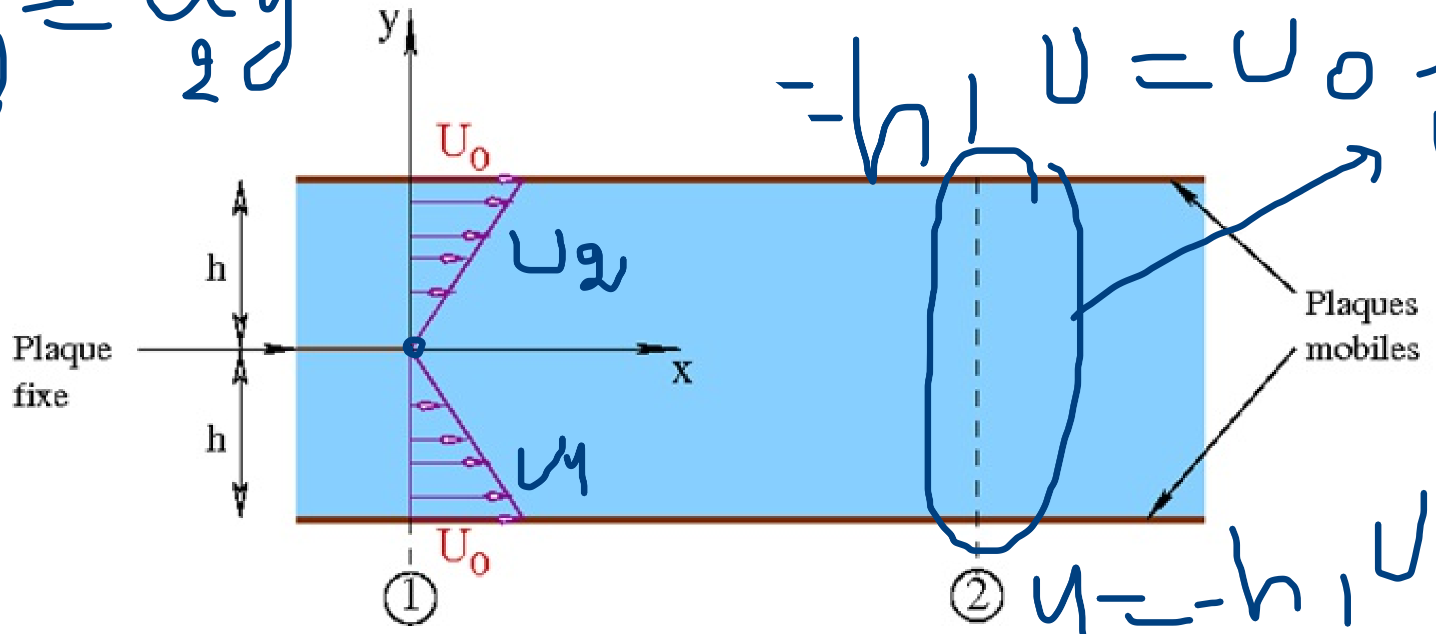
$C_2$ $C_4$

$C_3$

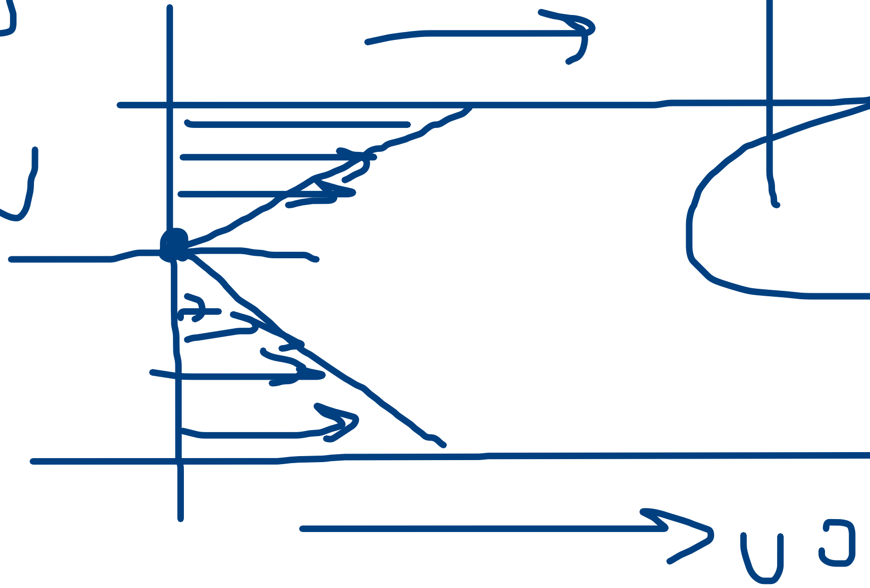
$$U_i = u_1(h) = u_2(h)$$

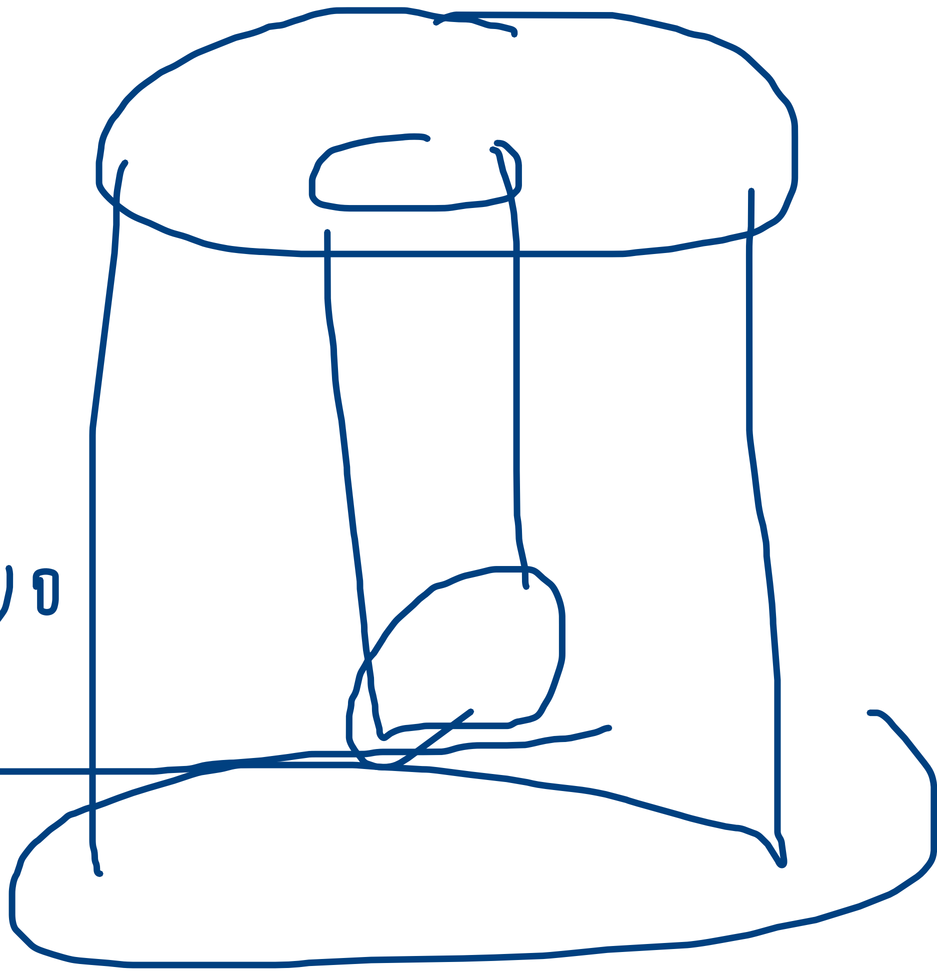
$U_2 = \frac{a}{2} y^2$

