Mathematical English

Arithmetic

Integers

0	zero	10	ten	20	twenty
1	one	11	eleven	30	thirty
2	two	12	twelve	40	forty
3	three	13	thirteen	50	fifty
4	four	14	fourteen	60	sixty
5	five	15	fifteen	70	seventy
6	six	16	sixteen	80	eighty
7	seven	17	seventeen	90	ninety
8	eight	18	eighteen	100	one hundred
9	nine	19	nineteen	1000	one thousand

-245	minus two hundred and forty-five
22731	twenty-two thousand seven hundred and thirty-one
1000000	one million
56000000	fifty-six million
1000000000	one billion [US usage, now universal]
7000000000	seven billion [US usage, now universal]
1000000000000	one trillion [US usage, now universal]
3000000000000	three trillion [US usage, now universal]

Fractions [= Rational Numbers]

$\frac{1}{2}$	one half	$\frac{3}{8}$	three eighths
$\frac{1}{3}$	one third	$\frac{26}{9}$	twenty-six ninths
$\frac{1}{4}$	one quarter [= one fourth]	$-\frac{5}{34}$	minus five thirty-fourths
$\frac{1}{5}$	one fifth	$2\frac{3}{7}$	two and three sevenths
$-\frac{1}{17}$	minus one seventeenth		

Real Numbers

-0.067	minus nought point zero six seven
81.59	eighty-one point five nine
$-2.3 \cdot 10^6$	minus two point three times ten to the six
$[= -2300000$	minus two million three hundred thousand]
$4 \cdot 10^{-3}$	four times ten to the minus three
$[= 0.004 = 4\ /1000$	four thousandths]
$\pi\,[= 3.14159\ldots]$	pi [pronounced as 'pie']
$e\,[= 2.71828\ldots]$	e [base of the natural logarithm]

Complex Numbers

i	i
$3 + 4i$	three plus four i
$1 - 2i$	one minus two i
$\overline{1-2i} = 1 + 2i$	the complex conjugate of one minus two i equals one plus two i

The real part and the imaginary part of $3 + 4i$ are equal, respectively, to 3 and 4.

Basic arithmetic operations

Addition :	$3 + 5 = 8$	three plus five equals [= is equal to] eight
Subtraction :	$3 - 5 = -2$	three minus five equals [= ...] minus two
Multiplication :	$3 \cdot 5 = 15$	three times five equals [= ...] fifteen
Division :	$3/5 = 0.6$	three divided by five equals [= ...] zero point six

$(2-3) \cdot 6 + 1 = -5$	two minus three in brackets times six plus one equals minus five
$\dfrac{1-3}{2+4} = -1/3$	one minus three over two plus four equals minus one third
$4![= 1 \cdot 2 \cdot 3 \cdot 4]$	four factorial

Exponentiation, Roots

5^2	$[= 5 \cdot 5 = 25]$	five squared
5^3	$[= 5 \cdot 5 \cdot 5 = 125]$	five cubed
5^4	$[= 5 \cdot 5 \cdot 5 \cdot 5 = 625]$	five to the (power of) four
5^{-1}	$[= 1/5 = 0.2]$	five to the minus one
5^{-2}	$[= 1/5^2 = 0.04]$	five to the minus two
$\sqrt{3}$	$[= 1.73205\ldots]$	the square root of three
$\sqrt[3]{64}$	$[= 4]$	the cube root of sixty four
$\sqrt[5]{32}$	$[= 2]$	the fifth root of thirty two

In the complex domain the notation $\sqrt[n]{a}$ is ambiguous, since any non-zero complex number has n different n -th roots. For example, $\sqrt[4]{-4}$ has four possible values : $\pm 1 \pm i$ (with all possible combinations of signs).

$(1+2)^{2+2}$	one plus two, all to the power of two plus two
$e^{\pi i} = -1$	e to the (power of) pi i equals minus one

Divisibility

The multiples of a positive integer a are the numbers $a, 2a, 3a, 4a, \ldots$ If b is a multiple of a, we also say that a divides b, or that a is a divisor of b (notation : $a \mid b$). This is equivalent to $\frac{b}{a}$ being an integer.

Division with remainder

If a, b are arbitrary positive integers, we can divide b by a, in general, only with a remainder. For example, 7 lies between the following two consecutive multiples of 3 :

$$2 \cdot 3 = 6 < 7 < 3 \cdot 3 = 9, \quad 7 = 2 \cdot 3 + 1 \quad \left(\Longleftrightarrow \frac{7}{3} = 2 + \frac{1}{3}\right)$$

In general, if qa is the largest multiple of a which is less than or equal to b, then

$$b = qa + r, \quad r = 0, 1, \ldots, a-1$$

The integer q (resp., r) is the quotient (resp., the remainder) of the division of b by a.

Euclid's algorithm

This algorithm computes the greatest common divisor (notation : $(a, b) = \gcd(a, b)$) of two positive integers a, b.

