السلسلة رقم 02

- التمرين الأول:

- الطاقة الحركية:

$T=\frac{1}{2}m\dot{x}^2$; $U=\frac{1}{2}kx^2$. $D=\frac{1}{2}\alpha\dot{x}^2$

- المعادلات التفاضلية للحركة.

* طريقة لاغرانج.

$$L=T-U \Leftrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=-\frac{\partial D}{\partial \dot{x}}$$

$$\Rightarrow m\ddot{x}+kx+\alpha\dot{x}=0 \Leftrightarrow \ddot{x}+\frac{\alpha}{m}\dot{x}+\frac{k}{m}x=0 \Rightarrow 2\delta=\frac{\alpha}{m};$$

$\omega_0=\sqrt{k/m}$;

* الشرط الذي يجب توفره في لو حتى تكون الحركة اهتزازية

$\Leftrightarrow \Delta=\sqrt{\delta^2-\omega_0^2}<0$

$$\frac{\alpha}{2m}<\sqrt{\frac{k}{m}} \Leftrightarrow \delta<\omega_0 \qquad \Delta=\delta^2-\omega_0^2<0 \quad \alpha<2\sqrt{km}.$$

* حلول المعادلة في هذا الحالة هي

$$x(t)=X_0e^{-\delta t}\cos(\omega t+\varphi)$$

$$x(t)=e^{-\delta t}\left(D_1e^{+i\sqrt{-\delta^2+\omega_0^2}}+D_2e^{-i\sqrt{-\delta^2+\omega_0^2}}\right)$$

التمرين الثاني:

1) المجموع الجبري لفرق الكمون يساوي الصفر

$$V_L(t)+V_R(t)+V_C(t)=0 \Leftrightarrow L\frac{di(t)}{dt}~~+R\dot{i}+\frac{1}{C}\int i(t)~~$$

$$L\frac{di(t)}{dt}+Ri(t)+\frac{1}{C}\int i(t)=0 \Leftrightarrow L\ddot{q}+R\dot{q}+\frac{1}{C}q=0$$

$$\Leftrightarrow \ddot{q}+\frac{R}{L}\dot{q}+\frac{1}{LC}q=0 \Leftrightarrow 2\delta=\frac{R}{L} \Rightarrow \delta=\frac{R}{2L};\ \sqrt{\frac{1}{LC}}=\omega_0$$

2) من اجل ان يهتز النظام يجب ان

$$R<2\sqrt{\frac{L}{C}} \Leftrightarrow \delta<\omega_0 \qquad \Leftrightarrow \delta^2-\omega_0^2<0 \qquad . R<2\sqrt{\frac{L}{C}}$$

$R_c=?$ المقاومة الحرجة $R=2\sqrt{\frac{L}{C}}=20k\Omega$.

3) $R=100k\Omega \Leftrightarrow$ بهذه المقاومة تكون $\delta>\omega_0 \Leftrightarrow$ تخامد ثقيل

والحل هو $q(t)=Ae^{r_1t}+Be^{r_2t}$ حيث $r_{1,2}=-\delta\pm\sqrt{\delta^2-\omega_0^2}$

$q(t) = A e^{-\delta t} \cos(\omega_a t + \varphi)$. تخامد خفيف $\Leftarrow \delta < \omega_0 \Leftarrow R = 500k\Omega$

حيث $\omega_a = \sqrt{\omega_0^2 - \delta^2}$.

- التمرين الثالث:

1- الطاقة الكامنة: $U = U_r + U_{M_1} + U_{M_2}(F)$

$$U_r = \frac{1}{2} k (\Delta\ell - a\theta)^2, \quad U_{M_2} = -M_2 g \left(\frac{L_2}{2} - \frac{L_2}{2}\cos\theta\right)$$

$$U_{M_1} = M_1 g \frac{L_1}{2} (1 - \cos\theta) \Leftarrow$$

$$U = \frac{1}{2} k (\Delta\ell - a\theta)^2 - M_2 g \frac{L_2}{4}\theta^2 + M_1 g \frac{L_1}{4}\theta^2$$