$-h,\ U = U_0$ $U_2 S$




$y = -h,\ U = U_0$



$$U_2(y) = \frac{A}{2} y^2 + C_1 y + C_2$$

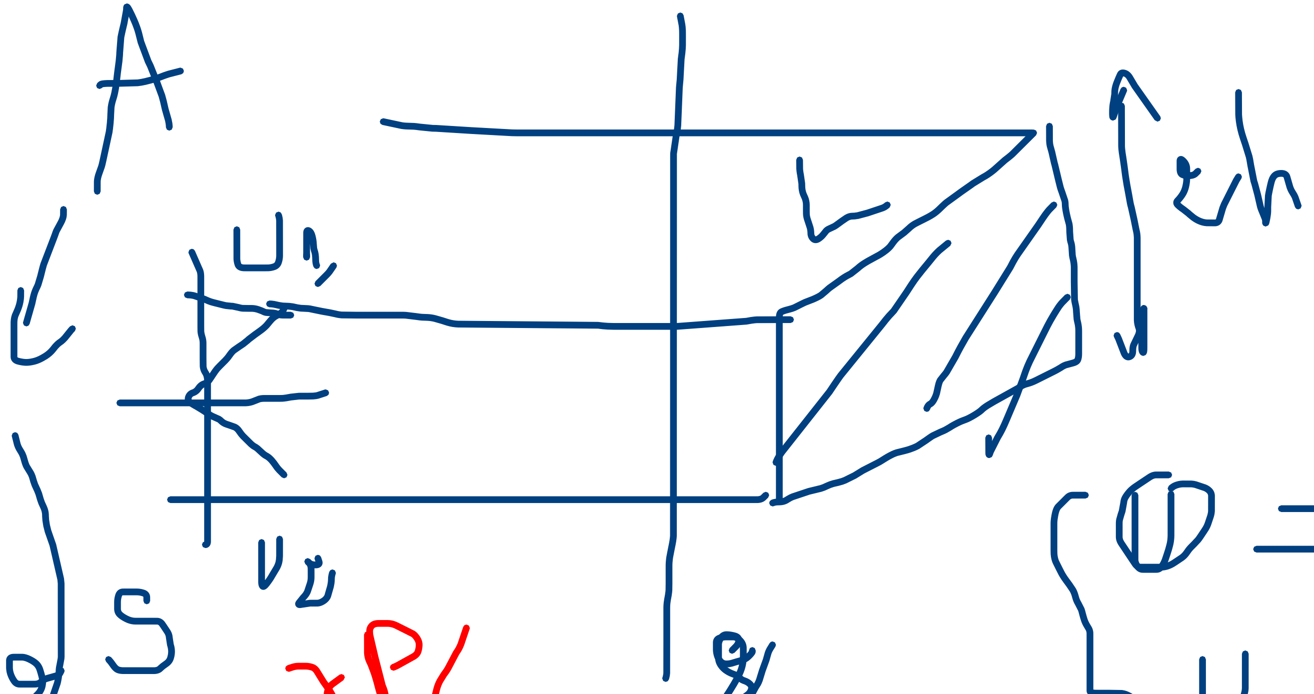
$$A = \frac{1}{\nu} \frac{\partial P}{\partial x}$$