It proceeds by replacing the pair a, b (say, with $a \leq b$) by r, a, where r is the remainder of the division of b by a. This procedure, which preserves the gcd, is repeated until we arrive at $r = 0$.
Example. Compute $\gcd(12, 44)$.

$$\begin{aligned} 44 &= 3 \cdot 12 + 8 \\ 12 &= 1 \cdot 8 + 4 \quad \gcd(12,44) = \gcd(8,12) = \gcd(4,8) = \gcd(0,4) = 4. \\ 8 &= 2 \cdot 4 + 0 \end{aligned}$$

This calculation allows us to write the fraction $\frac{44}{12}$ in its lowest terms, and also as a continued fraction :

$$\frac{44}{12} = \frac{44/4}{12/4} = \frac{11}{3} = 3 + \frac{1}{1 + \frac{1}{2}}$$

If $\gcd(a, b) = 1$, we say that a and b are relatively prime.

add	additionner
algorithm	algorithme
Euclid's algorithm	algorithme de division euclidienne
bracket	parenthèse
left bracket	parenthèse à gauche
right bracket	parenthèse à droite
curly bracket	accolade
denominator	denominateur
difference	différence
divide	diviser
divisibility	divisibilité
divisor	diviseur
exponent	exposant
factorial	factoriel
fraction	fraction
continued fraction	fraction continue
gcd [= **greatest common divisor]**	pgcd [= plus grand commun diviseur]
lcm [= **least common multiple]**	ppcm [= plus petit commun multiple]
infinity	l'infini
iterate	itérer
iteration	itération
multiple	multiple
multiply	multiplier
number	nombre
even number	nombre pair
odd number	nombre impair
numerator	numerateur
pair	couple
pairwise	deux à deux
power	puissance
product	produit
quotient	quotient
ratio	rapport ; raison
rational	rationnel(le)
irrational	irrationnel(le)

relatively prime	premiers entre eux
remainder	reste
root	racine
sum	somme
subtract	soustraire

Algebra

Algebraic Expressions

$A = a^2$	capital a equals small a squared
$a = x + y$	a equals x plus y
$b = x - y$	b equals x minus y
$c = x \cdot y \cdot z$	c equals x times y times z
$c = xyz$	c equals x y z
$(x + y)z + xy$	x plus y in brackets times z plus x y
$x^2 + y^3 + z^5$	x squared plus y cubed plus z to the (power of) five
$x^n + y^n = z^n$	x to the n plus y to the n equals z to the n
$(x - y)^{3m}$	x minus y in brackets to the (power of) three m
	x minus y, all to the (power of) three m
$2^x 3^y$	two to the x times three to the y
$ax^2 + bx + c$	a x squared plus b x plus c
$\sqrt{x} + \sqrt[3]{y}$	the square root of x plus the cube root of y
$\sqrt[n]{x + y}$	the n-th root of x plus y
$\frac{a + b}{c - d}$	a plus b over c minus d
$\binom{n}{m}$	(the binomial coefficient) n over m

Indices

x_0	x zero ; x nought
$x_1 + y_i$	x one plus y i
R_{ij}	(capital) R (subscript) i j ; (capital) R lower i j
M_{ij}^k	(capital) M upper k lower i j ;
	(capital) M superscript k subscript i j
$\sum_{i=0}^n a_i x^i$	sum of a i x to the i for i from nought [= zero] to n ;
	sum over i (ranging) from zero to n of a i (times) x to the i
$\prod_{m=1}^\infty b_m$	product of b m for m from one to infinity ;
	product over m (ranging) from one to infinity of b m
$\sum_{j=1}^n a_{ij} b_{jk}$	sum of a i j times b j k for j from one to n ;
	sum over j (ranging) from one to n of a i j times b j k
$\sum_{i=0}^n \begin{pmatrix} n \\ i \end{pmatrix} x^i y^{n-i}$	sum of n over i x to the i y to the n minus i for i
	from nought [= zero] to n

Matrices

column	colonne
column vector	vecteur colonne
determinant	déterminant
index (pl. indices)	indice
matrix	matrice
matrix entry (pl. entries)	coefficient d'une matrice
$m \times n$ **matrix [m by n matrix]**	matrice à m lignes et n colonnes
multi-index	multiindice
row	ligne
row vector	vecteur ligne
square	carré
square matrix	matrice carrée

Inequalities

$x > y$	x is greater than y
$x \geq y$	x is greater (than) or equal to y
$x < y$	x is smaller than y
$x \leq y$	x is smaller (than) or equal to y

$x > 0$	x is positive
$x \geq 0$	x is positive or zero ; x is non-negative
$x < 0$	x is negative
$x \leq 0$	x is negative or zero

The French terminology is different !

