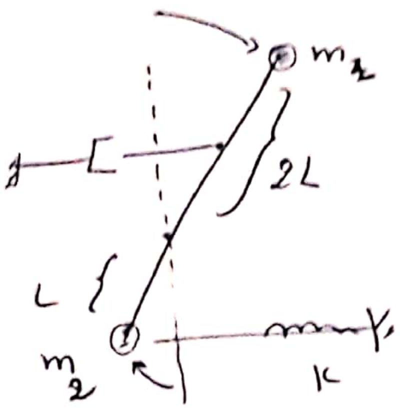
السلسلة الرابعة



$m_1 = m_2$

تمرين ①

$$E_p = \frac{1}{2}(\Delta L + L\theta)^2 + m_2 g(L - L\cos\theta)$$

$$+ m_1 g(2L - 2L\cos\theta)$$

$$E_p = \frac{1}{2}(\Delta L + L\theta)^2 + m_2 g \frac{L\theta^2}{2} - m_1 g L\theta^2$$

شرط التوازن $\Delta L = 0$

الطاقة الحركية :

$$E_c = \frac{1}{2}\left(m_2 L^2 + m_1 4L^2\right)\dot\theta^2 = \frac{5}{2} m L^2 \dot\theta^2$$

$m_1 = m_2$ لـ

- المعادلة التفاضلية للحركة :

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot\theta}\right) + \frac{\partial E_p}{\partial \theta} = -\frac{\partial D}{\partial \dot\theta} + \frac{\partial \vec r}{\partial \theta}\vec F \quad \text{I}$$

$$D = -\frac{1}{2}\alpha \dot x^2(t) = -\frac{1}{2}\alpha L^2 \dot\theta^2$$

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot\theta}\right) = 5mL^2\ddot\theta, \quad \frac{\partial E_p}{\partial \theta} = (kL - mg)L\theta, \quad \frac{\partial D}{\partial \dot\theta} = -\alpha L^2\dot\theta$$

$$\frac{\partial \vec r}{d\theta}\vec F \Rightarrow \quad r = 2L\theta, \quad \frac{dr}{d\theta} = 2L, \quad F = F_0\cos\omega t,$$

$$\frac{\partial \vec r}{d\theta} \cdot \vec F = 2LF_0\cos(\omega t) \rightarrow$$

$$\text{I} \Leftrightarrow 5mL^2\ddot\theta + (kL - mg)L\theta + \alpha L^2\dot\theta = \ominus 2LF_0\cos\omega t$$

لأن القوة تدور عكس الاتجاه الموجب للزوايا

$\ominus$ ومنه $\leftarrow +$

$$\Rightarrow \ddot\theta + \frac{\alpha}{5m}\dot\theta + \frac{(kL - mg)}{5mL}\theta = -\frac{2F_0}{5mL}\cos\omega t$$

$\ddot{\theta}\cancel{+2\delta\dot{\theta}+b\cos\omega t=}$

$$\ddot{\theta}+2\delta\dot{\theta}+\omega_0^2\theta=b\cos\omega t \qquad (*)$$

$$2\delta=\frac{\alpha}{5mL}\ ;\quad \omega_0^2=\frac{(kL-mg)}{5mL}\ ,\quad b=-\frac{2F_0}{5mL}$$

نبحث عن الحل العام :

$$x_p(t)=a\cos(\omega t+\varphi)\Rightarrow$$

$$X=Ae^{j(\omega t+\varphi)}\ ;\ b\cos\omega t=be^{j\omega t}$$

نعوض في المعادلة $(*)$ فنجد :

$$-A\omega^2 e^{j(\cancel{\omega t}+\varphi)}+2\delta\omega jAe^{j(\cancel{\omega t}+\varphi)}+\omega_0^2Ae^{j(\cancel{\omega t}+\varphi)}=be^{j\cancel{\omega t}}$$

$$((\omega_0^2-\omega^2)+2\delta\omega j)A=be^{-j\varphi}=b(\cos\varphi-j\sin\varphi)\Rightarrow$$