$\Delta\ell$ عند التوازن نطبق شرط التوازن $\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0 \Leftarrow$

$$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = -ka(\Delta\ell - \cancel{a\theta}) - \cancel{M_2 g \frac{L_2}{2}\theta} + \cancel{M_1 g \frac{L_1}{2}\theta} = 0$$

$$-ka\Delta\ell = 0 \Leftarrow \Delta\ell = 0$$

2- الطاقة الحركية للجملة:

$$T = T_p + T_{M_1} + T_{M_2} = \frac{1}{2} I_{/0}\dot\theta^2 + \frac{1}{2} I_{M_1/0}\dot\theta^2 + \frac{1}{2} I_{M_2/0}\dot\theta^2 \Leftarrow$$

$$\frac{1}{2}\left[\frac{MR^2}{2}\right]\dot\theta^2 + \frac{1}{2}\left[\frac{M_1L_1^2}{3}\right]\dot\theta^2 + \frac{1}{2}\left[\frac{M_2L_2^2}{3}\right]\dot\theta^2$$

$$= \frac{1}{2}\left[\frac{MR^2}{2} + \frac{M_1L_1^2}{3} + \frac{M_2L_2^2}{3}\right]\dot\theta^2$$

- معادلة الحركة: لاغرانج -

$$\mathcal{L} = T - U$$

$$L = \frac{1}{2}\left[\frac{MR^2}{2} + \frac{M_1L_1^2}{3} + \frac{M_2L_2^2}{3}\right]\dot\theta^2 - \frac{1}{2} a\theta^2 - M_2 g \frac{L_2}{4}\theta^2 + M_1 g \frac{L_1}{4}\theta^2$$

$$D = \frac{1}{2}\alpha \dot x^2 = \frac{1}{2}\alpha R^2 \dot\theta^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = -\frac{\partial D}{\partial \dot\theta} \Leftarrow \left[\frac{MR^2}{2} + \frac{M_1L_1^2}{3} + \frac{M_2L_2^2}{3}\right]\ddot\theta + \left[Ka^2 + M_2 g\frac{L_2}{2} - M_1 g \frac{L_1}{2}\right]\theta = \alpha R^2 \dot\theta$$

$$\ddot{\theta} + \frac{\left[Ka^2 + M_2 g\frac{L_2}{2} - M_1 g\frac{L_1}{2}\right]}{\left[\frac{MR^2}{2} + \frac{M_1L_1^2}{3} + \frac{M_2L_2^2}{3}\right]}\theta + \frac{\alpha R^2}{\left[\frac{MR^2}{2} + \frac{M_1L_1^2}{3} + \frac{M_2L_2^2}{3}\right]}\dot{\theta} = 0$$

3) متى تكون الحركة اهتزازية : $\Leftrightarrow \delta < \omega_0$

$$2\delta = \frac{\alpha R^2}{\left[\frac{MR^2}{2} + \frac{M_1L_1^2}{3} + \frac{M_2L_2^2}{3}\right]} \; ; \; \delta^2 < \omega_0^2 \Leftrightarrow$$

$$\frac{\alpha^2 R^4}{B^2} = \frac{\left[Ka^2 + M_2 g\frac{L_2}{2} - M_1 g\frac{L_1}{2}\right]}{B}$$

$$\boxed{K = \frac{\alpha^2 R^4}{4\left[\frac{MR^2}{2} + \frac{M_1L_1^2}{3} + \frac{M_2L_2^2}{3}\right]} - M_2 g\frac{L_2}{2} + M_1 g\frac{L_1}{2}}$$

4) = $\theta(t) = ?$

$$\theta(t) = \theta_0 e^{-\delta t}\cos(\omega_a t + \varphi) \Rightarrow \omega_a = \sqrt{+\omega_0^2 - \delta^2}$$

التمرين 4 :

$$T = \frac{1}{2} m 4a^2\dot{\theta}^2 + \frac{1}{2} M 4a^2\dot{\theta}^2 \Rightarrow$$

$$T = \frac{1}{2}\,2(m+M)a^2\dot{\theta}^2$$

$$U = U_m + U_M + U_r = -mg2a\left(\frac{\theta^2}{2}\right) + Mg\frac{2a\theta^2}{2}$$

$$= -mga\theta^2 + Mga\theta^2 = [Mga - mga]\theta^2$$

$$U_r = \frac{1}{2}Ka^2\theta^2 \Rightarrow U = \left[(Mga - mga) + \frac{1}{2}Ka^2\right]\theta^2$$

$$D = \frac{1}{2}\alpha \dot{y}^2 \Rightarrow \frac{1}{2}\alpha a^2\dot{\theta}^2$$

$$\omega_0 = \sqrt{\frac{2(M-m)g + ka}{4(m+M)a}}$$

$$\omega_a = \sqrt{\frac{2(M-m)g + ka}{4(m+M)a} - \frac{q^2 a^4}{32(m+M)a^2}} \; ?$$