$x > y$	x est strictement plus grand que y
$x \geq y$	x est supérieur ou égal à y
$x < y$	x est strictement plus petit que y
$x \leq y$	x est inférieur ou égal à y
$x > 0$	x est strictement positif
$x \geq 0$	x est positif ou nul
$x < 0$	x est strictement négatif
$x \leq 0$	x est négatif ou nul

Polynomial equations

A polynomial equation of degree $n \geq 1$ with complex coefficients

$$f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_n = 0 \quad (a_0 \neq 0)$$

has n complex solutions (= roots), provided that they are counted with multiplicities. For example, a quadratic equation

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

can be solved by completing the square, i.e., by rewriting the L.H.S. as $a(x+$ constant $)^2+$ another constant. This leads to an equivalent equation

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

whose solutions are

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

where $\Delta = b^2 - 4ac \left(= a^2 \left(x_1 - x_2\right)^2\right)$ is the discriminant of the original equation. More precisely,

$$ax^2 + bx + c = a\left(x - x_1\right)\left(x - x_2\right)$$

If all coefficients a, b, c are real, then the sign of Δ plays a crucial rôle :
if $\Delta = 0$, then $x_1 = x_2 (= -b/2a)$ is a double root ;
if $\Delta > 0$, then $x_1 \neq x_2$ are both real ;

if $\Delta < 0$, then $x_1 = \overline{x_2}$ are complex conjugates of each other (and non-real).

coefficient	coefficient
degree	degré
discriminant	discriminant
equation	équation
L.H.S. [= left hand side]	terme de gauche
R.H.S. [= right hand side]	terme de droite
polynomial adj.	polynomial(e)
polynomial n.	polynôme
provided that	à condition que
root	racine
simple root	racine simple
double root	racine double
triple root	racine triple
multiple root	racine multiple
root of multiplicity m	racine de multiplicité m
solution	solution
solve	résoudre

Congruences

Two integers a, b are congruent modulo a positive integer m if they have the same remainder when divided by m (equivalently, if their difference $a - b$ is a multiple of m).

$a \equiv b (\text{mod} m)$ a is congruent to b modulo m

$a \equiv b(m)$

Some people use the following, slightly horrible, notation : $a = b[m]$.

Fermat's Little Theorem. If p is a prime number and a is an integer, then $a^p \equiv a(\text{mod} p)$. In other words, $a^p - a$ is always divisible by p.

Chinese Remainder Theorem. If $m_1, \ldots, m_k$ are pairwise relatively prime integers, then the system of congruences

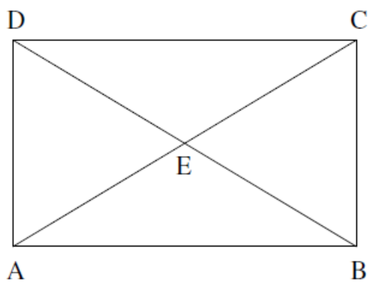
$$x \equiv a_1 \,(\text{mod} m_1) \quad \cdots \quad x \equiv a_k \,(\text{mod} m_k)$$

has a unique solution modulo $m_1 \cdots m_k$, for any integers $a_1, \ldots, a_k$.

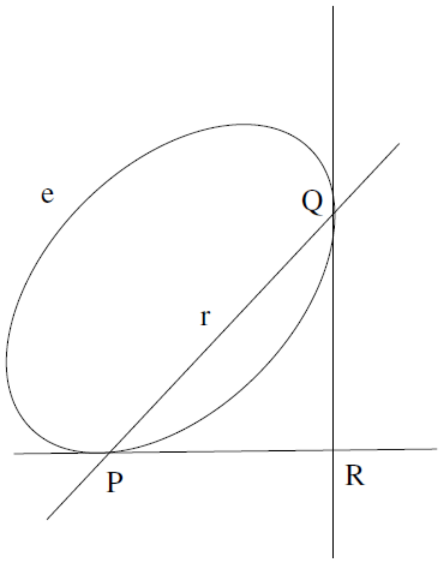
The definite article (and its absence)

measure theory	théorie de la mesure
number theory	théorie des nombres
Chapter one	le chapitre un
Equation (7)	l'équation (7)
Harnack's inequality	l'inégalité de Harnack
the Harnack inequality	
the Riemann hypothesis	l'hypothèse de Riemann
the Poincare conjecture	la conjecture de Poincaré
Minkowski's theorem	le théorème de Minkowski
the Minkowski theorem	
the Dirac delta function	la fonction delta de Dirac
Dirac's delta function	
the delta function	la fonction delta

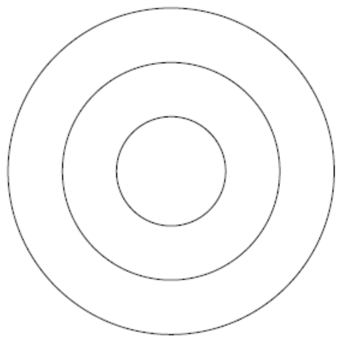
Geometry



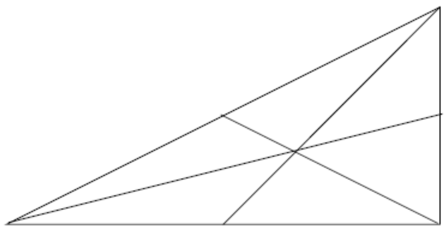
Let E be the intersection of the diagonals of the rectangle $ABCD$. The lines (AB) and (CD) are parallel to each other (and similarly for (BC) and (DA)). We can see on this picture several acute angles : $\angle EAD, \angle EAB, \angle EBA, \angle AED, \angle BEC \ldots$; right angles : $\angle ABC$, $\angle CDA, \angle DAB$ and obtuse angles : $\angle AEB, \angle CED$



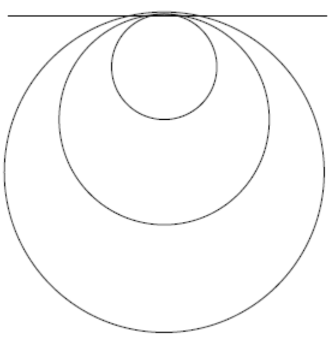
Let P and Q be two points lying on an ellipse e. Denote by R the intersection point of the respective tangent lines to e at P and Q. The line r passing through P and Q is called the polar of the point R w.r.t. the ellipse e.