$$(\omega_0^2-\omega^2)A=b\cos\varphi$$

$$2\delta\omega\cancel{j}A=-b\cancel{j}\sin\varphi$$

نربع و نجمع طرف لطرف فنجد

$$(\omega_0^2-\omega^2)^2A^2=b^2\cos^2\varphi$$

$$4\delta^2\omega^2A^2=b^2\sin^2\varphi$$

$$\Rightarrow((\omega_0^2-\omega^2)^2+4\delta^2\omega^2)A=b^2$$

$$A^2=b^2/(\omega_0^2-\omega^2)^2+4\delta^2\omega^2\Rightarrow$$

$$\cancel{A=}\sqrt{B}$$

$$\underline{\underline{A}}=\frac{b}{\sqrt{(\omega_0^2-\omega^2)^2+4\delta^2\omega^2}}$$

$$(\omega_0^2-\omega^2)A = b\cos\varphi$$

$$2\delta\omega jA = -bj\sin\varphi$$

نقسم طرفا على طرف فنجد :

$$\frac{(\omega_0^2-\omega^2)}{2\delta\omega A} = -\frac{\cos\varphi}{\sin\varphi} \Rightarrow -ctg\,\varphi = \frac{(\omega_0^2-\omega^2)}{2\delta\omega}$$

$$\Rightarrow \underline{\varphi} = \mathrm{arcctg}\left(-\left(\frac{\omega_0^2-\omega^2}{2\delta\omega}\right)\right)$$

الحل هو

$$x(t) = ce^{-\delta t}\cos(\omega_1 t+\varphi) +$$

$$\underline{\underline{A}}\cos(\omega t+\underline{\underline{\varphi}})$$

التمرين ② :

الطاقة الكامنة

$$E_p = \frac{1}{2}k(\Delta L+L\theta)^2 + m_2 gL\frac{\theta^2}{2} - m_1 gL\theta^2 - \underbrace{2mgL\theta}_{\Downarrow}$$

$$mgL\varphi \quad \text{حيث أن}$$

$$2\theta=\varphi \quad \Leftarrow \quad \cancel{2}L\theta = \cancel{L}\varphi$$

من شرط التوازن :

$$\Delta L = 0$$

الطاقة الحركية :

$$E_c = \frac{1}{2}\underbrace{\left[\frac{2mL^2}{2}\right]}_{\text{كرة}}\dot\varphi^2 + \frac{1}{2}\underbrace{mL^2\dot\varphi^2}_{\text{كتلة } m} + \frac{1}{2}\underbrace{[m4L^2]\dot\theta^2}_{\text{كتلة } m \text{ موزعة}} +$$

$$\frac{1}{2}\underbrace{[mL^2]}_{\text{كتلة } m \text{ نقطية}}\dot\theta^2$$

$$\varphi=2\theta \Rightarrow \dot\varphi = 2\dot\theta \Rightarrow$$

$$E_c = \frac{1}{2}[mL^2]4\dot\theta^2 + \frac{1}{2}mL^2 4\dot\theta^2 + \frac{1}{2}[4mL^2]\dot\theta^2 + \frac{1}{2}[mL^2]\dot\theta^2$$

$$= 18mL^2\dot\theta^2$$

$$D = +\frac{1}{2}\alpha L^2 \dot{\theta}^2$$

المعادلة التفاضلية للحركة :

$$\underbrace{\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{\theta}}\right)}_{I} + \underbrace{\frac{\partial E_p}{\partial \theta}}_{II} = \underbrace{-\frac{\partial D}{\partial \dot{\theta}}}_{III} + \underbrace{\frac{\partial \vec{r}}{d\theta}}_{V} \vec{F} \Rightarrow (*)$$

$$I \Leftrightarrow 36 m L^2 \ddot{\theta} \; ; \quad II \Leftrightarrow 2L(8KL - 4mg)\theta$$

$$2L(8KL - 4mg)\theta$$

$$\frac{\partial D}{\partial \dot{\theta}} = \alpha L^2 \dot{\theta} \, , \quad \frac{\partial \vec{r}}{d\theta} = ? \quad r = 2L\varphi \Rightarrow r = 2L(2\theta) = 4L\theta$$

$$\frac{\partial \vec{r}}{\partial \theta} = 4L. \qquad \vec{F} = F_0 \cos\omega t$$

$$(*) \Leftrightarrow 36 m L^2 \ddot{\theta} + 2L(8KL - 4mg)\theta + \alpha L^2 \dot{\theta} = 4L F_0 \cos\omega t$$

$$\ddot{\theta} + \frac{\alpha}{9m}\dot{\theta} + \frac{(4KL - 2mg)}{9mL}\theta = \frac{F_0}{9mL}\cos\omega t.$$