$$Q_1 = Q_2$$

$$Q_{S_1} + Q_{S_2} = \bar{U}_2 S$$

$$\int_{-h}^{0} u_1\,dy + \int_{0}^{h} u_2\,dy = \bar{U}_2 S$$

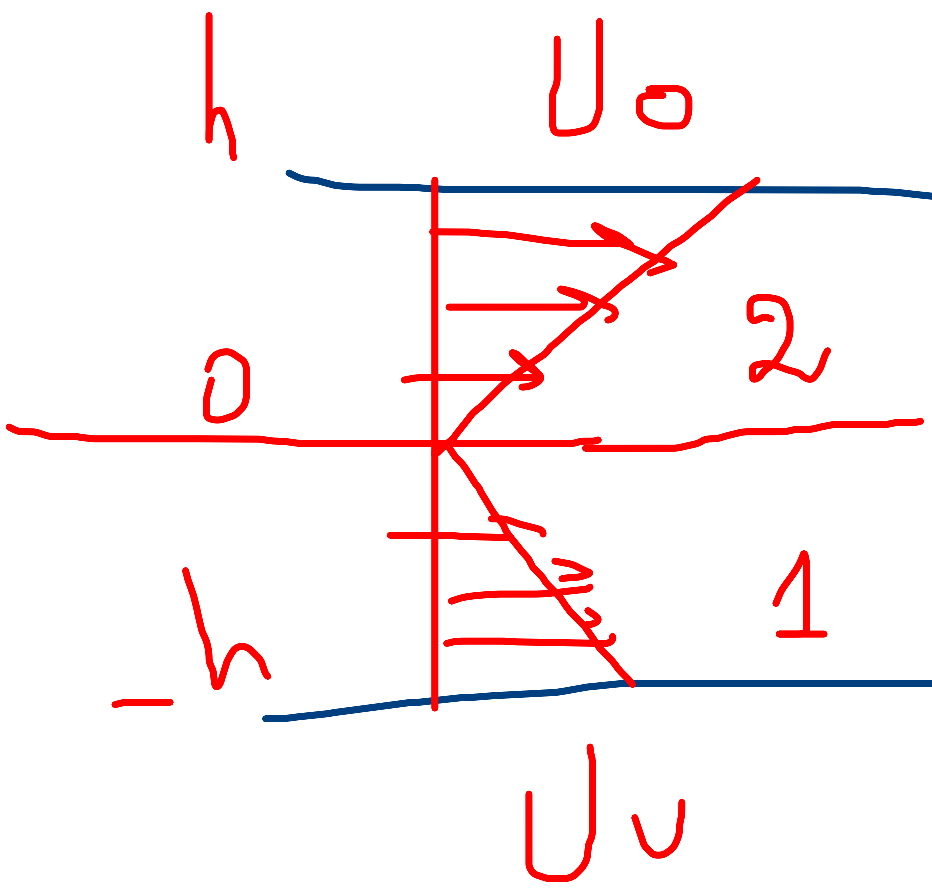
$$U_2 = a_2 y + b_2$$



$$A = \frac{\partial P}{\partial x}$$

$$\begin{cases} y = 0 \Rightarrow u_2 = 0 \\ y = h \Rightarrow U_2 = U_0 \end{cases}$$

$$\begin{cases} 0 = b_2 \\ U_0 = a_2 h \\ a_2 = \dfrac{U_0}{h} \end{cases}$$




$$U_2 = a_2 y + b_2$$

$$\begin{cases} 0 = b_2 \\ U_0 = a_2 h \\ a_2 = \dfrac{U_0}{h} \end{cases}$$

$$U_2 = \frac{U_0}{h} y$$

$$U_1 = -\frac{U_0}{h} y$$

$$\bar{U}_\ell S = L \int_{-h}^{h} u_\ell \, dy$$