Here we see three concentric circles with respective radii equal to 1, 2 and 3.



If we draw a line through each vertex of a given triangle and the midpoint of the opposite side, we obtain three lines which intersect at the barycentre (= the centre of gravity) of the triangle.



Above, three circles have a common tangent at their (unique) intersection point.

Euler's Formula

Let P be a convex polyhedron. Euler's formula asserts that

$$V - E + F = 2$$

$$V = \quad \text{the number of vertices of } P$$

$$E = \quad \text{the number of edges of } P$$

$$F = \quad thenumberoffacesofP$$

angle	angle
acute angle	angle aigu
obtuse angle	angle obtus
right angle	angle droit
area	aire
axis (pl. axes)	axe
coordinate axis	axe de coordonnées
horizontal axis	axe horisontal
vertical axis	axe vertical
centre [US : center]	centre
circle	cercle
colinear (points)	(points) alignés
conic (section)	(section) conique
cone	cône
convex	convexe
cube	cube
curve	courbe
dimension	dimension
distance	distance
dodecahedron	dodecaèdre
edge	arête
ellipse	ellipse
ellipsoid	ellipsoïde
face	face
hexagon	hexagone
hyperbola	hyperbole
hyperboloid	hyperboloïde
one-sheet (two-sheet) hyperboloid	hyperbole

icosahedron	icosaèdre
intersect	intersecter
intersection	intersection
lattice	réseau
lettuce	laitue
length	longeur
line	droite
midpoint of	milieu de
octahedron	octaèdre
orthogonal ; perpendicular	orthogonal(e) ; perpendiculaire
parabola	parabole
parallel	parallèl(e)
parallelogram	parallélogramme
pass through	passer par
pentagon	pentagone
plane	plan
point	point
(regular) polygon	polygone (régulier)
(regular) polyhedron (pl. polyhedra)	polyèdre (régulier)
projection	projection
central projection projection	conique ; projection centrale
orthogonal projection	projection orthogonale
parallel projection	projection parallèle
quadrilateral	quadrilatère
radius (pl. radii)	rayon
rectangle	rectangle
rectangular	rectangulaire
rotation	rotation
side	côté
slope	pente
sphere	sphère
square	carré
square lattice	réseau carré
surface	surface
tangent to	tangent(e) à
tangent line	droite tangente

tangent hyper(plane)	(hyper)plan tangent
tetrahedron	tetraèdre
triangle	triangle
equilateral triangle	triangle équilatéral
isosceles triangle	triangle isocèle
right-angled triangle	triangle rectangle
vertex	sommet

Linear Algebra

basis (pl. bases)	base
change of basis	changement de base
bilinear form	forme bilinéaire
coordinate	coordonnée
(non-)degenerate	(non) dégénéré(e)
dimension	dimension
codimension	codimension
finite dimension	dimension finie
infinite dimension	dimension infinie
dual space	espace dual
eigenvalue	valeur propre
eigenvector	vecteur propre
(hyper)plane	(hyper)plan
image	image
isometry	isométrie
kernel	noyau
linear	linéaire
linear form	forme linéaire
linear map	application linéaire
linearly dependent	liés ; linéairement dépendants
linearly independent	libres ; linéairement indépendants
multi-linear form	forme multilinéaire
origin	origine
orthogonal ; perpendicular	orthogonal(e) ; perpendiculaire
orthogonal complement	supplémentaire orthogonal
orthogonal matrix	matrice orthogonale

(orthogonal) projection	projection (orthogonale)
quadratic form	forme quadratique
reflection	réflexion
represent	représenter
rotation	rotation
scalar	scalaire
scalar product	produit scalaire
subspace	sous-espace
(direct) sum	somme (directe)
skew-symmetric	anti-symétrique
symmetric	symétrique
trilinear form	forme trilinéaire
vector	vecteur
vector space	espace vectoriel
vector subspace	sous-espace vectoriel
ctor space of dimension n	ve espace vectoriel de dimension n

Mathematical arguments

Set theory

$x \in A$	x is an element of A; x lies in A ;
	x belongs to A ; x is in A
$x \notin A$	x is not an element of A; x does not lie in A ;
	x does not belong to A; x is not in A
$x, y \in A$	(both) x and y are elements of A; . . . lie in A ;
	. . . belong to A ; ... are in A
$x, y \notin A$	(neither) x nor y is an element of A; . . . lies in A ;
	. . . belongs to A ; ...is in A
$\emptyset$	the empty set (= set with no elements)
$A = \emptyset$	A is an empty set
$A \neq \emptyset$	A is non-empty
$A \cup B$	the union of (the sets) A and B ; A union B
$A \cap B$	the intersection of (the sets) A and B ; A intersection B
$A \times B$	the product of (the sets) A and B ; A times B
$A \cap B = \emptyset$	A is disjoint from B ; the intersection of A and B is empty
$\{x \mid \ldots\}$	the set of all x such that ...
C	the set of all complex numbers
Q	the set of all rational numbers
R	the set of all real numbers

$A \cup B$ contains those elements that belong to A or to B (or to both).

$A \cap B$ contains those elements that belong to both A and B.

$A \times B$ contains the ordered pairs (a, b), where $a($ resp. $, b)$ belongs to A (resp., to B)

$A^n = \underbrace{A \times \cdots \times A}_{n \text{ times}}$ contains all ordered n -tuples of elements of A.

belong to appartenir à

disjoint from disjoint de

element élément

empty vide

non-empty non vide

intersection intersection

inverse l'inverse

the inverse map to f l'application réciproque de f

the inverse of f l'inverse de f

map application

bijective map application bijective

injective map application injective

surjective map application surjective

pair couple

ordered pair couple ordonné

triple triplet

quadruple quadruplet

n-tuple n-uplet

relation relation

equivalence relation relation d'équivalence

set ensemble

finite set ensemble fini

infinite set ensemble infini

union réunion

Logic

$S \vee T$	S or T
$S \wedge T$	S and T
$S \Longrightarrow T$	S implies T ; if S then T
$S \Longleftrightarrow T$	S is equivalent to T ; S iff T
$\neg S$	not S
$\forall x \in A \ldots$	for each [= for every] x in A ...
$\exists x \in A \ldots$	there exists [= there is] an x in A (such that) ...
$\exists! x \in A \ldots$	there exists [= there is] a unique x in A (such that) ...
$\nexists x \in A \ldots$	there is no x in A (such that)...

$x > 0 \wedge y > 0 \Longrightarrow x + y > 0$ if both x and y are positive, so is $x + y$

$\nexists x \in \mathrm{Q} \quad x^2 = 2$ no rational number has a square equal to two

$\forall x \in \mathbf{R} \exists y \in \mathbf{Q} \quad |x - y| < 2/3$ for every real number x there exists a rational number y such that the absolute value of x minus y is smaller than two thirds

Exercise. Read out the following statements.

$$x \in A \cap B \Longleftrightarrow (x \in A \wedge x \in B), \quad x \in A \cup B \Longleftrightarrow (x \in A \vee x \in B),$$
$$\forall x \in \mathbf{R} \quad x^2 \geq 0, \quad \neg \exists x \in \mathbf{R} \quad x^2 < 0, \quad \forall y \in \mathbf{C} \exists z \in \mathbf{C} \quad y = z^2$$

Basic arguments

It follows from . . . that . . .

We deduce from . . . that . . .

Conversely, ... implies that ...

Equality (1) holds, by Proposition 2.

By definition, ...

The following statements are equivalent.

Thanks to . . . , the properties . . . and . . . of . . . are equivalent to each other.

. . . has the following properties.

Theorem 1 holds unconditionally.

This result is conditional on Axiom A.

. . . is an immediate consequence of Theorem 3.

Note that . . . is well-defined, since . . .

As . . . satisfies . . . , formula (1) can be simplified as follows.

We conclude (the argument) by combining inequalities (2) and (3).

(Let us) denote by X the set of all . . .

Let X be the set of all . . .

Recall that . . . , by assumption.

It is enough to show that . . .

We are reduced to proving that . . .

The main idea is as follows.

We argue by contradiction. Assume that . . . exists.

The formal argument proceeds in several steps.

Consider first the special case when . . .

The assumptions . . . and . . . are independent (of each other), since . . .

. . . , which proves the required claim.

We use induction on n to show that . . .

On the other hand, . . .

. . . , which means that . . .

In other words, . . .

argument argument

assume supposer

assumption hypothèse

axiom axiome

case cas

special case cas particulier

claim v. affirmer

(the following) claim l'affirmation suivante ; l'assertion suivante

concept notion

conclude conclure

conclusion conclusion

condition condition

a necessary and sufficient condition une condition necessaire et suffisante

conjecture conjecture

consequence conséquence

consider considérer

contradict contredire

contradiction contradiction

conversely réciproquement

corollary corollaire

deduce déduire

define définir

well-defined bien défini(e)

definition définition

equivalent équivalent(e)

establish établir

example exemple

exercise exercice

explain expliquer

explanation explication

false faux, fausse

formal formel

hand main

on one hand d'une part

on the other hand d'autre part

iff [= if and only if] si et seulement si

imply impliquer, entraîner

induction on récurrence sur

lemma lemme

proof preuve ; démonstration

property propriété

satisfy property P satisfaire à la propriété P ; verifier la propriété P

proposition proposition

reasoning raisonnement

reduce to se ramener à

remark remarque(r)

required réquis(e)

result résultat

s.t. = such that

statement énoncé

t.f.a.e. = the following are equivalent

theorem théorème

true vrai

truth vérité

wlog = without loss of generality

word mot

in other words autrement dit

Functions

Formulas/Formulae

$f(x)$	f of x
$g(x, y)$	g of x (comma) y
$h(2x, 3y)$	h of two x (comma) three y
$\sin(x)$	sine x
$\cos(x)$	cosine x
$\tan(x)$	tan x
$\arcsin(x)$	arc sine x
$\arccos(x)$	arc cosine x
$\arctan(x)$	arc tan x
$\sinh(x)$	hyperbolic sine x
$\cosh(x)$	hyperbolic cosine x
$\tanh(x)$	hyperbolic tan x
$\sin(x^2)$	sine of x squared
$\sin(x)^2$	sine squared of x ; sine x, all squared
$\frac{x+1}{\tan(y^4)}$	x plus one, all over over tan of y to the four
$3^{x-\cos(2x)}$	three to the (power of) x minus cosine of two x
$\exp(x^3 + y^3)$	exponential of x cubed plus y cubed

Intervals

(a, b) open interval a b

$[a, b]$ closed interval a b

$(a, b]$ half open interval a b (open on the left, closed on the right)

$[a, b)$ half open interval a b (open on the right, closed on the left)

The French notation is different !

$]a, b[$ intervalle ouvert a b

$[a, b]$ intervalle fermé a b

$]a, b]$ intervalle demi ouvert a b (ouvert à gauche, fermé à droite)

$[a, b[$ intervalle demi ouvert a b (ouvert à droite, ferme à gauche)

Exercise. Which of the two notations do you prefer, and why ?

Derivatives

f' f dash ; f prime ; the first derivative of f

f'' f double dash ; f double prime ; the second derivative of f

$f^{(3)}$ the third derivative of f

$f^{(n)}$ the n-th derivative of f

$\frac{dy}{dx}$ dy by dx ; the derivative of y by x

$\frac{d^2y}{dx^2}$ the second derivative of y by x ; d squared y by d x squared

$\frac{\partial f}{\partial x}$ the partial derivative of f by x (with respect to x) ; partial df by dx

$\frac{\partial^2 f}{\partial x^2}$ the second partial derivative of f by x (with respect to x)

partial d squared f by d x squared

∇f nabla f; the gradient of f

Δf delta$f$

Example. The (total) differential of a function $f(x, y, z)$ in three real variables is equal to

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

The gradient of f is the vector whose components are the partial derivatives of f with respect to the three variables :

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

The Laplace operator Δ acts on f by taking the sum of the second partial derivatives with respect to the three variables :

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

The Jacobian matrix of a pair of functions $g(x, y), h(x, y)$ in two real variables is the 2×2 matrix whose entries are the partial derivatives of g and h, respectively, with respect to the two variables :

$$\begin{pmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} \end{pmatrix}$$

Integrals

$\int f(x)dx$ integral of f of xdx

$\int_a^b t^2 dt$ integral from a to b of t squared dt

$\int\int_S h(x, y)dxdy$ double integral over S of h of xydxdy

Differential equations

An ordinary (resp., a partial) differential equation, abbreviated as ODE (resp., PDE), is an equation involving an unknown function f of one (resp., more than one) variable together with its derivatives (resp., partial derivatives). Its order is the maximal order of derivatives that appear in the equation. The equation is linear if f and its derivatives appear linearly; otherwise it is non-linear.

$f' + xf = 0$	first order linear ODE
$f'' + \sin(f) = 0$	second order non-linear ODE
$(x^2 + y)\frac{\partial f}{\partial x} - (x + y^2)\frac{\partial f}{\partial y} + 1 = 0$	first order linear PDE

The classical linear PDEs arising from physics involve the Laplace operator

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$\Delta f = 0$ the Laplace equation

$\Delta f = \lambda f$ the Helmholtz equation

$\Delta g = \frac{\partial g}{\partial t}$ the heat equation

$\Delta g = \frac{\partial^2 g}{\partial t^2}$ the wave equation

Above, x, y, z are the standard coordinates on a suitable domain U in $\mathbf{R}^3, t$ is the time variable, $f = f(x, y, z)$ and $g = g(x, y, z, t)$. In addition, the function $f($ resp. $, g)$ is required to satisfy suitable boundary conditions (resp., initial conditions) on the boundary of $U($ resp., for $t = 0$).

act v. agir

action action

bound borne

bounded borné(e)

bounded above borné(e) supérieurement

bounded below borné(e) inférieurement

unbounded non borné(e)

comma virgule

concave function fonction concave

condition condition

boundary condition condition au bord

initial condition condition initiale

constant n. constante

constant adj. constant(e)

constant function fonction constant(e)

non-constant adj. non constant(e)

non-constant function fonction non constante

continuous continu(e)

continuous function fonction continue

convex function fonction convexe

decrease n. diminution

decrease v. décroître

decreasing function fonction décroissante

strictly decreasing function fonction strictement décroissante

derivative dérivée

second derivative dérivée seconde

n-th derivative dérivée n-ième

partial derivative dérivée partielle

differential n. différentielle

differential form forme différentielle

differentiable function fonction dérivable

twice differentiable function fonction deux fois dérivable

n-times continuously differentiable function fonction n fois continument dérivable

domain domaine

equation équation

the heat equation l'équation de la chaleur

the wave equation l'équation des ondes

function fonction

graph graphe

increase n. croissance

increase v. croître

increasing function fonction croissante

strictly increasing function fonction strictement croissante

integral intégrale

interval intervalle

closed interval intervalle fermé

open interval intervalle ouvert

half-open interval intervalle demi ouvert

Jacobian matrix matrice jacobienne

Jacobian le jacobien [= le déterminant de la matrice jacobienne]

linear linéaire

non-linear non linéaire

maximum maximum

global maximum maximum global

local maximum maximum local

minimum minimum

global minimum minimum global

local minimum minimum local

monotone function fonction monotone

strictly monotone function fonction strictement monotone

operator opérateur

the Laplace operator opérateur de Laplace

ordinary ordinaire

order ordre

otherwise autrement

partial partiel(le)

PDE [= partial differential equation] EDP

sign signe

value valeur

complex-valued function fonction à valeurs complexes

real-valued function fonction à valeurs réelles

variable variable

complex variable variable complexe

real variable variable réelle

function in three variables fonction en trois variables

with respect to [= w.r.t.] par rapport ‘

This is all Greek to me

Small Greek letters used in mathematics

α	alpha	β	beta	γ	gamma	δ	delta
ϵ, ε	epsilon	ζ	zeta	η	eta	θ, ϑ	theta
ι	iota	κ	kappa	λ	lambda	μ	mu
ν	nu	ξ	xi	o	omicron	π, ϖ	pi
ρ, ϱ	rho	σ	sigma	τ	tau	υ	upsilon
ϕ, φ	phi	χ	chi	ψ	psi	ω	omega

Capital Greek letters used in mathematics

B	Beta	Γ	Gamma	Δ	Delta	Θ	Theta
Λ	Lambda	Ξ	Xi	Π	Pi	Σ	Sigma
Υ	Upsilon	Φ	Phi	Ψ	Psi	Ω	Omega

Sequences, Series

Convergence criteria

By definition, an infinite series of complex numbers $\sum_{n=1}^{\infty} a_n$ converges (to a complex number s) if the sequence of partial sums $s_n = a_1 + \cdots + a_n$ has a finite limit (equal to s) ; otherwise it diverges. The simplest convergence criteria are based on the following two facts.

Fact 1. If $\sum_{n=1}^{\infty} |a_n|$ is convergent, so is $\sum_{n=1}^{\infty} a_n$ (in this case we say that the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent).

Fact 2 . If $0 \le a_n \le b_n$ for all sufficiently large n and if $\sum_{n=1}^{\infty} b_n$ converges, so does $\sum_{n=1}^{\infty} a_n$
Taking $b_n = r^n$ and using the fact that the geometric series $\sum_{n=1}^{\infty} r^n$ of ratio r is convergent iff $|r| < 1$, we deduce from Fact 2 the following statements.

The ratio test (d'Alembert). If there exists $0 < r < 1$ such that, for all sufficiently large n, $|a_{n+1}| \le r\,|a_n|$, then $\sum_{n=1}^{\infty} a_n$ is (absolutely) convergent.

The root test (Cauchy). If there exists $0 < r < 1$ such that, for all sufficiently large n, $\sqrt[n]{|a_n|} \le r$, then $\sum_{n=1}^{\infty} a_n$ is (absolutely) convergent.

What is the sum $1 + 2 + 3 + \cdots$ equal to ?

At first glance, the answer is easy and not particularly interesting : as the partial sums

$$1, \quad 1+2=3, \quad 1+2+3=6, \quad 1+2+3+4=10, \quad \ldots$$

tend towards plus infinity, we have

$$1+2+3+\cdots = +\infty$$

It turns out that something much more interesting is going on behind the scenes. In fact, there are several ways of "regularising" this divergent series and they all lead to the following surprising answer : (the regularised value of) $1+2+3+\cdots = -\frac{1}{12}$ How is this possible ? Let us pretend that the infinite sums

$$a = 1+2+3+4+\cdots$$
$$b = 1-2+3-4+\cdots$$
$$c = 1-1+1-1+\cdots$$

all make sense. What can we say about their values ? Firstly, adding c to itself yields

$$\left.\begin{aligned} c &= 1-1+1-1+\cdots \\ c &= 1-1+1-\cdots \\ c+c &= 1+0+0+0+\cdots = 1 \end{aligned}\right\} \Rightarrow c = \frac{1}{2}$$

Secondly, computing $c^2 = c(1-1+1-1+\cdots) = c-c+c-c+\cdots$ by adding the infinitely many rows in the following table

$$\begin{aligned} c &= 1 \quad -1+1-1+\cdots \\ -c &= \quad\;\; -1+1-1+\cdots \\ c &= \qquad\qquad 1-1+\cdots \\ -c &= \qquad\qquad\quad -1+\cdots \\ &\vdots \qquad\qquad\qquad\quad \ddots \end{aligned}$$

we obtain $b = c^2 = \frac{1}{4}$. Alternatively, adding b to itself gives

$$\left.\begin{aligned} b &= 1-2+3-4+\cdots \\ b &= 1-2+3-\cdots \\ b+b &= 1-1+1-1+\cdots = c \end{aligned}\right\} \Longrightarrow b = \frac{c}{2} = \frac{1}{4}$$

Finally, we can relate a to b, by adding up the following two rows :

$$\left.\begin{aligned} a &= 1+2+3+4+\cdots \\ -4a &= -4 \quad -8-\cdots \end{aligned}\right\} \Rightarrow -3a = b = \frac{1}{4} \Rightarrow a = -\frac{1}{12}$$

Exercise. Using the same method, "compute" the sum

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots$$

$$\lim_{x\to 1} f(x) = 2$$

the limit of f of x as x tends to one is equal to two

approach	approcher
close	proche
arbitrarily close to	arbitrairement proche de
compare	comparer
comparison	comparaison
converge	converger
convergence	convergence
criterion (pl. criteria)	critère
diverge	diverger
divergence	divergence
infinite	infini(e)
infinity	l'infini
minus infinity	moins l'infini
plus infinity	plus l'infini
large	grand
large enough	assez grand
sufficiently large	suffisamment grand
limit	limite
tend to a limit	admettre une limite
tends to $\sqrt{2}$	tends vers $\sqrt{2}$
neighbo(u)rhood	voisinage
sequence	suite
bounded sequence	suite bornée
convergent sequence	suite convergente
divergent sequence	suite divergente
unbounded sequence	suite non bornée
series	série
absolutely convergent series	série absolument convergente
geometric series	série géométrique
sum	somme
partial sum	somme partielle

Prime Numbers

An integer $n > 1$ is a prime (number) if it cannot be written as a product of two integers $a, b > 1$. If, on the contrary, $n = ab$ for integers $a, b > 1$, we say that n is a composite number. The list of primes begins as follows :

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61 \ldots$$

Note the presence of several "twin primes" (pairs of primes of the form $p, p+2$) in this sequence :

$$11, 13 \quad 17, 19 \quad 29, 31 \quad 41, 43 \quad 59, 61$$

Two fundamental properties of primes - with proofs - were already contained in Euclid's Elements : Proposition 1. There are infinitely many primes. Proposition 2. Every integer $n \geq 1$ can be written in a unique way (up to reordering of the factors) as a product of primes.

Recall the proof of Proposition 1 : given any finite set of primes $p_1, \ldots, p_j$, we must show that there is a prime p different from each p_i. Set $M = p_1 \cdots p_j$; the integer $N = M + 1 \geq 2$ is divisible by at least one prime p (namely, the smallest divisor of N greater than 1). If p was equal to p_i for some $i = 1, \ldots, j$, then it would divide both N and $M = p_i\,(M/p_i)$, hence also $N - M = 1$, which is impossible. This contradiction implies that $p \neq p_1, \ldots, p_j$, concluding the proof.
any single prime, since the proof works even for $j = 0$: in this case $N = 2$ (as the empty product M is equal to 1, by definition) and $p = 2$.
It is easy to adapt this proof in order to show that there are infinitely many primes of the form $4n + 3$ (resp., $6n + 5$). It is slightly more difficult, but still elementary, to do the same for the primes of the form $4n + 1$ (resp., $6n + 1$).

Questions About Prime Numbers

Q1. Given a large integer n (say, with several hundred decimal digits), is it possible to decide whether or not n is a prime ?
Yes, there are algorithms for "primality testing" which are reasonably fast both in theory (the Agrawal-Kayal-Saxena test) and in practice (the Miller-Rabin test).
Q2. Is it possible to find concrete large primes ?
Searching for huge prime numbers usually involves numbers of special form, such as the Mersenne numbers $M_n = 2^n - 1$ (if M_n is a prime, n is necessarily also a prime). The point is that there is a simple test (the Lucas-Lehmer criterion) for deciding whether M_n is a prime or not.
In practice, if we wish to generate a prime with several hundred decimal digits, it is computationally feasible to pick a number randomly and then apply a primality testing algorithm to numbers in its vicinity (having first eliminated those which are divisible by small primes).
Q3. Given a large integer n, is it possible to make explicit the factorisation of n into a product of primes ? [For example, $999999 = 3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 37$.]

At present, no (unless n has special form). It is an open question whether a fast "prime factorisation" algorithm exists (such an algorithm is known for a hypothetical quantum computer).

Q4. Are there infinitely many primes of special form ?

According to Dirichlet's theorem on primes in arithmetic progressions, there are infinitely many primes of the form $an + b$, for fixed integers $a, b \geq 1$ without a common factor. It is unknown whether there are infinitely many primes of the form $n^2 + 1$ (or, more generally, of the form $f(n)$, where $f(n)$ is a polynomial of degree $\deg(f) > 1$).

Similarly, it is unknown whether there are infinitely many primes of the form $2^n - 1$ (the Mersenne primes) or $2^n + 1$ (the Fermat primes).

Q5. Is there anything interesting about primes that one can actually prove ?

Green and Tao have recently shown that there are arbitrarily long arithmetic progressions consisting entirely of primes.

digit	chiffre
prime number	nombre premier
twin primes	nombres premiers jumeaux
progression	progression
arithmetic progression	progression arithmétique
geometric progression	progression géométrique